# Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/1.2.1.5-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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# Contents

# Chapter 1

# Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [ 123 ]. This is test number [ 36 ].

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | solved | Failed |
|---|---|---|
| Rubi | % 100. ( 123 ) | % 0. ( 0 ) |
| Mathematica | % 100. ( 123 ) | % 0. ( 0 ) |
| Maple | % 98.37 ( 121 ) | % 1.63 ( 2 ) |
| Maxima | % 54.47 ( 67 ) | % 45.53 ( 56 ) |
| Fricas | % 89.43 ( 110 ) | % 10.57 ( 13 ) |
| Sympy | % 34.96 ( 43 ) | % 65.04 ( 80 ) |
| Giac | % 70.73 ( 87 ) | % 29.27 ( 36 ) |

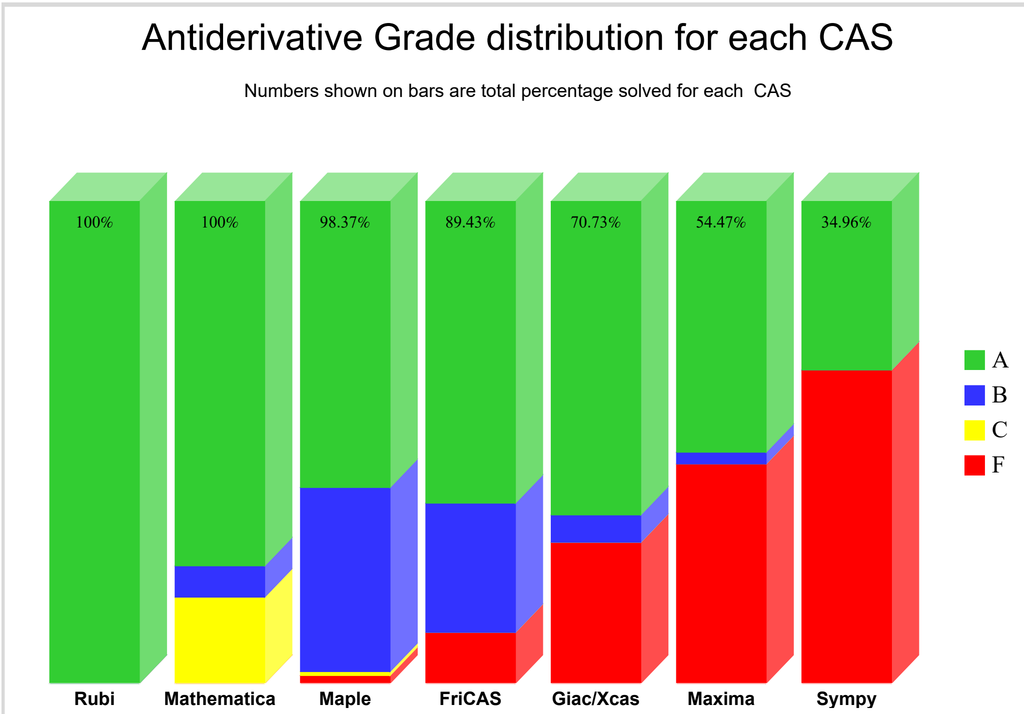
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

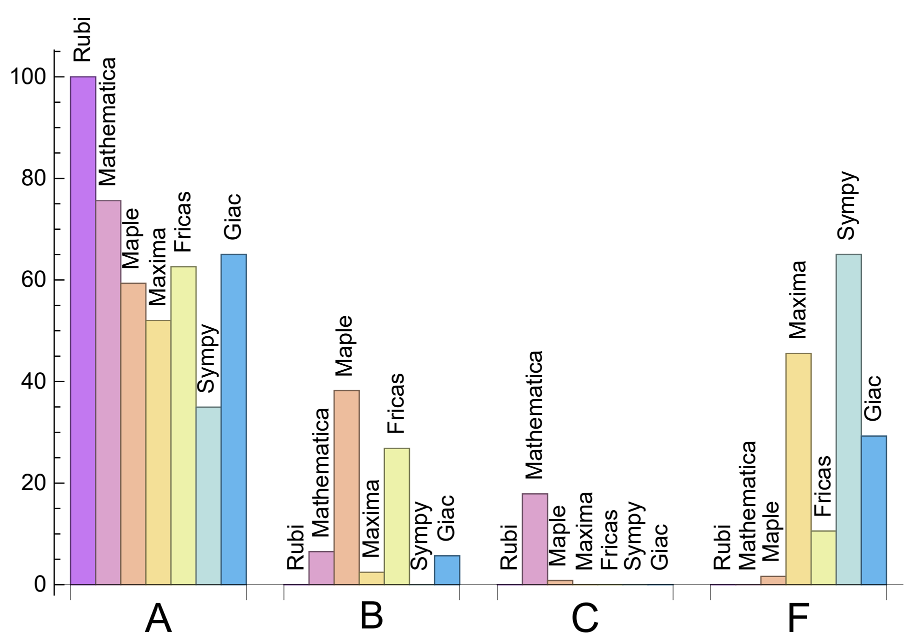
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 100. | 0. | 0. | 0. |
| Mathematica | 75.61 | 6.5 | 17.89 | 0. |
| Maple | 59.35 | 38.21 | 0.81 | 1.63 |
| Maxima | 52.03 | 2.44 | 0. | 45.53 |
| Fricas | 62.6 | 26.83 | 0. | 10.57 |
| Sympy | 34.96 | 0. | 0. | 65.04 |
| Giac | 65.04 | 5.69 | 0. | 29.27 |

The following is a Bar chart illustration of the data in the above table.




The figure below compares the CAS systems for each grade level.




## 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

| System | Mean time (sec) | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|---|
| Rubi | 0.71 | 205.62 | 1. | 128. | 1. |
| Mathematica | 0.94 | 322.93 | 1.34 | 95. | 1. |
| Maple | 0.16 | 6251.53 | 19.31 | 136. | 0.82 |
| Maxima | 1.39 | 121.69 | 1.22 | 97. | 1.12 |
| Fricas | 3.72 | 2411.01 | 11.67 | 395.5 | 4.02 |
| Sympy | 0.22 | 76.74 | 0.99 | 73. | 0.97 |
| Giac | 1.37 | 202.79 | 1.32 | 97. | 1.05 |

## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {6, 106, 107, 108, 113, 122, 123}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

### 1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

### 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

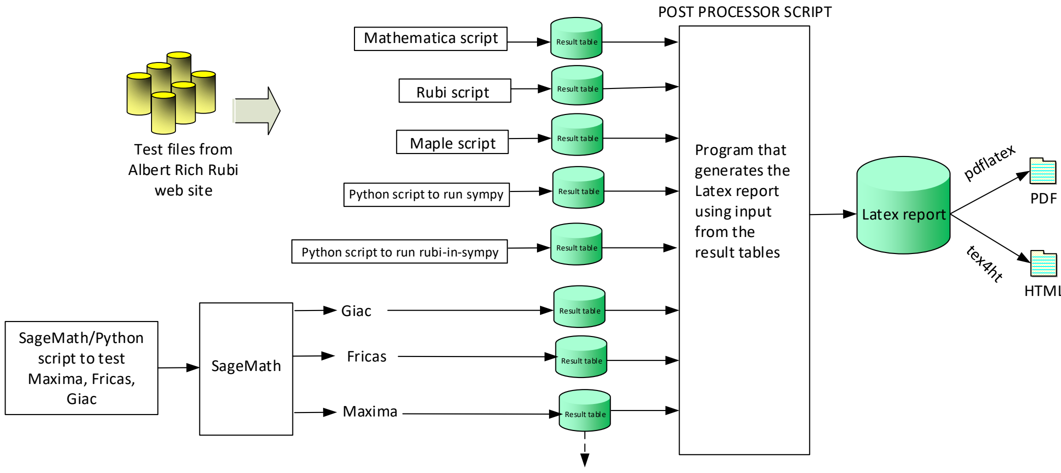
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.




**One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)**

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

**High level overview of the CAS independent integration test build system**

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# Chapter 2

# detailed summary tables of results

## 2.1 List of integrals sorted by grade for each CAS

### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

B grade: { }

C grade: { }

F grade: { }

### 2.1.2 Mathematica

A grade: { 1, 9, 10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123 }

B grade: { 2, 3, 4, 5, 6, 7, 107, 108 }

C grade: { 8, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 121 }

F grade: { }

### 2.1.3 Maple

A grade: { 1, 8, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 96, 111, 120, 122, 123 }

B grade: { 2, 3, 4, 5, 6, 7, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 121 }

C grade: { 11 }

F grade: { 9, 10 }

### 2.1.4 Maxima

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 96

B grade: { 93, 94, 95 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

### 2.1.5 FriCAS

A grade: { 1, 8, 11, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 93, 94, 96, 100, 101, 105, 109, 110, 111, 119 }

B grade: { 2, 3, 4, 5, 7, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 89, 90, 91, 92, 95, 97, 98, 99, 104, 112, 114, 115, 116, 118, 120, 121 }

C grade: { }

F grade: { 6, 9, 10, 13, 102, 103, 106, 107, 108, 113, 117, 122, 123 }

### 2.1.6 Sympy

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123 }

### 2.1.7 Giac

A grade: { 1, 11, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 72, 73, 74, 75, 79, 80, 81, 82, 86, 87, 88, 89, 93, 94, 95, 96, 100, 101, 105, 109, 110, 111, 114, 115, 116, 118, 119 }

B grade: { 3, 8, 12, 14, 104, 120, 121 }

C grade: { }

F grade: { 2, 4, 5, 6, 7, 9, 10, 13, 62, 63, 64, 69, 70, 71, 76, 77, 78, 83, 84, 85, 90, 91, 92, 97, 98, 99, 102, 103, 106, 107, 108, 112, 113, 117, 122, 123 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

| Problem 1 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 102 | 87 | 136 | 0 | 494 | 0 | 113 |
| normalized size | 1 | 1. | 0.85 | 1.33 | 0. | 4.84 | 0. | 1.11 |
| time (sec) | N/A | 0.094 | 0.191 | 0.052 | 0. | 1.157 | 0. | 1.302 |

| Problem 2 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 178 | 491 | 0 | 2226 | 0 | 0 |
| normalized size | 1 | 1. | 2.17 | 5.99 | 0. | 27.15 | 0. | 0. |
| time (sec) | N/A | 0.111 | 0.388 | 0.388 | 0. | 2.14 | 0. | 0. |

| Problem 3 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F(-2) | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 161 | 307 | 0 | 1755 | 0 | 620 |
| normalized size | 1 | 1. | 2.44 | 4.65 | 0. | 26.59 | 0. | 9.39 |
| time (sec) | N/A | 0.079 | 0.226 | 0.327 | 0. | 1.458 | 0. | 14.668 |

| Problem 4 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 129 | 129 | 296 | 829 | 0 | 3297 | 0 | 0 |
| normalized size | 1 | 1. | 2.29 | 6.43 | 0. | 25.56 | 0. | 0. |
| time (sec) | N/A | 0.169 | 0.91 | 0.301 | 0. | 3.384 | 0. | 0. |

| Problem 5 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 224 | 224 | 1748 | 1884 | 0 | 8058 | 0 | 0 |
| normalized size | 1 | 1. | 7.8 | 8.41 | 0. | 35.97 | 0. | 0. |
| time (sec) | N/A | 0.427 | 6.376 | 0.242 | 0. | 19.058 | 0. | 0. |

| Problem 6 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 328 | 328 | 3386 | 3695 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 10.32 | 11.27 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.97 | 6.58 | 0.259 | 0. | 0. | 0. | 0. |

| Problem 7 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 162 | 162 | 490 | 1377 | 0 | 4059 | 0 | 0 |
| normalized size | 1 | 1. | 3.02 | 8.5 | 0. | 25.06 | 0. | 0. |
| time (sec) | N/A | 0.306 | 2.127 | 0.352 | 0. | 8.127 | 0. | 0. |

| Problem 8 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 28 | 28 | 84 | 27 | 0 | 123 | 0 | 70 |
| normalized size | 1 | 1. | 3. | 0.96 | 0. | 4.39 | 0. | 2.5 |
| time (sec) | N/A | 0.017 | 0.068 | 0.048 | 0. | 1.034 | 0. | 1.142 |

| Problem 9 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 136 | 136 | 142 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.04 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.103 | 0.138 | 2.905 | 0. | 0. | 0. | 0. |

| Problem 10 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | F | F | F | F(-1) | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 200 | 200 | 172 | 0 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.86 | 0. | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.134 | 0.238 | 3.482 | 0. | 0. | 0. | 0. |

| Problem 11 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | C | F | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 48 | 48 | 27 | 16 | 0 | 55 | 0 | 66 |
| normalized size | 1 | 1. | 0.56 | 0.33 | 0. | 1.15 | 0. | 1.38 |
| time (sec) | N/A | 0.015 | 0.018 | 0.168 | 0. | 0.866 | 0. | 1.182 |

| Problem 12 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 66 | 84 | 0 | 235 | 0 | 193 |
| normalized size | 1 | 1. | 0.94 | 1.2 | 0. | 3.36 | 0. | 2.76 |
| time (sec) | N/A | 0.051 | 0.083 | 0.059 | 0. | 0.808 | 0. | 1.168 |

| Problem 13 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | A | F | F | F | F |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 1077 | 1077 | 600 | 661 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.56 | 0.61 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 3.099 | 1.528 | 0.618 | 0. | 0. | 0. | 0. |

| Problem 14 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | A | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 159 | 341 | 0 | 441 | 0 | 231 |
| normalized size | 1 | 1. | 1.62 | 3.48 | 0. | 4.5 | 0. | 2.36 |
| time (sec) | N/A | 0.21 | 0.401 | 0.112 | 0. | 0.975 | 0. | 1.177 |

| Problem 15 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 68 | 55 | 73 | 176 | 65 | 73 |
| normalized size | 1 | 1. | 1. | 0.81 | 1.07 | 2.59 | 0.96 | 1.07 |
| time (sec) | N/A | 0.049 | 0.003 | 0.044 | 0.972 | 0.847 | 0.084 | 1.154 |

| Problem 16 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 45 | 59 | 131 | 53 | 59 |
| normalized size | 1 | 1. | 1. | 0.8 | 1.05 | 2.34 | 0.95 | 1.05 |
| time (sec) | N/A | 0.038 | 0.002 | 0.044 | 0.983 | 0.717 | 0.084 | 1.175 |

| Problem 17 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 44 | 35 | 46 | 96 | 41 | 46 |
| normalized size | 1 | 1. | 1. | 0.8 | 1.05 | 2.18 | 0.93 | 1.05 |
| time (sec) | N/A | 0.029 | 0.001 | 0.042 | 0.961 | 0.712 | 0.079 | 1.185 |

| Problem 18 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 30 | 30 | 30 | 25 | 32 | 59 | 26 | 32 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.07 | 1.97 | 0.87 | 1.07 |
| time (sec) | N/A | 0.016 | 0.001 | 0.043 | 0.993 | 0.711 | 0.063 | 1.174 |

| Problem 19 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 34 | 45 | 119 | 49 | 45 |
| normalized size | 1 | 1. | 1. | 0.81 | 1.07 | 2.83 | 1.17 | 1.07 |
| time (sec) | N/A | 0.04 | 0.015 | 0.049 | 1.484 | 0.834 | 0.154 | 1.168 |

| Problem 20 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 43 | 34 | 49 | 146 | 42 | 49 |
| normalized size | 1 | 1. | 1. | 0.79 | 1.14 | 3.4 | 0.98 | 1.14 |
| time (sec) | N/A | 0.027 | 0.015 | 0.046 | 1.466 | 0.851 | 0.213 | 1.223 |

| Problem 21 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 53 | 47 | 76 | 242 | 63 | 62 |
| normalized size | 1 | 1. | 0.83 | 0.73 | 1.19 | 3.78 | 0.98 | 0.97 |
| time (sec) | N/A | 0.037 | 0.025 | 0.044 | 1.472 | 0.801 | 0.281 | 1.251 |

| Problem 22 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 80 | 80 | 65 | 86 | 221 | 76 | 86 |
| normalized size | 1 | 1. | 1. | 0.81 | 1.08 | 2.76 | 0.95 | 1.08 |
| time (sec) | N/A | 0.06 | 0.003 | 0.043 | 0.957 | 0.622 | 0.133 | 1.199 |

| Problem 23 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 66 | 66 | 66 | 55 | 73 | 167 | 63 | 73 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.11 | 2.53 | 0.95 | 1.11 |
| time (sec) | N/A | 0.048 | 0.002 | 0.042 | 0.964 | 0.734 | 0.137 | 1.16 |

| Problem 24 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 54 | 54 | 54 | 45 | 59 | 122 | 51 | 59 |
| normalized size | 1 | 1. | 1. | 0.83 | 1.09 | 2.26 | 0.94 | 1.09 |
| time (sec) | N/A | 0.042 | 0.002 | 0.044 | 0.967 | 0.904 | 0.108 | 1.195 |

| Problem 25 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 46 | 46 | 46 | 35 | 46 | 95 | 41 | 46 |
| normalized size | 1 | 1. | 1. | 0.76 | 1. | 2.07 | 0.89 | 1. |
| time (sec) | N/A | 0.03 | 0.001 | 0.045 | 0.964 | 0.829 | 0.097 | 1.171 |

| Problem 26 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 53 | 44 | 58 | 165 | 63 | 58 |
| normalized size | 1 | 1. | 0.95 | 0.79 | 1.04 | 2.95 | 1.12 | 1.04 |
| time (sec) | N/A | 0.051 | 0.019 | 0.044 | 1.488 | 0.993 | 0.189 | 1.16 |

| Problem 27 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 59 | 51 | 70 | 252 | 65 | 70 |
| normalized size | 1 | 1. | 0.94 | 0.81 | 1.11 | 4. | 1.03 | 1.11 |
| time (sec) | N/A | 0.061 | 0.031 | 0.046 | 1.46 | 0.998 | 0.214 | 1.185 |

| Problem 28 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 53 | 47 | 76 | 258 | 63 | 62 |
| normalized size | 1 | 1. | 0.83 | 0.73 | 1.19 | 4.03 | 0.98 | 0.97 |
| time (sec) | N/A | 0.052 | 0.023 | 0.046 | 1.488 | 0.948 | 0.298 | 1.239 |

| Problem 29 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 85 | 63 | 57 | 103 | 371 | 83 | 76 |
| normalized size | 1 | 1. | 0.74 | 0.67 | 1.21 | 4.36 | 0.98 | 0.89 |
| time (sec) | N/A | 0.063 | 0.045 | 0.05 | 1.446 | 0.963 | 0.251 | 1.182 |

| Problem 30 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 96 | 96 | 96 | 75 | 100 | 274 | 92 | 100 |
| normalized size | 1 | 1. | 1. | 0.78 | 1.04 | 2.85 | 0.96 | 1.04 |
| time (sec) | N/A | 0.071 | 0.003 | 0.044 | 0.983 | 0.823 | 0.126 | 1.177 |

| Problem 31 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 82 | 65 | 86 | 217 | 78 | 86 |
| normalized size | 1 | 1. | 1. | 0.79 | 1.05 | 2.65 | 0.95 | 1.05 |
| time (sec) | N/A | 0.056 | 0.002 | 0.045 | 0.97 | 0.832 | 0.085 | 1.14 |

| Problem 32 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 68 | 55 | 73 | 166 | 65 | 73 |
| normalized size | 1 | 1. | 1. | 0.81 | 1.07 | 2.44 | 0.96 | 1.07 |
| time (sec) | N/A | 0.051 | 0.002 | 0.043 | 0.957 | 0.85 | 0.086 | 1.194 |

| Problem 33 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 56 | 45 | 59 | 122 | 53 | 59 |
| normalized size | 1 | 1. | 1. | 0.8 | 1.05 | 2.18 | 0.95 | 1.05 |
| time (sec) | N/A | 0.032 | 0.001 | 0.043 | 0.967 | 0.882 | 0.108 | 1.204 |

| Problem 34 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 63 | 54 | 72 | 219 | 76 | 72 |
| normalized size | 1 | 1. | 0.9 | 0.77 | 1.03 | 3.13 | 1.09 | 1.03 |
| time (sec) | N/A | 0.053 | 0.022 | 0.046 | 1.466 | 0.991 | 0.149 | 1.187 |

| Problem 35 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 77 | 61 | 84 | 321 | 78 | 84 |
| normalized size | 1 | 1. | 1. | 0.79 | 1.09 | 4.17 | 1.01 | 1.09 |
| time (sec) | N/A | 0.072 | 0.027 | 0.046 | 1.479 | 1.03 | 0.172 | 1.208 |

| Problem 36 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 78 | 63 | 97 | 406 | 85 | 84 |
| normalized size | 1 | 1. | 0.93 | 0.75 | 1.15 | 4.83 | 1.01 | 1. |
| time (sec) | N/A | 0.087 | 0.037 | 0.048 | 1.442 | 1.012 | 0.216 | 1.212 |

| Problem 37 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 72 | 64 | 85 | 252 | 87 | 85 |
| normalized size | 1 | 1. | 0.86 | 0.76 | 1.01 | 3. | 1.04 | 1.01 |
| time (sec) | N/A | 0.057 | 0.027 | 0.046 | 1.435 | 0.975 | 0.156 | 1.25 |

| Problem 38 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 70 | 70 | 63 | 54 | 72 | 196 | 73 | 72 |
| normalized size | 1 | 1. | 0.9 | 0.77 | 1.03 | 2.8 | 1.04 | 1.03 |
| time (sec) | N/A | 0.056 | 0.021 | 0.047 | 1.472 | 0.935 | 0.144 | 1.167 |

| Problem 39 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 56 | 56 | 52 | 44 | 58 | 147 | 60 | 58 |
| normalized size | 1 | 1. | 0.93 | 0.79 | 1.04 | 2.62 | 1.07 | 1.04 |
| time (sec) | N/A | 0.05 | 0.017 | 0.049 | 1.44 | 0.994 | 0.197 | 1.126 |

| Problem 40 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 42 | 42 | 42 | 34 | 45 | 112 | 46 | 45 |
| normalized size | 1 | 1. | 1. | 0.81 | 1.07 | 2.67 | 1.1 | 1.07 |
| time (sec) | N/A | 0.035 | 0.01 | 0.044 | 1.431 | 1.051 | 0.124 | 1.198 |

| Problem 41 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 73 | 73 | 73 | 60 | 80 | 207 | 83 | 80 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.1 | 2.84 | 1.14 | 1.1 |
| time (sec) | N/A | 0.053 | 0.031 | 0.048 | 1.427 | 1. | 0.268 | 1.265 |

| Problem 42 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 94 | 77 | 105 | 377 | 102 | 105 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.12 | 4.01 | 1.09 | 1.12 |
| time (sec) | N/A | 0.089 | 0.078 | 0.049 | 1.483 | 0.985 | 0.298 | 1.223 |

| Problem 43 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 104 | 89 | 132 | 578 | 119 | 119 |
| normalized size | 1 | 1. | 0.9 | 0.77 | 1.15 | 5.03 | 1.03 | 1.03 |
| time (sec) | N/A | 0.124 | 0.144 | 0.049 | 1.439 | 1.027 | 0.4 | 1.26 |

| Problem 44 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 91 | 91 | 91 | 71 | 97 | 348 | 88 | 97 |
| normalized size | 1 | 1. | 1. | 0.78 | 1.07 | 3.82 | 0.97 | 1.07 |
| time (sec) | N/A | 0.086 | 0.05 | 0.048 | 1.425 | 0.974 | 0.245 | 1.154 |

| Problem 45 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 77 | 77 | 77 | 61 | 84 | 292 | 75 | 84 |
| normalized size | 1 | 1. | 1. | 0.79 | 1.09 | 3.79 | 0.97 | 1.09 |
| time (sec) | N/A | 0.073 | 0.027 | 0.047 | 1.435 | 1.033 | 0.254 | 1.183 |

| Problem 46 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 63 | 63 | 63 | 51 | 70 | 234 | 61 | 70 |
| normalized size | 1 | 1. | 1. | 0.81 | 1.11 | 3.71 | 0.97 | 1.11 |
| time (sec) | N/A | 0.063 | 0.03 | 0.045 | 1.424 | 1.026 | 0.171 | 1.19 |

| Problem 47 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 43 | 34 | 49 | 138 | 41 | 49 |
| normalized size | 1 | 1. | 1. | 0.79 | 1.14 | 3.21 | 0.95 | 1.14 |
| time (sec) | N/A | 0.026 | 0.014 | 0.047 | 1.432 | 0.988 | 0.206 | 1.139 |

| Problem 48 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 94 | 94 | 94 | 77 | 105 | 363 | 102 | 105 |
| normalized size | 1 | 1. | 1. | 0.82 | 1.12 | 3.86 | 1.09 | 1.12 |
| time (sec) | N/A | 0.088 | 0.055 | 0.049 | 1.445 | 1.002 | 0.307 | 1.133 |

| Problem 49 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 127 | 106 | 94 | 130 | 548 | 122 | 130 |
| normalized size | 1 | 1. | 0.83 | 0.74 | 1.02 | 4.31 | 0.96 | 1.02 |
| time (sec) | N/A | 0.123 | 0.058 | 0.052 | 1.447 | 0.991 | 0.449 | 1.185 |

| Problem 50 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 136 | 106 | 159 | 802 | 143 | 149 |
| normalized size | 1 | 1. | 0.92 | 0.72 | 1.07 | 5.42 | 0.97 | 1.01 |
| time (sec) | N/A | 0.161 | 0.064 | 0.053 | 1.491 | 1.041 | 0.466 | 1.132 |

| Problem 51 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 98 | 98 | 73 | 111 | 447 | 95 | 97 |
| normalized size | 1 | 1. | 1. | 0.74 | 1.13 | 4.56 | 0.97 | 0.99 |
| time (sec) | N/A | 0.113 | 0.037 | 0.054 | 1.454 | 0.905 | 0.278 | 1.148 |

| Problem 52 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 84 | 84 | 84 | 63 | 97 | 377 | 82 | 84 |
| normalized size | 1 | 1. | 1. | 0.75 | 1.15 | 4.49 | 0.98 | 1. |
| time (sec) | N/A | 0.087 | 0.036 | 0.05 | 1.454 | 0.985 | 0.3 | 1.131 |

| Problem 53 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 51 | 47 | 76 | 239 | 61 | 62 |
| normalized size | 1 | 1. | 0.8 | 0.73 | 1.19 | 3.73 | 0.95 | 0.97 |
| time (sec) | N/A | 0.053 | 0.029 | 0.047 | 1.439 | 1.007 | 0.276 | 1.291 |

| Problem 54 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 64 | 64 | 51 | 47 | 76 | 225 | 61 | 62 |
| normalized size | 1 | 1. | 0.8 | 0.73 | 1.19 | 3.52 | 0.95 | 0.97 |
| time (sec) | N/A | 0.033 | 0.028 | 0.046 | 1.452 | 1.05 | 0.192 | 1.202 |

| Problem 55 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 115 | 115 | 99 | 89 | 132 | 556 | 122 | 119 |
| normalized size | 1 | 1. | 0.86 | 0.77 | 1.15 | 4.83 | 1.06 | 1.03 |
| time (sec) | N/A | 0.124 | 0.155 | 0.053 | 1.432 | 1.025 | 0.397 | 1.16 |

| Problem 56 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 160 | 136 | 106 | 157 | 763 | 143 | 149 |
| normalized size | 1 | 1. | 0.85 | 0.66 | 0.98 | 4.77 | 0.89 | 0.93 |
| time (sec) | N/A | 0.16 | 0.102 | 0.056 | 1.466 | 1.03 | 0.535 | 1.156 |

| Problem 57 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | A | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 181 | 151 | 118 | 186 | 1035 | 163 | 157 |
| normalized size | 1 | 1. | 0.83 | 0.65 | 1.03 | 5.72 | 0.9 | 0.87 |
| time (sec) | N/A | 0.204 | 0.085 | 0.053 | 1.447 | 1.172 | 0.539 | 1.164 |

| Problem 58 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 208 | 208 | 85 | 166 | 239 | 439 | 0 | 126 |
| normalized size | 1 | 1. | 0.41 | 0.8 | 1.15 | 2.11 | 0. | 0.61 |
| time (sec) | N/A | 0.309 | 0.291 | 0.069 | 1.484 | 1.408 | 0. | 1.137 |

| Problem 59 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 75 | 132 | 193 | 352 | 0 | 112 |
| normalized size | 1 | 1. | 0.45 | 0.8 | 1.16 | 2.12 | 0. | 0.67 |
| time (sec) | N/A | 0.181 | 0.172 | 0.059 | 1.472 | 1.523 | 0. | 1.327 |

| Problem 60 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 65 | 98 | 147 | 257 | 0 | 99 |
| normalized size | 1 | 1. | 0.52 | 0.79 | 1.19 | 2.07 | 0. | 0.8 |
| time (sec) | N/A | 0.099 | 0.106 | 0.053 | 1.534 | 1.529 | 0. | 1.196 |

| Problem 61 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 55 | 64 | 101 | 207 | 0 | 85 |
| normalized size | 1 | 1. | 0.67 | 0.78 | 1.23 | 2.52 | 0. | 1.04 |
| time (sec) | N/A | 0.04 | 0.053 | 0.048 | 1.504 | 1.649 | 0. | 1.164 |

| Problem 62 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 174 | 174 | 185 | 2065 | 0 | 7056 | 0 | 0 |
| normalized size | 1 | 1. | 1.06 | 11.87 | 0. | 40.55 | 0. | 0. |
| time (sec) | N/A | 0.441 | 0.463 | 0.276 | 0. | 4.393 | 0. | 0. |

| Problem 63 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 214 | 16357 | 0 | 8142 | 0 | 0 |
| normalized size | 1 | 1. | 1.14 | 87.01 | 0. | 43.31 | 0. | 0. |
| time (sec) | N/A | 0.393 | 1.083 | 0.292 | 0. | 5.824 | 0. | 0. |

| Problem 64 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 299 | 44343 | 0 | 10298 | 0 | 0 |
| normalized size | 1 | 1. | 1.34 | 198.85 | 0. | 46.18 | 0. | 0. |
| time (sec) | N/A | 0.459 | 2.069 | 0.428 | 0. | 6.156 | 0. | 0. |

| Problem 65 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 231 | 95 | 185 | 278 | 532 | 0 | 139 |
| normalized size | 1 | 1. | 0.41 | 0.8 | 1.2 | 2.3 | 0. | 0.6 |
| time (sec) | N/A | 0.342 | 0.367 | 0.072 | 1.546 | 1.585 | 0. | 1.174 |

| Problem 66 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 85 | 151 | 232 | 423 | 0 | 126 |
| normalized size | 1 | 1. | 0.45 | 0.8 | 1.23 | 2.24 | 0. | 0.67 |
| time (sec) | N/A | 0.19 | 0.234 | 0.059 | 1.503 | 1.593 | 0. | 1.16 |

| Problem 67 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 75 | 117 | 186 | 342 | 0 | 112 |
| normalized size | 1 | 1. | 0.51 | 0.8 | 1.27 | 2.33 | 0. | 0.76 |
| time (sec) | N/A | 0.122 | 0.144 | 0.055 | 1.5 | 1.517 | 0. | 1.154 |

| Problem 68 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 65 | 83 | 140 | 257 | 0 | 99 |
| normalized size | 1 | 1. | 0.62 | 0.79 | 1.33 | 2.45 | 0. | 0.94 |
| time (sec) | N/A | 0.05 | 0.084 | 0.058 | 1.499 | 1.602 | 0. | 1.248 |

| Problem 69 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 197 | 197 | 310 | 3460 | 0 | 7468 | 0 | 0 |
| normalized size | 1 | 1. | 1.57 | 17.56 | 0. | 37.91 | 0. | 0. |
| time (sec) | N/A | 0.488 | 0.695 | 0.19 | 0. | 4.324 | 0. | 0. |

| Problem 70 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 232 | 232 | 530 | 28185 | 0 | 8892 | 0 | 0 |
| normalized size | 1 | 1. | 2.28 | 121.49 | 0. | 38.33 | 0. | 0. |
| time (sec) | N/A | 0.575 | 2.584 | 0.355 | 0. | 5.108 | 0. | 0. |

| Problem 71 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 1262 | 81552 | 0 | 9586 | 0 | 0 |
| normalized size | 1 | 1. | 5.66 | 365.7 | 0. | 42.99 | 0. | 0. |
| time (sec) | N/A | 0.433 | 5.388 | 0.607 | 0. | 4.943 | 0. | 0. |

| Problem 72 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 254 | 105 | 204 | 317 | 657 | 0 | 153 |
| normalized size | 1 | 1. | 0.41 | 0.8 | 1.25 | 2.59 | 0. | 0.6 |
| time (sec) | N/A | 0.373 | 0.454 | 0.08 | 1.534 | 1.407 | 0. | 1.155 |

| Problem 73 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 212 | 95 | 170 | 271 | 512 | 0 | 139 |
| normalized size | 1 | 1. | 0.45 | 0.8 | 1.28 | 2.42 | 0. | 0.66 |
| time (sec) | N/A | 0.22 | 0.288 | 0.061 | 1.48 | 1.358 | 0. | 1.152 |

| Problem 74 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 170 | 170 | 85 | 136 | 225 | 409 | 0 | 126 |
| normalized size | 1 | 1. | 0.5 | 0.8 | 1.32 | 2.41 | 0. | 0.74 |
| time (sec) | N/A | 0.13 | 0.19 | 0.053 | 1.474 | 1.377 | 0. | 1.163 |

| Problem 75 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 75 | 102 | 180 | 327 | 0 | 112 |
| normalized size | 1 | 1. | 0.59 | 0.8 | 1.41 | 2.55 | 0. | 0.88 |
| time (sec) | N/A | 0.062 | 0.108 | 0.049 | 1.438 | 1.386 | 0. | 1.145 |

| Problem 76 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 222 | 222 | 229 | 4860 | 0 | 7899 | 0 | 0 |
| normalized size | 1 | 1. | 1.03 | 21.89 | 0. | 35.58 | 0. | 0. |
| time (sec) | N/A | 0.539 | 1.07 | 0.179 | 0. | 3.907 | 0. | 0. |

| Problem 77 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 255 | 685 | 40028 | 0 | 9335 | 0 | 0 |
| normalized size | 1 | 1. | 2.69 | 156.97 | 0. | 36.61 | 0. | 0. |
| time (sec) | N/A | 0.66 | 1.699 | 0.414 | 0. | 5.257 | 0. | 0. |

| Problem 78 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 281 | 1009 | 119458 | 0 | 10927 | 0 | 0 |
| normalized size | 1 | 1. | 3.59 | 425.12 | 0. | 38.89 | 0. | 0. |
| time (sec) | N/A | 0.655 | 2.116 | 0.69 | 0. | 5.495 | 0. | 0. |

| Problem 79 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 185 | 185 | 75 | 147 | 200 | 355 | 0 | 112 |
| normalized size | 1 | 1. | 0.41 | 0.79 | 1.08 | 1.92 | 0. | 0.61 |
| time (sec) | N/A | 0.312 | 0.246 | 0.065 | 1.532 | 1.36 | 0. | 1.18 |

| Problem 80 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 143 | 143 | 65 | 113 | 154 | 266 | 0 | 99 |
| normalized size | 1 | 1. | 0.45 | 0.79 | 1.08 | 1.86 | 0. | 0.69 |
| time (sec) | N/A | 0.168 | 0.132 | 0.054 | 1.449 | 1.34 | 0. | 1.195 |

| Problem 81 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 101 | 101 | 55 | 79 | 108 | 211 | 0 | 85 |
| normalized size | 1 | 1. | 0.54 | 0.78 | 1.07 | 2.09 | 0. | 0.84 |
| time (sec) | N/A | 0.088 | 0.076 | 0.051 | 1.489 | 1.377 | 0. | 1.165 |

| Problem 82 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 59 | 59 | 45 | 45 | 62 | 163 | 0 | 72 |
| normalized size | 1 | 1. | 0.76 | 0.76 | 1.05 | 2.76 | 0. | 1.22 |
| time (sec) | N/A | 0.033 | 0.038 | 0.051 | 1.532 | 1.321 | 0. | 1.156 |

| Problem 83 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 148 | 148 | 176 | 684 | 0 | 7101 | 0 | 0 |
| normalized size | 1 | 1. | 1.19 | 4.62 | 0. | 47.98 | 0. | 0. |
| time (sec) | N/A | 0.314 | 0.347 | 0.108 | 0. | 4.685 | 0. | 0. |

| Problem 84 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 188 | 188 | 287 | 5225 | 0 | 8625 | 0 | 0 |
| normalized size | 1 | 1. | 1.53 | 27.79 | 0. | 45.88 | 0. | 0. |
| time (sec) | N/A | 0.429 | 1.007 | 0.153 | 0. | 5.141 | 0. | 0. |

| Problem 85 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 223 | 1279 | 13040 | 0 | 9927 | 0 | 0 |
| normalized size | 1 | 1. | 5.74 | 58.48 | 0. | 44.52 | 0. | 0. |
| time (sec) | N/A | 0.469 | 6.238 | 0.191 | 0. | 5.178 | 0. | 0. |

| Problem 86 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 166 | 95 | 166 | 200 | 385 | 0 | 111 |
| normalized size | 1 | 1. | 0.57 | 1. | 1.2 | 2.32 | 0. | 0.67 |
| time (sec) | N/A | 0.204 | 0.377 | 0.066 | 1.495 | 1.416 | 0. | 1.198 |

| Problem 87 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 124 | 65 | 132 | 154 | 311 | 0 | 97 |
| normalized size | 1 | 1. | 0.52 | 1.06 | 1.24 | 2.51 | 0. | 0.78 |
| time (sec) | N/A | 0.127 | 0.229 | 0.059 | 1.526 | 1.327 | 0. | 1.206 |

| Problem 88 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 82 | 82 | 55 | 98 | 108 | 251 | 0 | 84 |
| normalized size | 1 | 1. | 0.67 | 1.2 | 1.32 | 3.06 | 0. | 1.02 |
| time (sec) | N/A | 0.071 | 0.141 | 0.052 | 1.484 | 1.3 | 0. | 1.181 |

| Problem 89 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 45 | 45 | 64 | 62 | 209 | 0 | 72 |
| normalized size | 1 | 1. | 1. | 1.42 | 1.38 | 4.64 | 0. | 1.6 |
| time (sec) | N/A | 0.029 | 0.077 | 0.049 | 1.509 | 1.301 | 0. | 1.229 |

| Problem 90 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 176 | 176 | 202 | 718 | 0 | 7618 | 0 | 0 |
| normalized size | 1 | 1. | 1.15 | 4.08 | 0. | 43.28 | 0. | 0. |
| time (sec) | N/A | 0.408 | 1.364 | 0.124 | 0. | 4.791 | 0. | 0. |

| Problem 91 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 211 | 740 | 5942 | 0 | 9287 | 0 | 0 |
| normalized size | 1 | 1. | 3.51 | 28.16 | 0. | 44.01 | 0. | 0. |
| time (sec) | N/A | 0.473 | 1.488 | 0.179 | 0. | 5.232 | 0. | 0. |

| Problem 92 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 246 | 246 | 231 | 18981 | 0 | 11105 | 0 | 0 |
| normalized size | 1 | 1. | 0.94 | 77.16 | 0. | 45.14 | 0. | 0. |
| time (sec) | N/A | 0.525 | 2.25 | 0.266 | 0. | 5.473 | 0. | 0. |

| Problem 93 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 147 | 75 | 214 | 342 | 441 | 0 | 109 |
| normalized size | 1 | 1. | 0.51 | 1.46 | 2.33 | 3. | 0. | 0.74 |
| time (sec) | N/A | 0.167 | 0.518 | 0.071 | 1.92 | 1.373 | 0. | 1.15 |

| Problem 94 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 105 | 105 | 65 | 180 | 296 | 370 | 0 | 97 |
| normalized size | 1 | 1. | 0.62 | 1.71 | 2.82 | 3.52 | 0. | 0.92 |
| time (sec) | N/A | 0.105 | 0.335 | 0.057 | 1.501 | 1.371 | 0. | 1.224 |

| Problem 95 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | B | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 68 | 68 | 55 | 146 | 250 | 302 | 0 | 82 |
| normalized size | 1 | 1. | 0.81 | 2.15 | 3.68 | 4.44 | 0. | 1.21 |
| time (sec) | N/A | 0.061 | 0.241 | 0.052 | 1.776 | 1.371 | 0. | 1.269 |

| Problem 96 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | A | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 47 | 33 | 30 | 80 | 132 | 0 | 39 |
| normalized size | 1 | 1. | 0.7 | 0.64 | 1.7 | 2.81 | 0. | 0.83 |
| time (sec) | N/A | 0.022 | 0.104 | 0.045 | 1.077 | 1.341 | 0. | 1.157 |

| Problem 97 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 199 | 218 | 751 | 0 | 8280 | 0 | 0 |
| normalized size | 1 | 1. | 1.1 | 3.77 | 0. | 41.61 | 0. | 0. |
| time (sec) | N/A | 0.456 | 0.91 | 0.121 | 0. | 4.947 | 0. | 0. |

| Problem 98 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 234 | 234 | 296 | 5975 | 0 | 8498 | 0 | 0 |
| normalized size | 1 | 1. | 1.26 | 25.53 | 0. | 36.32 | 0. | 0. |
| time (sec) | N/A | 0.543 | 1.162 | 0.175 | 0. | 4.836 | 0. | 0. |

| Problem 99 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 269 | 269 | 242 | 19014 | 0 | 11142 | 0 | 0 |
| normalized size | 1 | 1. | 0.9 | 70.68 | 0. | 41.42 | 0. | 0. |
| time (sec) | N/A | 0.589 | 1.971 | 0.277 | 0. | 5.318 | 0. | 0. |

| Problem 100 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 436 | 436 | 657 | 1429 | 0 | 2866 | 0 | 861 |
| normalized size | 1 | 1. | 1.51 | 3.28 | 0. | 6.57 | 0. | 1.97 |
| time (sec) | N/A | 0.789 | 0.965 | 0.063 | 0. | 2.833 | 0. | 1.302 |

| Problem 101 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 175 | 175 | 173 | 453 | 0 | 1061 | 0 | 286 |
| normalized size | 1 | 1. | 0.99 | 2.59 | 0. | 6.06 | 0. | 1.63 |
| time (sec) | N/A | 0.164 | 0.293 | 0.052 | 0. | 1.495 | 0. | 1.16 |

| Problem 102 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 431 | 431 | 417 | 6019 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.97 | 13.97 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.051 | 1.255 | 0.433 | 0. | 0. | 0. | 0. |

| Problem 103 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 488 | 488 | 555 | 22287 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.14 | 45.67 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 2.929 | 5.449 | 0.371 | 0. | 0. | 0. | 0. |

| Problem 104 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 564 | 564 | 829 | 2458 | 0 | 5115 | 0 | 1553 |
| normalized size | 1 | 1. | 1.47 | 4.36 | 0. | 9.07 | 0. | 2.75 |
| time (sec) | N/A | 0.935 | 1.828 | 0.063 | 0. | 9.817 | 0. | 1.339 |

| Problem 105 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 236 | 236 | 392 | 862 | 0 | 1947 | 0 | 563 |
| normalized size | 1 | 1. | 1.66 | 3.65 | 0. | 8.25 | 0. | 2.39 |
| time (sec) | N/A | 0.23 | 0.703 | 0.053 | 0. | 2.634 | 0. | 1.294 |

| Problem 106 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-2) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 679 | 678 | 1232 | 22523 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.81 | 33.17 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 11.033 | 4.857 | 0.329 | 0. | 0. | 0. | 0. |

| Problem 107 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F(-1) | F(-1) | F(-2) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 704 | 704 | 2843 | 72576 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 4.04 | 103.09 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 11.95 | 6.82 | 0.369 | 0. | 0. | 0. | 0. |

| Problem 108 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | B | B | F | F(-1) | F(-1) | F(-2) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 671 | 669 | 4727 | 178044 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 7.04 | 265.34 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 11.597 | 7.296 | 0.36 | 0. | 0. | 0. | 0. |

| Problem 109 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 717 | 717 | 615 | 1930 | 0 | 3661 | 0 | 1112 |
| normalized size | 1 | 1. | 0.86 | 2.69 | 0. | 5.11 | 0. | 1.55 |
| time (sec) | N/A | 2.71 | 1.405 | 0.073 | 0. | 7.39 | 0. | 1.369 |

| Problem 110 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 316 | 316 | 251 | 706 | 0 | 1457 | 0 | 410 |
| normalized size | 1 | 1. | 0.79 | 2.23 | 0. | 4.61 | 0. | 1.3 |
| time (sec) | N/A | 0.626 | 0.537 | 0.062 | 0. | 2.845 | 0. | 1.396 |

| Problem 111 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | A | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 116 | 116 | 96 | 185 | 0 | 549 | 0 | 132 |
| normalized size | 1 | 1. | 0.83 | 1.59 | 0. | 4.73 | 0. | 1.14 |
| time (sec) | N/A | 0.111 | 0.151 | 0.062 | 0. | 1.929 | 0. | 1.291 |

| Problem 112 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 374 | 374 | 376 | 761 | 0 | 22873 | 0 | 0 |
| normalized size | 1 | 1. | 1.01 | 2.03 | 0. | 61.16 | 0. | 0. |
| time (sec) | N/A | 0.579 | 1.508 | 0.359 | 0. | 45.018 | 0. | 0. |

| Problem 113 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 789 | 787 | 1377 | 3858 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.75 | 4.89 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 8.21 | 6.734 | 0.329 | 0. | 0. | 0. | 0. |

| Problem 114 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 649 | 649 | 745 | 2827 | 0 | 6743 | 0 | 1484 |
| normalized size | 1 | 1. | 1.15 | 4.36 | 0. | 10.39 | 0. | 2.29 |
| time (sec) | N/A | 2.106 | 1.685 | 0.069 | 0. | 15.911 | 0. | 1.382 |

| Problem 115 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 309 | 288 | 1011 | 0 | 2750 | 0 | 549 |
| normalized size | 1 | 1. | 0.93 | 3.27 | 0. | 8.9 | 0. | 1.78 |
| time (sec) | N/A | 0.447 | 0.757 | 0.067 | 0. | 9.595 | 0. | 1.658 |

| Problem 116 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 111 | 111 | 113 | 249 | 0 | 941 | 0 | 165 |
| normalized size | 1 | 1. | 1.02 | 2.24 | 0. | 8.48 | 0. | 1.49 |
| time (sec) | N/A | 0.08 | 0.328 | 0.052 | 0. | 4.45 | 0. | 1.502 |

| Problem 117 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 666 | 666 | 700 | 4099 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 1.05 | 6.15 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 1.829 | 6.792 | 0.355 | 0. | 0. | 0. | 0. |

| Problem 118 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | B | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 891 | 891 | 872 | 4635 | 0 | 8338 | 0 | 1891 |
| normalized size | 1 | 1. | 0.98 | 5.2 | 0. | 9.36 | 0. | 2.12 |
| time (sec) | N/A | 1.768 | 2.376 | 0.071 | 0. | 48.355 | 0. | 1.541 |

| Problem 119 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | B | F(-2) | A | F(-1) | A |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 444 | 444 | 387 | 1786 | 0 | 3252 | 0 | 792 |
| normalized size | 1 | 1. | 0.87 | 4.02 | 0. | 7.32 | 0. | 1.78 |
| time (sec) | N/A | 0.451 | 1.315 | 0.061 | 0. | 27.422 | 0. | 1.388 |

| Problem 120 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F(-2) | B | F(-1) | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 131 | 131 | 147 | 185 | 0 | 614 | 0 | 356 |
| normalized size | 1 | 1. | 1.12 | 1.41 | 0. | 4.69 | 0. | 2.72 |
| time (sec) | N/A | 0.085 | 0.395 | 0.052 | 0. | 23.233 | 0. | 1.364 |

| Problem 121 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | C | B | F | B | F | B |
| verified | N/A | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 81 | 144 | 0 | 424 | 0 | 277 |
| normalized size | 1 | 1. | 1.59 | 2.82 | 0. | 8.31 | 0. | 5.43 |
| time (sec) | N/A | 0.066 | 0.04 | 0.108 | 0. | 1.91 | 0. | 1.494 |

| Problem 122 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 1432 | 1432 | 670 | 929 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.47 | 0.65 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 6.219 | 2.487 | 0.836 | 0. | 0. | 0. | 0. |

| Problem 123 | Optimal | Rubi | Mathematica | Maple | Maxima | Fricas | Sympy | Giac |
|---|---|---|---|---|---|---|---|---|
| grade | A | A | A | A | F | F | F | F |
| verified | N/A | Yes | NO | TBD | TBD | TBD | TBD | TBD |
| size | 652 | 652 | 390 | 420 | 0 | 0 | 0 | 0 |
| normalized size | 1 | 1. | 0.6 | 0.64 | 0. | 0. | 0. | 0. |
| time (sec) | N/A | 0.677 | 0.62 | 3.535 | 0. | 0. | 0. | 0. |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [14] had the largest ratio of [ 0.4444 ]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 4 | 4 | 1. | 29 | 0.138 |
| 2 | A | 2 | 2 | 1. | 31 | 0.065 |
| 3 | A | 2 | 2 | 1. | 27 | 0.074 |
| 4 | A | 4 | 4 | 1. | 27 | 0.148 |
| 5 | A | 5 | 5 | 1. | 27 | 0.185 |
| 6 | A | 6 | 5 | 1. | 27 | 0.185 |
| 7 | A | 4 | 4 | 1. | 31 | 0.129 |
| 8 | A | 2 | 2 | 1. | 23 | 0.087 |
| 9 | A | 2 | 2 | 1. | 31 | 0.065 |
| 10 | A | 2 | 2 | 1. | 34 | 0.059 |
| 11 | A | 3 | 3 | 1. | 22 | 0.136 |
| 12 | A | 6 | 5 | 1. | 18 | 0.278 |
| 13 | A | 3 | 3 | 1. | 26 | 0.115 |
| 14 | A | 16 | 12 | 1. | 27 | 0.444 |
| 15 | A | 2 | 1 | 1. | 23 | 0.043 |
| 16 | A | 2 | 1 | 1. | 23 | 0.043 |
| 17 | A | 2 | 1 | 1. | 23 | 0.043 |
| 18 | A | 2 | 1 | 1. | 21 | 0.048 |
| 19 | A | 6 | 5 | 1. | 23 | 0.217 |
| 20 | A | 4 | 4 | 1. | 23 | 0.174 |
| 21 | A | 5 | 5 | 1. | 23 | 0.217 |
| 22 | A | 2 | 1 | 1. | 25 | 0.04 |
| 23 | A | 2 | 1 | 1. | 25 | 0.04 |
| 24 | A | 2 | 1 | 1. | 25 | 0.04 |
| 25 | A | 2 | 1 | 1. | 23 | 0.043 |
| 26 | A | 6 | 5 | 1. | 25 | 0.2 |
| 27 | A | 7 | 6 | 1. | 25 | 0.24 |
| 28 | A | 5 | 4 | 1. | 25 | 0.16 |
| 29 | A | 6 | 5 | 1. | 25 | 0.2 |
| 30 | A | 2 | 1 | 1. | 25 | 0.04 |
| 31 | A | 2 | 1 | 1. | 25 | 0.04 |
| 32 | A | 2 | 1 | 1. | 25 | 0.04 |
| 33 | A | 2 | 1 | 1. | 23 | 0.043 |
| 34 | A | 6 | 5 | 1. | 25 | 0.2 |
| 35 | A | 7 | 6 | 1. | 25 | 0.24 |
| 36 | A | 8 | 6 | 1. | 25 | 0.24 |
| 37 | A | 6 | 5 | 1. | 25 | 0.2 |
| 38 | A | 6 | 5 | 1. | 25 | 0.2 |
| 39 | A | 6 | 5 | 1. | 25 | 0.2 |
| 40 | A | 6 | 5 | 1. | 23 | 0.217 |
| 41 | A | 9 | 5 | 1. | 25 | 0.2 |
| 42 | A | 10 | 6 | 1. | 25 | 0.24 |
| 43 | A | 11 | 7 | 1. | 25 | 0.28 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | number of rules / integrand leaf size |
|---|---|---|---|---|---|---|
| 44 | A | 7 | 6 | 1. | 25 | 0.24 |
| 45 | A | 7 | 6 | 1. | 25 | 0.24 |
| 46 | A | 7 | 6 | 1. | 25 | 0.24 |
| 47 | A | 4 | 4 | 1. | 23 | 0.174 |
| 48 | A | 10 | 6 | 1. | 25 | 0.24 |
| 49 | A | 11 | 7 | 1. | 25 | 0.28 |
| 50 | A | 12 | 7 | 1. | 25 | 0.28 |
| 51 | A | 8 | 6 | 1. | 25 | 0.24 |
| 52 | A | 8 | 6 | 1. | 25 | 0.24 |
| 53 | A | 5 | 4 | 1. | 25 | 0.16 |
| 54 | A | 5 | 5 | 1. | 23 | 0.217 |
| 55 | A | 11 | 7 | 1. | 25 | 0.28 |
| 56 | A | 12 | 7 | 1. | 25 | 0.28 |
| 57 | A | 13 | 7 | 1. | 25 | 0.28 |
| 58 | A | 11 | 5 | 1. | 27 | 0.185 |
| 59 | A | 9 | 5 | 1. | 27 | 0.185 |
| 60 | A | 7 | 5 | 1. | 27 | 0.185 |
| 61 | A | 5 | 5 | 1. | 25 | 0.2 |
| 62 | A | 8 | 7 | 1. | 27 | 0.259 |
| 63 | A | 6 | 5 | 1. | 27 | 0.185 |
| 64 | A | 7 | 6 | 1. | 27 | 0.222 |
| 65 | A | 12 | 5 | 1. | 27 | 0.185 |
| 66 | A | 10 | 5 | 1. | 27 | 0.185 |
| 67 | A | 8 | 5 | 1. | 27 | 0.185 |
| 68 | A | 6 | 5 | 1. | 25 | 0.2 |
| 69 | A | 9 | 8 | 1. | 27 | 0.296 |
| 70 | A | 10 | 9 | 1. | 27 | 0.333 |
| 71 | A | 7 | 6 | 1. | 27 | 0.222 |
| 72 | A | 13 | 5 | 1. | 27 | 0.185 |
| 73 | A | 11 | 5 | 1. | 27 | 0.185 |
| 74 | A | 9 | 5 | 1. | 27 | 0.185 |
| 75 | A | 7 | 5 | 1. | 25 | 0.2 |
| 76 | A | 10 | 9 | 1. | 27 | 0.333 |
| 77 | A | 11 | 9 | 1. | 27 | 0.333 |
| 78 | A | 11 | 10 | 1. | 27 | 0.37 |
| 79 | A | 10 | 4 | 1. | 27 | 0.148 |
| 80 | A | 8 | 4 | 1. | 27 | 0.148 |
| 81 | A | 6 | 4 | 1. | 27 | 0.148 |
| 82 | A | 4 | 4 | 1. | 25 | 0.16 |
| 83 | A | 5 | 4 | 1. | 27 | 0.148 |
| 84 | A | 6 | 5 | 1. | 27 | 0.185 |
| 85 | A | 7 | 6 | 1. | 27 | 0.222 |
| 86 | A | 9 | 5 | 1. | 27 | 0.185 |
| 87 | A | 7 | 5 | 1. | 27 | 0.185 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 88 | A | 5 | 5 | 1. | 27 | 0.185 |
| 89 | A | 4 | 4 | 1. | 25 | 0.16 |
| 90 | A | 6 | 5 | 1. | 27 | 0.185 |
| 91 | A | 7 | 6 | 1. | 27 | 0.222 |
| 92 | A | 8 | 6 | 1. | 27 | 0.222 |
| 93 | A | 8 | 5 | 1. | 27 | 0.185 |
| 94 | A | 6 | 5 | 1. | 27 | 0.185 |
| 95 | A | 5 | 4 | 1. | 27 | 0.148 |
| 96 | A | 3 | 3 | 1. | 25 | 0.12 |
| 97 | A | 7 | 6 | 1. | 27 | 0.222 |
| 98 | A | 8 | 6 | 1. | 27 | 0.222 |
| 99 | A | 9 | 6 | 1. | 27 | 0.222 |
| 100 | A | 7 | 5 | 1. | 27 | 0.185 |
| 101 | A | 5 | 5 | 1. | 25 | 0.2 |
| 102 | A | 8 | 5 | 1. | 27 | 0.185 |
| 103 | A | 6 | 4 | 1. | 27 | 0.148 |
| 104 | A | 8 | 5 | 1. | 27 | 0.185 |
| 105 | A | 6 | 5 | 1. | 25 | 0.2 |
| 106 | A | 9 | 6 | 1. | 27 | 0.222 |
| 107 | A | 10 | 7 | 1. | 27 | 0.259 |
| 108 | A | 7 | 5 | 1. | 27 | 0.185 |
| 109 | A | 8 | 4 | 1. | 27 | 0.148 |
| 110 | A | 6 | 4 | 1. | 27 | 0.148 |
| 111 | A | 4 | 4 | 1. | 25 | 0.16 |
| 112 | A | 5 | 3 | 1. | 27 | 0.111 |
| 113 | A | 6 | 4 | 1. | 27 | 0.148 |
| 114 | A | 7 | 5 | 1. | 27 | 0.185 |
| 115 | A | 5 | 5 | 1. | 27 | 0.185 |
| 116 | A | 4 | 4 | 1. | 25 | 0.16 |
| 117 | A | 6 | 4 | 1. | 27 | 0.148 |
| 118 | A | 6 | 5 | 1. | 27 | 0.185 |
| 119 | A | 5 | 4 | 1. | 27 | 0.148 |
| 120 | A | 3 | 3 | 1. | 25 | 0.12 |
| 121 | A | 5 | 4 | 1. | 27 | 0.148 |
| 122 | A | 3 | 3 | 1. | 29 | 0.103 |
| 123 | A | 3 | 3 | 1. | 29 | 0.103 |

# Chapter 3

# Listing of integrals

## 3.1 $\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx$

**Optimal**. Leaf size=102

$$\frac{\left(8af-b\left(\frac{4df}{e}+e\right)\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{b\sqrt{d+ex+fx^2}}{4f}$$

```
[Out] (b*Sqrt[d + e*x + f*x^2])/(4*f) + (b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((8*a
*f - b*(e + (4*d*f)/e))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2
])])/(8*f^(3/2))
```

---

**Rubi [A]** time = 0.0944484, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1661, 640, 621, 206}

$$\frac{\left(8af-b\left(\frac{4df}{e}+e\right)\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{b\sqrt{d+ex+fx^2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (b*Sqrt[d + e*x + f*x^2])/(4*f) + (b*x*Sqrt[d + e*x + f*x^2])/(2*e) + ((8*a
*f - b*(e + (4*d*f)/e))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + e*x + f*x^2
])])/(8*f^(3/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx+\frac{bfx^2}{e}}{\sqrt{d+ex+fx^2}}\,dx &= \frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\int \frac{\left(2a-\frac{bd}{e}\right)f+\frac{bfx}{2}}{\sqrt{d+ex+fx^2}}\,dx}{2f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(-be+8af-\frac{4bdf}{e}\right)\int \frac{1}{\sqrt{d+ex+fx^2}}\,dx}{8f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}+\frac{\left(-be+8af-\frac{4bdf}{e}\right)\operatorname{Subst}\left(\int \frac{1}{4f-x^2}\,dx,x,\frac{e+2fx}{\sqrt{d+ex+fx^2}}\right)}{4f}\\
&= \frac{b\sqrt{d+ex+fx^2}}{4f}+\frac{bx\sqrt{d+ex+fx^2}}{2e}-\frac{\left(be-8af+\frac{4bdf}{e}\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+ex+fx^2}}\right)}{8f^{3/2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.191387, size = 87, normalized size = 0.85

$$
\frac{2b\sqrt{f}(e+2fx)\sqrt{d+x(e+fx)}-\left(b\left(4df+e^2\right)-8aef\right)\tanh^{-1}\left(\frac{e+2fx}{2\sqrt{f}\sqrt{d+x(e+fx)}}\right)}{8ef^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + (b*f*x^2)/e)/Sqrt[d + e*x + f*x^2],x]
```

```
[Out] (2*b*Sqrt[f]*(e + 2*f*x)*Sqrt[d + x*(e + f*x)] - (-8*a*e*f + b*(e^2 + 4*d*f
))*ArcTanh[(e + 2*f*x)/(2*Sqrt[f]*Sqrt[d + x*(e + f*x)])])/(8*e*f^(3/2))
```

---

**Maple [A]** time = 0.052, size = 136, normalized size = 1.3

$$
\frac{bx}{2e}\sqrt{fx^2+ex+d}+\frac{b}{4f}\sqrt{fx^2+ex+d}-\frac{be}{8}\ln\left(\left(\frac{e}{2}+fx\right)\frac{1}{\sqrt{f}}+\sqrt{fx^2+ex+d}\right)f^{-\frac{3}{2}}-\frac{bd}{2e}\ln\left(\left(\frac{e}{2}+fx\right)\frac{1}{\sqrt{f}}+\sqrt{fx^2+e
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 1/2*b*x*(f*x^2+e*x+d)^(1/2)/e+1/4*b*(f*x^2+e*x+d)^(1/2)/f-1/8*e*b/f^(3/2)*l
n((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))-1/2/e*b/f^(1/2)*d*ln((1/2*e+f*x)
/f^(1/2)+(f*x^2+e*x+d)^(1/2))+a*ln((1/2*e+f*x)/f^(1/2)+(f*x^2+e*x+d)^(1/2))
/f^(1/2)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.15693, size = 494, normalized size = 4.84

$$\left[-\frac{\left(be^2+4\,(bd-2\,ae)f\right)\sqrt{f}\log\left(-8\,f^2x^2-8\,efx-e^2-4\,\sqrt{fx^2+ex+d}\left(2\,fx+e\right)\sqrt{f}-4\,df\right)-4\left(2\,bf^2x+bef\right)\sqrt{fx^2}}{16\,ef^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqrt(f)*log(-8*f^2*x^2 - 8*e*f*x - e^2
- 4*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(f) - 4*d*f) - 4*(2*b*f^2*x + b*e
*f)*sqrt(f*x^2 + e*x + d))/(e*f^2), 1/8*((b*e^2 + 4*(b*d - 2*a*e)*f)*sqrt(-
f)*arctan(1/2*sqrt(f*x^2 + e*x + d)*(2*f*x + e)*sqrt(-f)/(f^2*x^2 + e*f*x +
 d*f)) + 2*(2*b*f^2*x + b*e*f)*sqrt(f*x^2 + e*x + d))/(e*f^2)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{ae}{\sqrt{d+ex+fx^2}}\,dx + \int \frac{bex}{\sqrt{d+ex+fx^2}}\,dx + \int \frac{bfx^2}{\sqrt{d+ex+fx^2}}\,dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] (Integral(a*e/sqrt(d + e*x + f*x**2), x) + Integral(b*e*x/sqrt(d + e*x + f*
x**2), x) + Integral(b*f*x**2/sqrt(d + e*x + f*x**2), x))/e
```

---

**Giac [A]** time = 1.30205, size = 113, normalized size = 1.11

$$\frac{1}{4}\sqrt{fx^2+xe+d}\left(2\,bxe^{(-1)}+\frac{b}{f}\right)+\frac{\left(4\,bdf-8\,afe+be^2\right)e^{(-1)}\log\left(\left|-2\left(\sqrt{f}x-\sqrt{fx^2+xe+d}\right)\sqrt{f}-e\right|\right)}{8\,f^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(f*x^2 + x*e + d)*(2*b*x*e^(-1) + b/f) + 1/8*(4*b*d*f - 8*a*f*e + b
*e^2)*e^(-1)*log(abs(-2*(sqrt(f)*x - sqrt(f*x^2 + x*e + d))*sqrt(f) - e))/f
^(3/2)
```

## 3.2 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)} \, dx$

**Optimal**. Leaf size=82

$$-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

```
[Out] (-2*Sqrt[e]*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f
]*Sqrt[d + e*x + f*x^2])])/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

---

**Rubi [A]** time = 0.110623, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {982, 208}

$$-\frac{2\sqrt{e}\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] (-2*Sqrt[e]*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f
]*Sqrt[d + e*x + f*x^2])])/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{d+ex+fx^2}\left(a+bx+\frac{bfx^2}{e}\right)} \, dx &= -\left((2e)\,\text{Subst}\left(\int \frac{1}{e(be-4af)-(bd-ae)x^2} \, dx, x, \frac{e+2fx}{\sqrt{d+ex+fx^2}}\right)\right)\\ &= -\frac{2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{bd-ae}\sqrt{be-4af}} \end{aligned}$$

**Mathematica [B]** time = 0.388, size = 178, normalized size = 2.17

$$\frac{\sqrt{e}\left(\tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}\left(e^2-4df\right)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)+\tanh^{-1}\left(\frac{\sqrt{b}\left(e^2-4df\right)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)\right)}{\sqrt{bd-ae}\sqrt{be-4af}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a + b*x + (b*f*x^2)/e)),x]
```

```
[Out] (Sqrt[e]*(ArcTanh[(-(Sqrt[b]*(e^2 - 4*d*f)) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e
+ 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])] + ArcTanh[(Sqrt[b]*(
e^2 - 4*d*f) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*
Sqrt[d + x*(e + f*x)])]))/(Sqrt[b*d - a*e]*Sqrt[b*e - 4*a*f])
```

---

**Maple [B]** time = 0.388, size = 491, normalized size = 6.

$$-e\ln\left(\left(\left(-2\frac{ae-bd}{b}+\frac{1}{b}\sqrt{-be\left(4af-be\right)}\left(x-\frac{1}{2bf}\left(-be+\sqrt{-be\left(4af-be\right)}\right)\right)+2\sqrt{-\frac{ae-bd}{b}}\sqrt{\left(x-1/2\frac{-be+\sqrt{-be\left(4a\cdots\right.}}{bf}\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -e/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(
4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*
d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b
*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)
)/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))+e/(-b*e*(4*a*f-b*e))^(1/2)/(
-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b
*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-b*e
*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*
(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))
^(1/2))/b/f))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 2.13968, size = 2226, normalized size = 27.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*log((8*b^2*d^2*e^4 -
 8*a*b*d*e^5 + a^2*e^6 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 -
```

```
8*a*b*d*e + 8*a^2*e^2)*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*
e^5*f + 16*(b^2*d^2*e - 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a
*b*e^4)*f^2)*x^3 + (b^2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^
2*d^2*e^2 - 13*a*b*d*e^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*
f)*x^2 - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^6 - 1
6*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 + 2*a^
2*e^5)*f)*x - 4*(2*b^3*d^2*e^4 - 3*a*b^2*d*e^5 + a^2*b*e^6 - 2*(16*(a*b^2*d
^2 - 3*a^2*b*d*e + 2*a^3*e^2)*f^4 - 4*(b^3*d^2*e - 4*a*b^2*d*e^2 + 3*a^2*b*
e^3)*f^3 - (b^3*d*e^3 - a*b^2*e^4)*f^2)*x^3 + 16*(a^2*b*d^2*e^2 - a^3*d*e^3
)*f^2 - 3*(16*(a*b^2*d^2*e - 3*a^2*b*d*e^2 + 2*a^3*e^3)*f^3 - 4*(b^3*d^2*e^
2 - 4*a*b^2*d*e^3 + 3*a^2*b*e^4)*f^2 - (b^3*d*e^4 - a*b^2*e^5)*f)*x^2 - 4*(
3*a*b^2*d^2*e^3 - 4*a^2*b*d*e^4 + a^3*e^5)*f + (b^3*d*e^5 - a*b^2*e^6 + 32*
(a^2*b*d^2*e - a^3*d*e^2)*f^3 - 40*(a*b^2*d^2*e^2 - 2*a^2*b*d*e^3 + a^3*e^4
)*f^2 + 2*(4*b^3*d^2*e^3 - 11*a*b^2*d*e^4 + 7*a^2*b*e^5)*f)*x)*sqrt(f*x^2 +
 e*x + d)*sqrt(e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f)))/(b^2*f^2*x^4 +
 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b^2*e^2 + 2*a*b*e*f)*x^2)), -sqrt
(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d - a^2*e)*f))*arctan(-1/2*(2*b*d*e^2 - a*e
^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 + (b*e^3 + 4*(b*d*e -
2*a*e^2)*f)*x)*sqrt(f*x^2 + e*x + d)*sqrt(-e/(b^2*d*e - a*b*e^2 - 4*(a*b*d
- a^2*e)*f))/(2*e*f^2*x^3 + 3*e^2*f*x^2 + d*e^2 + (e^3 + 2*d*e*f)*x))]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$e\int \frac{1}{ae\sqrt{d+ex+fx^2}+bex\sqrt{d+ex+fx^2}+bfx^2\sqrt{d+ex+fx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x**2/e)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] e*Integral(1/(a*e*sqrt(d + e*x + f*x**2) + b*e*x*sqrt(d + e*x + f*x**2) + b
*f*x**2*sqrt(d + e*x + f*x**2)), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x+b*f*x^2/e)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

## 3.3 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)} \, dx$

**Optimal**. Leaf size=66

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

```
[Out] (-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

---

**Rubi [A]** time = 0.0790939, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {982, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```


```
[Out] (-2*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```


Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)} \, dx &= -\left((2b)\operatorname{Subst}\left(\int \frac{1}{b\left(b^2-4cd\right)-(ab-bd)x^2} \, dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)\right)\\ &= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}} \end{aligned}$$

**Mathematica [B]** time = 0.225642, size = 161, normalized size = 2.44

$$\frac{\tanh^{-1}\left(\frac{4ac-2cx\sqrt{b^2-4cd}-b\left(\sqrt{b^2-4cd}+b\right)}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)+\tanh^{-1}\left(\frac{-2c\left(2a+x\sqrt{b^2-4cd}\right)-b\sqrt{b^2-4cd}+b^2}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right)}{\sqrt{a-d}\sqrt{b^2-4cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)),x]
```

```
[Out] (ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c
*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])] + ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] -
 2*c*(2*a + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])
/(Sqrt[a - d]*Sqrt[b^2 - 4*c*d])
```

---

**Maple [B]** time = 0.327, size = 307, normalized size = 4.7

$$\ln\left(\left(\left(2a-2d-\sqrt{b^2-4cd}\left(x+\frac{1}{2c}\left(\sqrt{b^2-4cd}+b\right)\right)+2\sqrt{a-d}\sqrt{\left(x+1/2\,\frac{\sqrt{b^2-4cd}+b}{c}\right)^2 c-\sqrt{b^2-4cd}\left(x+1/2\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-
4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-
4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d
)^(1/2)+b)/c))-1/(b^2-4*c*d)^(1/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2
)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1
/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/
(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 1.45844, size = 1755, normalized size = 26.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 -
32*(b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 12
8*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*
c^2)*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^
2 - 2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*
c^2 + 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3
```

```
 + 4*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 +
 4*c*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)
*d + 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a
*b^3*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)
*x^2 + d^2))/sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d), -sqrt(-a*b^2 - 4*c*d^
2 + (b^2 + 4*a*c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2
 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (
b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2
 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3
*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(
b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x))/(a*b^2 + 4*c*d^
2 - (b^2 + 4*a*c)*d)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(bx+cx^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(b*x + c*x**2 + d)), x)
```

---

**Giac [B]** time = 14.6685, size = 620, normalized size = 9.39

$$\frac{\log\left(\left|-3\,ab^2c+4\,a^2c^2+2\,b^2cd-\left(b^2c+4\,ac^2-8\,c^2d+4\,\sqrt{ab^2-b^2d-4\,acd+4\,cd^2}c^{\frac{3}{2}}\right)\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)^2-\sqrt{ab^2 -}\right.\right.}{\sqrt{ab^2-b^2d-4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(-3*a*b^2*c + 4*a^2*c^2 + 2*b^2*c*d - (b^2*c + 4*a*c^2 - 8*c^2*d + 4
*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*c^(3/2))*(sqrt(c)*x - sqrt(c*x^2 +
 b*x + a))^2 - sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^2*sqrt(c) - (b^3*s
qrt(c) + 4*a*b*c^(3/2) - 8*b*c^(3/2)*d + 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4
*c*d^2)*b*c)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))))/sqrt(a*b^2 - b^2*d - 4*a
*c*d + 4*c*d^2) - log(abs(-3*a*b^2*c + 4*a^2*c^2 + 2*b^2*c*d - (b^2*c + 4*a
*c^2 - 8*c^2*d - 4*sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*c^(3/2))*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))^2 + sqrt(a*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)*b^
2*sqrt(c) - (b^3*sqrt(c) + 4*a*b*c^(3/2) - 8*b*c^(3/2)*d - 4*sqrt(a*b^2 - b
^2*d - 4*a*c*d + 4*c*d^2)*b*c)*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))))/sqrt(a
*b^2 - b^2*d - 4*a*c*d + 4*c*d^2)
```

## 3.4 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx$

**Optimal**. Leaf size=129

$$\frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

```
[Out] -(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*x
^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
- 4*c*d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2))
```

---

**Rubi [A]** time = 0.169022, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 12, 982, 208}

$$\frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(bx+cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] -(((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/((a - d)*(b^2 - 4*c*d)*(d + b*x + c*x
^2))) + ((b^2 + 4*c*(a - 2*d))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
- 4*c*d]*Sqrt[a + b*x + c*x^2])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3/2))
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
```

```
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\int -\frac{c^2\left(b^2+4c(a-2d)\right)(a-d)}{2\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)}\,dx}{c^2(a-d)^2\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} - \frac{\left(b^2+4c(a-2d)\right)\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)}\,dx}{2(a-d)\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\left(b\left(b^2+4c(a-2d)\right)\right)\text{Subst}\left(\int \frac{1}{b\left(b^2-4cd\right)-(ab-\cdots}\right.}{(a-d)\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)} + \frac{\left(b^2+4c(a-2d)\right)\tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}$$

**Mathematica [B]** time = 0.909816, size = 296, normalized size = 2.29

$$\frac{1}{2}\left(-\frac{8c(b+2cx)\sqrt{a+x(b+cx)}}{(a-d)\left(4cd-b^2\right)\left(\sqrt{b^2-4cd}-b-2cx\right)\left(\sqrt{b^2-4cd}+b+2cx\right)} - \frac{\left(4c(a-2d)+b^2\right)\tanh^{-1}\left(\frac{4ac-2cx\sqrt{b^2-4cd}-b\left(\sqrt{b^2-4cd}+\cdots\right.}{4c\sqrt{a-d}\sqrt{a+x(b+cx)}}\right.}{(a-d)^{3/2}\left(b^2-4cd\right)^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^2),x]
```

```
[Out] ((-8*c*(b + 2*c*x)*Sqrt[a + x*(b + c*x)])/((a - d)*(-b^2 + 4*c*d)*(-b + Sqr
t[b^2 - 4*c*d] - 2*c*x)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)) - ((b^2 + 4*c*(a -
 2*d))*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x
)/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/((a - d)^(3/2)*(b^2 - 4*c*d)^(3
/2)) - ((b^2 + 4*c*(a - 2*d))*ArcTanh[(b^2 - b*Sqrt[b^2 - 4*c*d] - 2*c*(2*a
 + Sqrt[b^2 - 4*c*d]*x))/(4*c*Sqrt[a - d]*Sqrt[a + x*(b + c*x)])])/((a - d)
^(3/2)*(b^2 - 4*c*d)^(3/2)))/2
```

---

**Maple [B]** time = 0.301, size = 829, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b
^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b
^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*
c*d)^(1/2)+b)/c))+2/(b^2-4*c*d)^(3/2)*c/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)
^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1
/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/(b^2-4*c*d)/(a-d)/(x-1/2/c*(b^2-4*
c*d)^(1/2)+1/2*b/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)
*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2/(b^2-4*c*d)^(1/2)/(a-d)^(3
/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^
(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b
^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-1/(b^2-4*
c*d)/(a-d)/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)*((x+1/2*((b^2-4*c*d)^(1/2)+b
)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2)-1/2/(
b^2-4*c*d)^(1/2)/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c
*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c
*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d)^(
1/2)+b)/c))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\left(cx^2 + bx + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^2), x)
```

---

**Fricas [B]** time = 3.38433, size = 3297, normalized size = 25.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 -
 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d - (b^3 + 4*a*b*c - 8*b*c*d)*x)*log((8*a^2*b
^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(b^2*c^3 + 4*a
*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*c^3*d^2 - 32*
(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)*d^2 + (b^6 +
 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 - 2*(19*b^4*c +
 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2 + 4*a*c^3 - 8
*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4*a*b*c)*d + (
b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 +
 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d + 2*(4*a*b^5 +
 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3*c + 48*a^2*b
*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2 + d^2)) - 4*
(a*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c +
4*a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c +
4*a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c
)*d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a
```

```
*b^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5
+ 16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c
^2)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x), -1/2*(sqrt(-a*b^2 - 4*c*d^2 + (b^2
 + 4*a*c)*d)*(8*c*d^2 - (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c)*d -
 (b^3 + 4*a*b*c - 8*b*c*d)*x)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c
^2*d)*x^2 - (b^2 + 4*a*c)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*
c*d^2 + (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(
a*b^2*c^2 + 4*c^3*d^2 - (b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d
^2 - (b^3*c + 4*a*b*c^2)*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^
2*c + 4*(b^2*c + 2*a*c^2)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(a
*b^3 + 4*b*c*d^2 - (b^3 + 4*a*b*c)*d + 2*(a*b^2*c + 4*c^2*d^2 - (b^2*c + 4*
a*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^2*b^4*d + 16*c^2*d^5 - 8*(b^2*c + 4*
a*c^2)*d^4 + (b^4 + 16*a*b^2*c + 16*a^2*c^2)*d^3 - 2*(a*b^4 + 4*a^2*b^2*c)*
d^2 + (a^2*b^4*c + 16*c^3*d^4 - 8*(b^2*c^2 + 4*a*c^3)*d^3 + (b^4*c + 16*a*b
^2*c^2 + 16*a^2*c^3)*d^2 - 2*(a*b^4*c + 4*a^2*b^2*c^2)*d)*x^2 + (a^2*b^5 +
16*b*c^2*d^4 - 8*(b^3*c + 4*a*b*c^2)*d^3 + (b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 2*(a*b^5 + 4*a^2*b^3*c)*d)*x)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{sage}_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

## 3.5 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}\,dx$

**Optimal**. Leaf size=224

$$-\frac{\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\left(b^2-4cd\right)^{5/2}}+\frac{3(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{4(a-d)^2\left(b^2-4cd\right)^2\left(bx+cx^2+d\right)}$$

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(2*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2)^2) + (3*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*(a
 - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)) - ((3*b^4 + 8*b^2*c*(a - 4*d) +
16*c^2*(3*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(4*(a - d)^(5/2)*(b^2 - 4*c*d)^(5/2))
```

---

**Rubi [A]** time = 0.426872, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1060, 12, 982, 208}

$$-\frac{\left(16c^2\left(3a^2-8ad+8d^2\right)+8b^2c(a-4d)+3b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-d}(b+2cx)}{\sqrt{b^2-4cd}\sqrt{a+bx+cx^2}}\right)}{4(a-d)^{5/2}\left(b^2-4cd\right)^{5/2}}+\frac{3(b+2cx)\left(4c(a-2d)+b^2\right)\sqrt{a+bx+cx^2}}{4(a-d)^2\left(b^2-4cd\right)^2\left(bx+cx^2+d\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(2*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2)^2) + (3*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*(a
 - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)) - ((3*b^4 + 8*b^2*c*(a - 4*d) +
16*c^2*(3*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt[a - d]*(b + 2*c*x))/(Sqrt[b^2
 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(4*(a - d)^(5/2)*(b^2 - 4*c*d)^(5/2))
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
```

```
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{\int \frac{-\frac{1}{2}c^2(a-d)\left(3b^2+12ac-16cd\right)-4bc^3(a-d)x-4c^4(a-d)x}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^2}}{2c^2(a-d)^2\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+cx}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+cx}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+cx}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{2(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^2}+\frac{3\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+cx}}{4(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\right)}$$

**Mathematica [B]** time = 6.37555, size = 1748, normalized size = 7.8

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^3),x]
```

```
[Out] (-2*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b - Sqrt[b^2 - 4*c
*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*
(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (2
*c^2*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(3/2)*(b + Sqrt[b^2 - 4*c*d]
 + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) + (6*c^2*(a + b*x + c*x^2))/((a - d)*(b^
2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (6*c^
2*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sq
rt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a - d]*(
b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) + (3*c*Sqrt[a + b*x + c*x^2]*ArcT
anh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt
[a - d]*Sqrt[a + b*x + c*x^2])])/(2*(a - d)^(3/2)*(b^2 - 4*c*d)^(3/2)*Sqrt[
a + x*(b + c*x)]) + (6*c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sq
rt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x +
 c*x^2])])/(Sqrt[a - d]*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) + (3*c*S
qrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[
b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/(2*(a - d)^(3/2)*
(b^2 - 4*c*d)^(3/2)*Sqrt[a + x*(b + c*x)]) + (4*c^3*Sqrt[a + b*x + c*x^2]*(
-(((2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d]))*Sqrt[
a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt
[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x))) + (4*c*Sqrt[a - d]*(b*
(-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d])) - 2*(4*
a*c^3 - c^2*(-b + Sqrt[b^2 - 4*c*d])*(b + 2*Sqrt[b^2 - 4*c*d])))*ArcTanh[(-
4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + Sqrt[b^2 - 4*c*d]))
*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 + 2*b*c*(-b + Sqrt[
b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + Sqrt[
b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(3/2)*(4*
a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*Sqrt
[a + x*(b + c*x)]) + (4*c^3*Sqrt[a + b*x + c*x^2]*(-(((2*c^2*(b - 2*Sqrt[b^
2 - 4*c*d]) - 2*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2])/((4*a*c
^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(b + Sqrt
[b^2 - 4*c*d] + 2*c*x))) + (4*c*Sqrt[a - d]*(b*(2*c^2*(b - 2*Sqrt[b^2 - 4*c
*d]) + 2*c^2*(b + Sqrt[b^2 - 4*c*d])) - 2*(4*a*c^3 + c^2*(b - 2*Sqrt[b^2 -
4*c*d])*(b + Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]
) - (-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x
 + c*x^2])])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 -
4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4*c*d]) + 4*c*(b + Sqrt[b^2 -
4*c*d])^2))))/((b^2 - 4*c*d)^(3/2)*(4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d])
 + c*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)])
```

---

**Maple [B]** time = 0.242, size = 1884, normalized size = 8.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -1/2/(b^2-4*c*d)^(3/2)/(a-d)/(x-1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)^2*((x-1/2*
(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
```

```
))/c)+a-d)^(1/2)+3/4/(b^2-4*c*d)/(a-d)^2/(x-1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c
)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4
*c*d)^(1/2))/c)+a-d)^(1/2)-3/8/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*ln((2*a-2*d+(b
^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c
)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*b^2+3/2/(b^2-4*c*d)^(3/2)/(
a-d)^(5/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2
*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2
*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*c*
d-1/(b^2-4*c*d)^(3/2)*c/(a-d)^(3/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c
+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+
(b^2-4*c*d)^(1/2))/c))+6*c^2/(b^2-4*c*d)^(5/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2
-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4
*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d
)^(1/2))/(x+1/2*((b^2-4*c*d)^(1/2)+b)/c))+3/(b^2-4*c*d)^2*c/(a-d)/(x-1/2/c*
(b^2-4*c*d)^(1/2)+1/2*b/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d
)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+1/2/(b^2-4*c*d)^(3/2)/(
a-d)/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)^2*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)
^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2)+3/4/(b^2-
4*c*d)/(a-d)^2/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)*((x+1/2*((b^2-4*c*d)^(1/
2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2)+3
/8/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2
-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2
-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*
d)^(1/2)+b)/c))*b^2-3/2/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*ln((2*a-2*d-(b^2-4*c*
d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)
^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/
2))/(x+1/2*((b^2-4*c*d)^(1/2)+b)/c))*c*d+1/(b^2-4*c*d)^(3/2)*c/(a-d)^(3/2)*
ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)
*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)
^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d)^(1/2)+b)/c))-6*c^2/(b^2-4*c*d)
^(5/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/
2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2
)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2)
)/c))+3/(b^2-4*c*d)^2*c/(a-d)/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)*((x+1/2*(
(b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/
c)+a-d)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^3), x)
```

---

**Fricas [B]** time = 19.0583, size = 8058, normalized size = 35.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [1/16*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d^2 -
 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c + 8*
a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3
+ (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^2*c^
2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c^2 -
 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (3*b^
5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(a*b^2 + 4*c*d^2 - (b^2 + 4*a*c)*d)
*log((8*a^2*b^4 + (b^4*c^2 + 24*a*b^2*c^3 + 16*a^2*c^4 + 128*c^4*d^2 - 32*(
b^2*c^3 + 4*a*c^4)*d)*x^4 + 2*(b^5*c + 24*a*b^3*c^2 + 16*a^2*b*c^3 + 128*b*
c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*x^3 + (b^4 + 24*a*b^2*c + 16*a^2*c^2)
*d^2 + (b^6 + 32*a*b^4*c + 48*a^2*b^2*c^2 + 32*(5*b^2*c^2 + 4*a*c^3)*d^2 -
2*(19*b^4*c + 104*a*b^2*c^2 + 48*a^2*c^3)*d)*x^2 - 4*(2*a*b^3 + 2*(b^2*c^2
+ 4*a*c^3 - 8*c^3*d)*x^3 + 3*(b^3*c + 4*a*b*c^2 - 8*b*c^2*d)*x^2 - (b^3 + 4
*a*b*c)*d + (b^4 + 8*a*b^2*c - 2*(5*b^2*c + 4*a*c^2)*d)*x)*sqrt(a*b^2 + 4*c
*d^2 - (b^2 + 4*a*c)*d)*sqrt(c*x^2 + b*x + a) - 8*(a*b^4 + 4*a^2*b^2*c)*d +
 2*(4*a*b^5 + 16*a^2*b^3*c + 16*(b^3*c + 4*a*b*c^2)*d^2 - (3*b^5 + 40*a*b^3
*c + 48*a^2*b*c^2)*d)*x)/(c^2*x^4 + 2*b*c*x^3 + 2*b*d*x + (b^2 + 2*c*d)*x^2
 + d^2)) - 4*(2*a^2*b^5 + 128*b*c^2*d^4 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a
*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 + 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c
^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3 + 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2
)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3
)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3
*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 +
 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*
b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c
^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*
a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*
a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 +
 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 1
44*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b
^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b
^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12
*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*
a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*
c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 -
32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 +
 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a
*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8
*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b
*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2
*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(
a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)
*x), -1/8*((128*c^2*d^4 + (3*b^4*c^2 + 8*a*b^2*c^3 + 48*a^2*c^4 + 128*c^4*d
^2 - 32*(b^2*c^3 + 4*a*c^4)*d)*x^4 - 32*(b^2*c + 4*a*c^2)*d^3 + 2*(3*b^5*c
+ 8*a*b^3*c^2 + 48*a^2*b*c^3 + 128*b*c^3*d^2 - 32*(b^3*c^2 + 4*a*b*c^3)*d)*
x^3 + (3*b^4 + 8*a*b^2*c + 48*a^2*c^2)*d^2 + (3*b^6 + 8*a*b^4*c + 48*a^2*b^
2*c^2 + 256*c^3*d^3 + 64*(b^2*c^2 - 4*a*c^3)*d^2 - 2*(13*b^4*c + 56*a*b^2*c
^2 - 48*a^2*c^3)*d)*x^2 + 2*(128*b*c^2*d^3 - 32*(b^3*c + 4*a*b*c^2)*d^2 + (
3*b^5 + 8*a*b^3*c + 48*a^2*b*c^2)*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*
c)*d)*arctan(-1/2*(2*a*b^2 + (b^2*c + 4*a*c^2 - 8*c^2*d)*x^2 - (b^2 + 4*a*c
)*d + (b^3 + 4*a*b*c - 8*b*c*d)*x)*sqrt(-a*b^2 - 4*c*d^2 + (b^2 + 4*a*c)*d)
*sqrt(c*x^2 + b*x + a)/(a^2*b^3 + 4*a*b*c*d^2 + 2*(a*b^2*c^2 + 4*c^3*d^2 -
(b^2*c^2 + 4*a*c^3)*d)*x^3 + 3*(a*b^3*c + 4*b*c^2*d^2 - (b^3*c + 4*a*b*c^2)
*d)*x^2 - (a*b^3 + 4*a^2*b*c)*d + (a*b^4 + 2*a^2*b^2*c + 4*(b^2*c + 2*a*c^2
)*d^2 - (b^4 + 6*a*b^2*c + 8*a^2*c^2)*d)*x)) + 2*(2*a^2*b^5 + 128*b*c^2*d^4
 - 52*(b^3*c + 4*a*b*c^2)*d^3 - 6*(a*b^4*c^2 + 4*a^2*b^2*c^3 - 32*c^4*d^3 +
 12*(b^2*c^3 + 4*a*c^4)*d^2 - (b^4*c^2 + 16*a*b^2*c^3 + 16*a^2*c^4)*d)*x^3
```

```
+ 5*(b^5 + 16*a*b^3*c + 16*a^2*b*c^2)*d^2 - 9*(a*b^5*c + 4*a^2*b^3*c^2 - 32
*b*c^3*d^3 + 12*(b^3*c^2 + 4*a*b*c^3)*d^2 - (b^5*c + 16*a*b^3*c^2 + 16*a^2*
b*c^3)*d)*x^2 - 7*(a*b^5 + 4*a^2*b^3*c)*d - (3*a*b^6 + 8*a^2*b^4*c - 256*c^
3*d^4 + 8*(b^2*c^2 + 52*a*c^3)*d^3 + 2*(13*b^4*c - 8*a*b^2*c^2 - 80*a^2*c^3
)*d^2 - (3*b^6 + 34*a*b^4*c - 8*a^2*b^2*c^2)*d)*x)*sqrt(c*x^2 + b*x + a))/(
a^3*b^6*d^2 + 64*c^3*d^8 - 48*(b^2*c^2 + 4*a*c^3)*d^7 + 12*(b^4*c + 12*a*b^
2*c^2 + 16*a^2*c^3)*d^6 - (b^6 + 36*a*b^4*c + 144*a^2*b^2*c^2 + 64*a^3*c^3)
*d^5 + 3*(a*b^6 + 12*a^2*b^4*c + 16*a^3*b^2*c^2)*d^4 + (a^3*b^6*c^2 + 64*c^
5*d^6 - 48*(b^2*c^4 + 4*a*c^5)*d^5 + 12*(b^4*c^3 + 12*a*b^2*c^4 + 16*a^2*c^
5)*d^4 - (b^6*c^2 + 36*a*b^4*c^3 + 144*a^2*b^2*c^4 + 64*a^3*c^5)*d^3 + 3*(a
*b^6*c^2 + 12*a^2*b^4*c^3 + 16*a^3*b^2*c^4)*d^2 - 3*(a^2*b^6*c^2 + 4*a^3*b^
4*c^3)*d)*x^4 - 3*(a^2*b^6 + 4*a^3*b^4*c)*d^3 + 2*(a^3*b^7*c + 64*b*c^4*d^6
 - 48*(b^3*c^3 + 4*a*b*c^4)*d^5 + 12*(b^5*c^2 + 12*a*b^3*c^3 + 16*a^2*b*c^4
)*d^4 - (b^7*c + 36*a*b^5*c^2 + 144*a^2*b^3*c^3 + 64*a^3*b*c^4)*d^3 + 3*(a*
b^7*c + 12*a^2*b^5*c^2 + 16*a^3*b^3*c^3)*d^2 - 3*(a^2*b^7*c + 4*a^3*b^5*c^2
)*d)*x^3 + (a^3*b^8 + 128*c^4*d^7 - 32*(b^2*c^3 + 12*a*c^4)*d^6 - 24*(b^4*c
^2 - 4*a*b^2*c^3 - 16*a^2*c^4)*d^5 + 2*(5*b^6*c + 36*a*b^4*c^2 - 48*a^2*b^2
*c^3 - 64*a^3*c^4)*d^4 - (b^8 + 30*a*b^6*c + 72*a^2*b^4*c^2 - 32*a^3*b^2*c^
3)*d^3 + 3*(a*b^8 + 10*a^2*b^6*c + 8*a^3*b^4*c^2)*d^2 - (3*a^2*b^8 + 10*a^3
*b^6*c)*d)*x^2 + 2*(a^3*b^7*d + 64*b*c^3*d^7 - 48*(b^3*c^2 + 4*a*b*c^3)*d^6
 + 12*(b^5*c + 12*a*b^3*c^2 + 16*a^2*b*c^3)*d^5 - (b^7 + 36*a*b^5*c + 144*a
^2*b^3*c^2 + 64*a^3*b*c^3)*d^4 + 3*(a*b^7 + 12*a^2*b^5*c + 16*a^3*b^3*c^2)*
d^3 - 3*(a^2*b^7 + 4*a^3*b^5*c)*d^2)*x)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

## 3.6 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4} dx$

**Optimal**. Leaf size=328

$$-\frac{(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-22d)+15b^4\right)\sqrt{a+bx+cx^2}}{24(a-d)^3\left(b^2-4cd\right)^3\left(bx+cx^2+d\right)}+\frac{\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad\right.\right.}{8(}$$

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(3*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2)^3) + (5*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(12*(
a - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)^2) - ((15*b^4 + 8*b^2*c*(7*a - 2
2*d) + 16*c^2*(15*a^2 - 44*a*d + 44*d^2))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(24*(a - d)^3*(b^2 - 4*c*d)^3*(d + b*x + c*x^2)) + ((b^2 + 4*c*(a - 2*d))
*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2*(5*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt
[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(8*(a - d)
^(7/2)*(b^2 - 4*c*d)^(7/2))
```

---

**Rubi [A]** time = 0.970484, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1060, 12, 982, 208}

$$-\frac{(b+2cx)\left(16c^2\left(15a^2-44ad+44d^2\right)+8b^2c(7a-22d)+15b^4\right)\sqrt{a+bx+cx^2}}{24(a-d)^3\left(b^2-4cd\right)^3\left(bx+cx^2+d\right)}+\frac{\left(4c(a-2d)+b^2\right)\left(16c^2\left(5a^2-8ad\right.\right.}{8(}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] -((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(3*(a - d)*(b^2 - 4*c*d)*(d + b*x + c*
x^2)^3) + (5*(b^2 + 4*c*(a - 2*d))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(12*(
a - d)^2*(b^2 - 4*c*d)^2*(d + b*x + c*x^2)^2) - ((15*b^4 + 8*b^2*c*(7*a - 2
2*d) + 16*c^2*(15*a^2 - 44*a*d + 44*d^2))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(24*(a - d)^3*(b^2 - 4*c*d)^3*(d + b*x + c*x^2)) + ((b^2 + 4*c*(a - 2*d))
*(5*b^4 - 8*b^2*c*(a + 4*d) + 16*c^2*(5*a^2 - 8*a*d + 8*d^2))*ArcTanh[(Sqrt
[a - d]*(b + 2*c*x))/(Sqrt[b^2 - 4*c*d]*Sqrt[a + b*x + c*x^2])])/(8*(a - d)
^(7/2)*(b^2 - 4*c*d)^(7/2))
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^4}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{\int \frac{-\frac{1}{2}c^2(a-d)\left(5b^2+20ac-24cd\right)-8bc^3(a-d)x-8c^4(a\cdots}{\sqrt{a+bx+cx^2}\left(d+bx+cx^2\right)^3}}{3c^2(a-d)^2\left(b^2-4cd\right)}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+\cdots}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+\cdots}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+\cdots}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+\cdots}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\cdots\right.}$$

$$= -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{3(a-d)\left(b^2-4cd\right)\left(d+bx+cx^2\right)^3} + \frac{5\left(b^2+4c(a-2d)\right)(b+2cx)\sqrt{a+bx+\cdots}}{12(a-d)^2\left(b^2-4cd\right)^2\left(d+bx+cx^2\cdots\right.}$$

**Mathematica [B]** time = 6.58038, size = 3386, normalized size = 10.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + b*x + c*x^2)^4),x]
```

```
[Out] (-8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b^2 - 4*c*d)^2*(b - Sqrt[b^2 - 4*c*d
] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) + (8*c^3*(a + b*x + c*x^2))/((a - d)*(b
^2 - 4*c*d)^(5/2)*(b - Sqrt[b^2 - 4*c*d] + 2*c*x)^2*Sqrt[a + x*(b + c*x)])
- (20*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^3*(b - Sqrt[b^2 - 4*c*d
] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (8*c^3*(a + b*x + c*x^2))/(3*(a - d)*(b
^2 - 4*c*d)^2*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)^3*Sqrt[a + x*(b + c*x)]) - (8
*c^3*(a + b*x + c*x^2))/((a - d)*(b^2 - 4*c*d)^(5/2)*(b + Sqrt[b^2 - 4*c*d]
 + 2*c*x)^2*Sqrt[a + x*(b + c*x)]) - (20*c^3*(a + b*x + c*x^2))/((a - d)*(b
^2 - 4*c*d)^3*(b + Sqrt[b^2 - 4*c*d] + 2*c*x)*Sqrt[a + x*(b + c*x)]) - (20*
c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*
Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/(Sqrt[a - d]
*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*c^2*Sqrt[a + b*x + c*x^2]*
ArcTanh[(b^2 - 4*a*c - b*Sqrt[b^2 - 4*c*d] - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*
Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/((a - d)^(3/2)*(b^2 - 4*c*d)^(5/2)*Sqr
t[a + x*(b + c*x)]) - (20*c^3*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b +
 Sqrt[b^2 - 4*c*d]) - 2*c*Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*
x + c*x^2])]])/(Sqrt[a - d]*(b^2 - 4*c*d)^(7/2)*Sqrt[a + x*(b + c*x)]) - (5*
c^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*c - b*(b + Sqrt[b^2 - 4*c*d]) - 2*c*
Sqrt[b^2 - 4*c*d]*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])]])/((a - d)^(3/
2)*(b^2 - 4*c*d)^(5/2)*Sqrt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^
2]*(-(((2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d]))*S
qrt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b +
Sqrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x))) + (4*c*Sqrt[a - d]
*(b*(-2*c^2*(-b + Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + 2*Sqrt[b^2 - 4*c*d])) - 2
*(4*a*c^3 - c^2*(-b + Sqrt[b^2 - 4*c*d])*(b + 2*Sqrt[b^2 - 4*c*d])))*ArcTan
```

```
h[(-4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(-b + Sqrt[b^2 - 4*c*
d]))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 + 2*b*c*(-b + S
qrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 + 8*b*c*(-b + S
qrt[b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(5/2)
*(4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*
Sqrt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^2]*(-((4*c^2*(-b + Sqrt
[b^2 - 4*c*d]) + 2*c^2*(2*b + 3*Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2])/
(2*(4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2
)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x)^2) - (-(((-10*c^3*Sqrt[b^2 - 4*c*d]*(-b
+ Sqrt[b^2 - 4*c*d]) - 2*c^3*(10*b^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^2 - 4*c
*d]))*Sqrt[a + b*x + c*x^2])/((4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c
*(-b + Sqrt[b^2 - 4*c*d])^2)*(-b + Sqrt[b^2 - 4*c*d] - 2*c*x))) + (4*c*Sqrt
[a - d]*(b*(10*c^3*Sqrt[b^2 - 4*c*d]*(-b + Sqrt[b^2 - 4*c*d]) - 2*c^3*(10*b
^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^2 - 4*c*d])) - 2*(-20*a*c^4*Sqrt[b^2 - 4*
c*d] + c^3*(-b + Sqrt[b^2 - 4*c*d])*(10*b^2 - 16*a*c - 24*c*d + 5*b*Sqrt[b^
2 - 4*c*d])))*ArcTanh[(-4*a*c - b*(-b + Sqrt[b^2 - 4*c*d]) - (2*b*c + 2*c*(
-b + Sqrt[b^2 - 4*c*d]))*x)/(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a
*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)*(16*a
*c^2 + 8*b*c*(-b + Sqrt[b^2 - 4*c*d]) + 4*c*(-b + Sqrt[b^2 - 4*c*d])^2)))/(
2*(4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b + Sqrt[b^2 - 4*c*d])^2)
)))/(3*(b^2 - 4*c*d)^2*(4*a*c^2 + 2*b*c*(-b + Sqrt[b^2 - 4*c*d]) + c*(-b +
Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b + c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^2
]*(-(((2*c^2*(b - 2*Sqrt[b^2 - 4*c*d]) - 2*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqr
t[a + b*x + c*x^2])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt
[b^2 - 4*c*d])^2)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x))) + (4*c*Sqrt[a - d]*(b*(
2*c^2*(b - 2*Sqrt[b^2 - 4*c*d]) + 2*c^2*(b + Sqrt[b^2 - 4*c*d])) - 2*(4*a*c
^3 + c^2*(b - 2*Sqrt[b^2 - 4*c*d])*(b + Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a*c
 - b*(b + Sqrt[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/(4
*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4
*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4*c
*d]) + 4*c*(b + Sqrt[b^2 - 4*c*d])^2))))/((b^2 - 4*c*d)^(5/2)*(4*a*c^2 - 2*
b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b +
c*x)]) - (16*c^4*Sqrt[a + b*x + c*x^2]*(-((2*c^2*(2*b - 3*Sqrt[b^2 - 4*c*d]
) - 4*c^2*(b + Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c*x^2])/(2*(4*a*c^2 - 2*b
*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(b + Sqrt[b^2 - 4
*c*d] + 2*c*x)^2) - (-(((10*c^3*Sqrt[b^2 - 4*c*d]*(b + Sqrt[b^2 - 4*c*d]) +
 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5*b*Sqrt[b^2 - 4*c*d]))*Sqrt[a + b*x + c
*x^2])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d]
)^2)*(b + Sqrt[b^2 - 4*c*d] + 2*c*x))) + (4*c*Sqrt[a - d]*(b*(-10*c^3*Sqrt[
b^2 - 4*c*d]*(b + Sqrt[b^2 - 4*c*d]) + 2*c^3*(10*b^2 - 16*a*c - 24*c*d - 5*
b*Sqrt[b^2 - 4*c*d])) - 2*(-20*a*c^4*Sqrt[b^2 - 4*c*d] + c^3*(b + Sqrt[b^2
- 4*c*d])*(10*b^2 - 16*a*c - 24*c*d - 5*b*Sqrt[b^2 - 4*c*d])))*ArcTanh[(4*a
*c - b*(b + Sqrt[b^2 - 4*c*d]) - (-2*b*c + 2*c*(b + Sqrt[b^2 - 4*c*d]))*x)/
(4*c*Sqrt[a - d]*Sqrt[a + b*x + c*x^2])])/((4*a*c^2 - 2*b*c*(b + Sqrt[b^2 -
 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*(16*a*c^2 - 8*b*c*(b + Sqrt[b^2 - 4
*c*d]) + 4*c*(b + Sqrt[b^2 - 4*c*d])^2)))/(2*(4*a*c^2 - 2*b*c*(b + Sqrt[b^2
 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2))))/(3*(b^2 - 4*c*d)^2*(4*a*c^2 -
2*b*c*(b + Sqrt[b^2 - 4*c*d]) + c*(b + Sqrt[b^2 - 4*c*d])^2)*Sqrt[a + x*(b
+ c*x)])
```

---

**Maple [B]** time = 0.259, size = 3695, normalized size = 11.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -5/8/(b^2-4*c*d)^2/(a-d)^3/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)*((x+1/2*((b^
2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+
a-d)^(1/2)*b^2-7/3/(b^2-4*c*d)^2*c/(a-d)^2/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2*b
/c)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c
*d)^(1/2)+b)/c)+a-d)^(1/2)-2/(b^2-4*c*d)^(5/2)*c/(a-d)/(x+1/2/c*(b^2-4*c*d)
^(1/2)+1/2*b/c)^2*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x
+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2)-3/2/(b^2-4*c*d)^(5/2)*c/(a-d)^(5/2
)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/
2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*
d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d)^(1/2)+b)/c))*b^2+6/(b^2-4*c*
d)^(5/2)*c^2/(a-d)^(5/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^
(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^
(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d)^(1/2)
+b)/c))*d+5/4/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(
x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/
c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2
*((b^2-4*c*d)^(1/2)+b)/c))*c*d-5/4/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*
d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*
(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*c*d+2/(b^2-4*c*d)^(5/2)
*c/(a-d)/(x-1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)^2*((x-1/2*(-b+(b^2-4*c*d)^(1/2
))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)-3/(
b^2-4*c*d)^(5/2)*c^2/(a-d)^(3/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2
-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2
-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((b^2-4*c*
d)^(1/2)+b)/c))-20*c^3/(b^2-4*c*d)^(7/2)/(a-d)^(1/2)*ln((2*a-2*d-(b^2-4*c*d
)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^
(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2
))/(x+1/2*((b^2-4*c*d)^(1/2)+b)/c))+5/12/(b^2-4*c*d)^(3/2)/(a-d)^2/(x-1/2/c
*(b^2-4*c*d)^(1/2)+1/2*b/c)^2*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*
c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+5/16/(b^2-4*c*d)^(3/
2)/(a-d)^(7/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/
c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x
-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)
)*b^2-3/4/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*c*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-
1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/
c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/
2*(-b+(b^2-4*c*d)^(1/2))/c))-1/3/(b^2-4*c*d)^2/(a-d)/(x-1/2/c*(b^2-4*c*d)^(
1/2)+1/2*b/c)^3*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-
1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+3/(b^2-4*c*d)^(5/2)*c^2/(a-d)^(3/2
)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1
/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2
-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))-5/12/(b^2-4
*c*d)^(3/2)/(a-d)^2/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)^2*((x+1/2*((b^2-4*c
*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^
(1/2)-5/16/(b^2-4*c*d)^(3/2)/(a-d)^(7/2)*ln((2*a-2*d-(b^2-4*c*d)^(1/2)*(x+1
/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^
2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2))/(x+1/2*((
b^2-4*c*d)^(1/2)+b)/c))*b^2+3/4/(b^2-4*c*d)^(3/2)/(a-d)^(5/2)*c*ln((2*a-2*d
-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+2*(a-d)^(1/2)*((x+1/2*((
b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c
)+a-d)^(1/2))/(x+1/2*((b^2-4*c*d)^(1/2)+b)/c))-1/3/(b^2-4*c*d)^2/(a-d)/(x+1
/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)^3*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2
-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2)+20*c^3/(b^2-4*c*d)
^(7/2)/(a-d)^(1/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/
2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2
)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2)
)/c))-5/8/(b^2-4*c*d)^2/(a-d)^3/(x-1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)*((x-1/2
*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/
```

```
2))/c)+a-d)^(1/2)*b^2-10/(b^2-4*c*d)^3*c^2/(a-d)/(x-1/2/c*(b^2-4*c*d)^(1/2)
+1/2*b/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-
b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)+5/2/(b^2-4*c*d)^2/(a-d)^3/(x+1/2/c*(b^2-
4*c*d)^(1/2)+1/2*b/c)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c)^2*c-(b^2-4*c*d)^(1/2
)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2)*c*d+5/2/(b^2-4*c*d)^2/(a-d)^3/
(x-1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(
b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2)*c*d-7/3/(b^2-4
*c*d)^2*c/(a-d)^2/(x-1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)*((x-1/2*(-b+(b^2-4*c*
d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1
/2)+3/2/(b^2-4*c*d)^(5/2)*c/(a-d)^(5/2)*ln((2*a-2*d+(b^2-4*c*d)^(1/2)*(x-1/
2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)
^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+a-d)^(1/2))/(x-1/2*
(-b+(b^2-4*c*d)^(1/2))/c))*b^2-6/(b^2-4*c*d)^(5/2)*c^2/(a-d)^(5/2)*ln((2*a-
2*d+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c)+2*(a-d)^(1/2)*((x-1/
2*(-b+(b^2-4*c*d)^(1/2))/c)^2*c+(b^2-4*c*d)^(1/2)*(x-1/2*(-b+(b^2-4*c*d)^(1
/2))/c)+a-d)^(1/2))/(x-1/2*(-b+(b^2-4*c*d)^(1/2))/c))*d-10/(b^2-4*c*d)^3*c^
2/(a-d)/(x+1/2/c*(b^2-4*c*d)^(1/2)+1/2*b/c)*((x+1/2*((b^2-4*c*d)^(1/2)+b)/c
)^2*c-(b^2-4*c*d)^(1/2)*(x+1/2*((b^2-4*c*d)^(1/2)+b)/c)+a-d)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(cx^2+bx+d\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(c*x^2 + b*x + d)^4), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+d)**4/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+d)^4/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

## 3.7 $\int \frac{1}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$

**Optimal**. Leaf size=162

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

```
[Out] -((b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e +
 b*e*x + b*f*x^2))) - ((8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]
*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(e^(3/2)*
(b*d - a*e)^(3/2)*(b*e - 4*a*f)^(3/2))
```

---

**Rubi [A]** time = 0.305681, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {974, 12, 982, 208}

$$-\frac{\left(8aef-b\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{e^{3/2}(bd-ae)^{3/2}(be-4af)^{3/2}}-\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```


```
[Out] -((b*(e + 2*f*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d - a*e)*(b*e - 4*a*f)*(a*e +
 b*e*x + b*f*x^2))) - ((8*a*e*f - b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]
*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e - 4*a*f]*Sqrt[d + e*x + f*x^2])])/(e^(3/2)*
(b*d - a*e)^(3/2)*(b*e - 4*a*f)^(3/2))
```


Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx = -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{\int \frac{b(bd-ae)f^2\left(8aef-b\left(e^2+4df\right)\right)}{2\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)}\,dx}{be(bd-ae)^2f^2(be-4af)}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} + \frac{\left(8aef-b\left(e^2+4df\right)\right)\int \frac{\cdots}{\sqrt{d+ex+\cdots}}}{2e(bd-ae)(be-\cdots}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\left(8aef-b\left(e^2+4df\right)\right)\operatorname{Subst}\left(\int\cdots\right.}{(bd-\cdots}$$

$$= -\frac{b(e+2fx)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} - \frac{\left(8aef-b\left(e^2+4df\right)\right)\tanh^{-1}\left(\cdots\right.}{e^{3/2}(bd-ae)^{3/2}(be-\cdots}$$

**Mathematica [B]** time = 2.12731, size = 490, normalized size = 3.02

$$\frac{2f\left(-\frac{\left(b\left(4df+e^2\right)-8aef\right)\tanh^{-1}\left(\frac{-\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}\left(e^2-4df\right)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}(be-4af)^{3/2}} - \frac{e\tanh^{-1}\left(\frac{\sqrt{b}\left(e^2-4df\right)-\sqrt{e}(e+2fx)\sqrt{be-4af}}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{4f(bd-ae)^{3/2}\sqrt{be-4af}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}(e+2fx)\sqrt{be-4af}-\sqrt{b}\left(e^2-4df\right)}{4f\sqrt{bd-ae}\sqrt{d+x(e+fx)}}\right)}{\sqrt{bd-ae}(be-4af)^{3/2}} - \cdots\right.}{e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

```
[Out] (2*f*(-((Sqrt[b]*Sqrt[e]*Sqrt[d + x*(e + f*x)])/((b*d - a*e)*(b*e - 4*a*f)*
(-(Sqrt[e]*Sqrt[b*e - 4*a*f]) + Sqrt[b]*(e + 2*f*x)))) - (Sqrt[b]*Sqrt[e]*S
qrt[d + x*(e + f*x)])/((b*d - a*e)*(b*e - 4*a*f)*(Sqrt[e]*Sqrt[b*e - 4*a*f]
 + Sqrt[b]*(e + 2*f*x))) - ((-8*a*e*f + b*(e^2 + 4*d*f))*ArcTanh[(-(Sqrt[b]
*(e^2 - 4*d*f)) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(4*Sqrt[b*d - a*e]
*f*Sqrt[d + x*(e + f*x)])]])/(4*(b*d - a*e)^(3/2)*f*(b*e - 4*a*f)^(3/2)) - (
e*ArcTanh[(Sqrt[b]*(e^2 - 4*d*f) - Sqrt[e]*Sqrt[b*e - 4*a*f]*(e + 2*f*x))/(
4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])]])/(4*(b*d - a*e)^(3/2)*f*Sqrt[b*
e - 4*a*f]) + ArcTanh[(-(Sqrt[b]*(e^2 - 4*d*f)) + Sqrt[e]*Sqrt[b*e - 4*a*f]
*(e + 2*f*x))/(4*Sqrt[b*d - a*e]*f*Sqrt[d + x*(e + f*x)])]]/(Sqrt[b*d - a*e]
*(b*e - 4*a*f)^(3/2))))/e^(3/2)
```

---

**Maple [B]** time = 0.352, size = 1377, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] -2/e/(4*a*f-b*e)*f/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*
e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*
f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a
*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))-1/e/(4*a*f-b
*e)/(a*e-b*d)/(x+1/2*e/f-1/2/b/f*(-b*e*(4*a*f-b*e))^(1/2))*((x-1/2*(-b*e+(-
b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-
b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)+1/2/b/e/(4*a*f-b*e)*(-b*e*(
4*a*f-b*e))^(1/2)/(a*e-b*d)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b+(-b*e*(
4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*
d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f+(-b*e*(4*a*f-b
*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)
)/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))+2/e/(4*a*f-b*e)*f/(-b*e*(4*a
*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(
1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((
x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x
+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-
b*e*(4*a*f-b*e))^(1/2))/b/f))-1/e/(4*a*f-b*e)/(a*e-b*d)/(x+1/2*e/f+1/2/b/f*
(-b*e*(4*a*f-b*e))^(1/2))*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(
-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b
*d)/b)^(1/2)-1/2/b/e/(4*a*f-b*e)*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)/(-(a*e-
b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b
*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*
f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f
-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2)
)/b/f))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(bfx^2 + bex + ae\right)^2 \sqrt{fx^2 + ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

---

**Fricas [B]** time = 8.12704, size = 4059, normalized size = 25.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(sqrt(b^2*d*e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*
e^2*f + 4*(b^2*d - 2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3
```

```
 + 4*(b^2*d*e - 2*a*b*e^2)*f)*x)*log((8*b^2*d^2*e^4 - 8*a*b*d*e^5 + a^2*e^6
 + 16*a^2*d^2*e^2*f^2 + (b^2*e^4*f^2 + 16*(b^2*d^2 - 8*a*b*d*e + 8*a^2*e^2)
*f^4 + 8*(3*b^2*d*e^2 - 4*a*b*e^3)*f^3)*x^4 + 2*(b^2*e^5*f + 16*(b^2*d^2*e
- 8*a*b*d*e^2 + 8*a^2*e^3)*f^3 + 8*(3*b^2*d*e^3 - 4*a*b*e^4)*f^2)*x^3 + (b^
2*e^6 - 32*(3*a*b*d^2*e - 4*a^2*d*e^2)*f^3 + 16*(3*b^2*d^2*e^2 - 13*a*b*d*e
^3 + 10*a^2*e^4)*f^2 + 2*(16*b^2*d*e^4 - 19*a*b*e^5)*f)*x^2 - 4*sqrt(b^2*d*
e^2 - a*b*e^3 - 4*(a*b*d*e - a^2*e^2)*f)*(2*b*d*e^3 - a*e^4 - 4*a*d*e^2*f +
 2*(b*e^2*f^2 + 4*(b*d - 2*a*e)*f^3)*x^3 + 3*(b*e^3*f + 4*(b*d*e - 2*a*e^2)
*f^2)*x^2 + (b*e^4 - 8*a*d*e*f^2 + 2*(4*b*d*e^2 - 5*a*e^3)*f)*x)*sqrt(f*x^2
 + e*x + d) - 8*(4*a*b*d^2*e^3 - 3*a^2*d*e^4)*f + 2*(4*b^2*d*e^5 - 3*a*b*e^
6 - 16*(3*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^2 + 8*(2*b^2*d^2*e^3 - 5*a*b*d*e^4 +
 2*a^2*e^5)*f)*x)/(b^2*f^2*x^4 + 2*b^2*e*f*x^3 + 2*a*b*e^2*x + a^2*e^2 + (b
^2*e^2 + 2*a*b*e*f)*x^2)) + 4*(b^3*d*e^3 - a*b^2*e^4 - 4*(a*b^2*d*e^2 - a^2
*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d*e^2 - a*b^2*e^3)*f)*x
)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d*e^6 + a^3*b^2*e^7 + 1
6*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (16*(a^2*b^3*d^2*e^2 -
2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 - 2*a^2*b^3*d*e^4 + a^3
*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^3*e^6)*f)*x^2 - 8*(a^2
*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*d^2*e^5 - 2*a*b^4*d*e^
6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e^4 + a^4*b*e^5)*f^2 -
8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*x), 1/2*(sqrt(-b^2*d*e
^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)*f)*(a*b*e^3 + (b^2*e^2*f + 4*(b^2*d -
2*a*b*e)*f^2)*x^2 + 4*(a*b*d*e - 2*a^2*e^2)*f + (b^2*e^3 + 4*(b^2*d*e - 2*a
*b*e^2)*f)*x)*arctan(-1/2*sqrt(-b^2*d*e^2 + a*b*e^3 + 4*(a*b*d*e - a^2*e^2)
*f)*(2*b*d*e^2 - a*e^3 - 4*a*d*e*f + (b*e^2*f + 4*(b*d - 2*a*e)*f^2)*x^2 +
(b*e^3 + 4*(b*d*e - 2*a*e^2)*f)*x)*sqrt(f*x^2 + e*x + d)/(b^2*d^2*e^3 - a*b
*d*e^4 - 2*(4*(a*b*d*e - a^2*e^2)*f^3 - (b^2*d*e^2 - a*b*e^3)*f^2)*x^3 - 3*
(4*(a*b*d*e^2 - a^2*e^3)*f^2 - (b^2*d*e^3 - a*b*e^4)*f)*x^2 - 4*(a*b*d^2*e^
2 - a^2*d*e^3)*f + (b^2*d*e^4 - a*b*e^5 - 8*(a*b*d^2*e - a^2*d*e^2)*f^2 + 2
*(b^2*d^2*e^2 - 3*a*b*d*e^3 + 2*a^2*e^4)*f)*x)) - 2*(b^3*d*e^3 - a*b^2*e^4
- 4*(a*b^2*d*e^2 - a^2*b*e^3)*f - 2*(4*(a*b^2*d*e - a^2*b*e^2)*f^2 - (b^3*d
*e^2 - a*b^2*e^3)*f)*x)*sqrt(f*x^2 + e*x + d))/(a*b^4*d^2*e^5 - 2*a^2*b^3*d
*e^6 + a^3*b^2*e^7 + 16*(a^3*b^2*d^2*e^3 - 2*a^4*b*d*e^4 + a^5*e^5)*f^2 + (
16*(a^2*b^3*d^2*e^2 - 2*a^3*b^2*d*e^3 + a^4*b*e^4)*f^3 - 8*(a*b^4*d^2*e^3 -
 2*a^2*b^3*d*e^4 + a^3*b^2*e^5)*f^2 + (b^5*d^2*e^4 - 2*a*b^4*d*e^5 + a^2*b^
3*e^6)*f)*x^2 - 8*(a^2*b^3*d^2*e^4 - 2*a^3*b^2*d*e^5 + a^4*b*e^6)*f + (b^5*
d^2*e^5 - 2*a*b^4*d*e^6 + a^2*b^3*e^7 + 16*(a^2*b^3*d^2*e^3 - 2*a^3*b^2*d*e
^4 + a^4*b*e^5)*f^2 - 8*(a*b^4*d^2*e^4 - 2*a^2*b^3*d*e^5 + a^3*b^2*e^6)*f)*
x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + e*x + f*x**2)*(a*e + b*e*x + b*f*x**2)**2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.8 $\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

**Optimal**. Leaf size=28

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

```
[Out] ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]
```

---

**Rubi [A]** time = 0.0172824, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {982, 204}

$$\frac{\tan^{-1}\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])]/Sqrt[3]
```

Rule 982

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)
*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0
]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\left(4\,\text{Subst}\left(\int \frac{1}{-24-2x^2}\,dx, x, \frac{2+2x}{\sqrt{5+2x+x^2}}\right)\right)\\ &= \frac{\tan^{-1}\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)}{\sqrt{3}}\end{aligned}$$

**Mathematica [C]** time = 0.0677789, size = 84, normalized size = 3.

$$-\frac{i\left(\tanh^{-1}\left(\frac{-i\sqrt{3}x-i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)-\tanh^{-1}\left(\frac{i\sqrt{3}x+i\sqrt{3}+4}{\sqrt{x^2+2x+5}}\right)\right)}{2\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

```
[Out] ((-I/2)*(ArcTanh[(4 - I*Sqrt[3] - I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]] - ArcTa
nh[(4 + I*Sqrt[3] + I*Sqrt[3]*x)/Sqrt[5 + 2*x + x^2]]))/Sqrt[3]
```

---

**Maple [A]** time = 0.048, size = 27, normalized size = 1.

$$\frac{\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}\,(2\,x+2)}{6}\frac{1}{\sqrt{x^2+2\,x+5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(2*x+2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+2\,x+5}\left(x^2+2\,x+4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

---

**Fricas [A]** time = 1.03401, size = 123, normalized size = 4.39

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\sqrt{x^2+2\,x+5}(x+1)-\frac{1}{3}\sqrt{3}\left(x^2+2\,x+4\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)*(x + 1) - 1/3*sqrt(3)*(x
^2 + 2*x + 4))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(x^2+2x+4\right)\sqrt{x^2+2x+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

```
[Out] Integral(1/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

---

**Giac [B]** time = 1.14215, size = 70, normalized size = 2.5

$$-\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}+2\right)\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2x+5}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + 1/3*sqrt(
3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5)))
```

## 3.9 $\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q\,dx$

**Optimal**. Leaf size=136

$$-\frac{2^{q+1}\left(-\frac{-\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q-1}\left(2a+2cx^2+ex\right)^{p+q+1}\,{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}}\right)}{(p+q+1)\sqrt{e^2-16ac}}$$

```
[Out] -((2^(1 + q)*(-((e - Sqrt[-16*a*c + e^2] + 4*c*x)/Sqrt[-16*a*c + e^2]))^(-1
 - p - q)*(2*a + e*x + 2*c*x^2)^(1 + p + q)*Hypergeometric2F1[-p - q, 1 + p
 + q, 2 + p + q, (e + Sqrt[-16*a*c + e^2] + 4*c*x)/(2*Sqrt[-16*a*c + e^2])]
)/(Sqrt[-16*a*c + e^2]*(1 + p + q)))
```

---

**Rubi [A]** time = 0.102988, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {967, 624}

$$-\frac{2^{q+1}\left(-\frac{-\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q-1}\left(2a+2cx^2+ex\right)^{p+q+1}\,{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{e+4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}}\right)}{(p+q+1)\sqrt{e^2-16ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

```
[Out] -((2^(1 + q)*(-((e - Sqrt[-16*a*c + e^2] + 4*c*x)/Sqrt[-16*a*c + e^2]))^(-1
 - p - q)*(2*a + e*x + 2*c*x^2)^(1 + p + q)*Hypergeometric2F1[-p - q, 1 + p
 + q, 2 + p + q, (e + Sqrt[-16*a*c + e^2] + 4*c*x)/(2*Sqrt[-16*a*c + e^2])]
)/(Sqrt[-16*a*c + e^2]*(1 + p + q)))
```

Rule 967

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.)*((d_) + (e_.)*(x_) + (f_.)*(x_
)^2)^(q_.), x_Symbol] :> Dist[(c/f)^p, Int[(d + e*x + f*x^2)^(p + q), x], x
] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a
*e, 0] && (IntegerQ[p] || GtQ[c/f, 0]) && ( !IntegerQ[q] || LeafCount[d + e
*x + f*x^2] <= LeafCount[a + b*x + c*x^2])
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[4*p]
```

Rubi steps

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p \left(2a+ex+2cx^2\right)^q\,dx = 2^{-p}\int \left(2a+ex+2cx^2\right)^{p+q}\,dx$$

$$= -\frac{2^{1+q}\left(-\frac{e-\sqrt{-16ac+e^2}+4cx}{\sqrt{-16ac+e^2}}\right)^{-1-p-q}\left(2a+ex+2cx^2\right)^{1+p+q}\,{}_2F_1\left(-p-q,1+p+q;2\right.}{\sqrt{-16ac+e^2}(1+p+q)}$$

**Mathematica [A]** time = 0.138064, size = 142, normalized size = 1.04

$$\frac{2^{q-2}\left(-\sqrt{e^2-16ac}+4cx+e\right)\left(\frac{\sqrt{e^2-16ac}+4cx+e}{\sqrt{e^2-16ac}}\right)^{-p-q}(2a+x(2cx+e))^{p+q}\,{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{-e-4cx+\sqrt{e^2-16ac}}{2\sqrt{e^2-16ac}}\right)}{c(p+q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (e*x)/2 + c*x^2)^p*(2*a + e*x + 2*c*x^2)^q,x]
```

```
[Out] (2^(-2 + q)*(e - Sqrt[-16*a*c + e^2] + 4*c*x)*((e + Sqrt[-16*a*c + e^2] + 4
*c*x)/Sqrt[-16*a*c + e^2])^(-p - q)*(2*a + x*(e + 2*c*x))^(p + q)*Hypergeom
etric2F1[-p - q, 1 + p + q, 2 + p + q, (-e + Sqrt[-16*a*c + e^2] - 4*c*x)/(
2*Sqrt[-16*a*c + e^2])])/(c*(1 + p + q))
```

---

**Maple [F]** time = 2.905, size = 0, normalized size = 0.

$$\int \left(a+\frac{ex}{2}+cx^2\right)^p\left(2\,cx^2+ex+2\,a\right)^q\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

```
[Out] int((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(2\,cx^2+ex+2\,a\right)^q\left(cx^2+\frac{1}{2}\,ex+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(2\,cx^2+ex+2\,a\right)^q\left(cx^2+\frac{1}{2}\,ex+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="fricas")
```

```
[Out] integral((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x**2)**p*(2*c*x**2+e*x+2*a)**q,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(2\,cx^2 + ex + 2\,a\right)^q \left(cx^2 + \frac{1}{2}\,ex + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+1/2*e*x+c*x^2)^p*(2*c*x^2+e*x+2*a)^q,x, algorithm="giac")
```

```
[Out] integrate((2*c*x^2 + e*x + 2*a)^q*(c*x^2 + 1/2*e*x + a)^p, x)
```

# 3.10 $\int \left(a+\frac{cex}{f}+cx^2\right)^p \left(\frac{af}{c}+ex+fx^2\right)^q \, dx$

**Optimal**. Leaf size=200

$$-\frac{\sqrt{c}2^{p+q+1}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{q+1}\left(-\frac{\sqrt{c}\left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+e+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-p-q-1}{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{\sqrt{c}\left(e+2fx\right.}{2\sqrt{c}}\right.}{(p+q+1)\sqrt{ce^2-4af^2}}$$

```
[Out] -((2^(1 + p + q)*Sqrt[c]*(-((Sqrt[c]*(e - Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2
*f*x))/Sqrt[c*e^2 - 4*a*f^2]))^(-1 - p - q)*(a + (c*e*x)/f + c*x^2)^p*((a*f
)/c + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q,
(Sqrt[c]*(e + Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f
^2])])/(Sqrt[c*e^2 - 4*a*f^2]*(1 + p + q)))
```

---

**Rubi [A]** time = 0.134029, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {968, 624}

$$-\frac{\sqrt{c}2^{p+q+1}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{q+1}\left(-\frac{\sqrt{c}\left(-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+e+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-p-q-1}{}_2F_1\left(-p-q,p+q+1;p+q+2;\frac{\sqrt{c}\left(e+2fx\right.}{2\sqrt{c}}\right.}{(p+q+1)\sqrt{ce^2-4af^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

```
[Out] -((2^(1 + p + q)*Sqrt[c]*(-((Sqrt[c]*(e - Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2
*f*x))/Sqrt[c*e^2 - 4*a*f^2]))^(-1 - p - q)*(a + (c*e*x)/f + c*x^2)^p*((a*f
)/c + e*x + f*x^2)^(1 + q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q,
(Sqrt[c]*(e + Sqrt[c*e^2 - 4*a*f^2]/Sqrt[c] + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f
^2])])/(Sqrt[c*e^2 - 4*a*f^2]*(1 + p + q)))
```

Rule 968

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x + c*x^2)^FracPart[p])/(
d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]), Int[(d + e*x + f*x^2)^(p + q),
 x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b
*d - a*e, 0] && !IntegerQ[p] && !IntegerQ[q] && !GtQ[c/f, 0]
```

Rule 624

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, -Simp[((a + b*x + c*x^2)^(p + 1)*Hypergeometric2F1[-p, p + 1
, p + 2, (b + q + 2*c*x)/(2*q)])/(q*(p + 1)*((q - b - 2*c*x)/(2*q))^(p + 1)
), x]] /; FreeQ[{a, b, c, p}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[4*p]
```

Rubi steps

$$\int\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^q dx=\left(\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{-p}\right)\int\left(\frac{af}{c}+ex+fx^2\right)^{p+q} dx$$

$$=-\frac{2^{1+p+q}\sqrt{c}\left(-\frac{\sqrt{c}\left(e-\frac{\sqrt{ce^2-4af^2}}{\sqrt{c}}+2fx\right)}{\sqrt{ce^2-4af^2}}\right)^{-1-p-q}\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^{1+q} 2}{\sqrt{ce^2-4af^2}(1+p+q)}$$

**Mathematica [A]** time = 0.238339, size = 172, normalized size = 0.86

$$\frac{2^{p+q-1}\left(\sqrt{c}(e+2fx)-\sqrt{ce^2-4af^2}\right)\left(a+\frac{cx(e+fx)}{f}\right)^p\left(\frac{af}{c}+x(e+fx)\right)^q\left(\frac{\sqrt{c}(e+2fx)}{\sqrt{ce^2-4af^2}}+1\right)^{-p-q}{}_2F_1\left(-p-q,p+q+1;p+q+2\right.}{\sqrt{c}f(p+q+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (c*e*x)/f + c*x^2)^p*((a*f)/c + e*x + f*x^2)^q,x]
```

```
[Out] (2^(-1 + p + q)*((a*f)/c + x*(e + f*x))^q*(a + (c*x*(e + f*x))/f)^p*(-Sqrt[
c*e^2 - 4*a*f^2] + Sqrt[c]*(e + 2*f*x))*(1 + (Sqrt[c]*(e + 2*f*x))/Sqrt[c*e
^2 - 4*a*f^2])^(-p - q)*Hypergeometric2F1[-p - q, 1 + p + q, 2 + p + q, 1/2
 - (Sqrt[c]*(e + 2*f*x))/(2*Sqrt[c*e^2 - 4*a*f^2])])/(Sqrt[c]*f*(1 + p + q)
)
```

---

**Maple [F]** time = 3.482, size = 0, normalized size = 0.

$$\int\left(a+\frac{cex}{f}+cx^2\right)^p\left(\frac{af}{c}+ex+fx^2\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

```
[Out] int((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(cx^2+\frac{cex}{f}+a\right)^p\left(fx^2+ex+\frac{af}{c}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\frac{cfx^2+cex+af}{c}\right)^q\left(\frac{cfx^2+cex+af}{f}\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="fricas")
```

```
[Out] integral((((c*f*x^2 + c*e*x + a*f)/c)^q*((c*f*x^2 + c*e*x + a*f)/f)^p, x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x**2)**p*(a*f/c+e*x+f*x**2)**q,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + \frac{cex}{f} + a\right)^p \left(fx^2 + ex + \frac{af}{c}\right)^q \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c*e*x/f+c*x^2)^p*(a*f/c+e*x+f*x^2)^q,x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + c*e*x/f + a)^p*(f*x^2 + e*x + a*f/c)^q, x)
```

## 3.11 $\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} \, dx$

**Optimal**. Leaf size=48

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1}+\frac{\sqrt{x^2+2x+1}\sinh^{-1}(x)}{x+1}$$

```
[Out] (Sqrt[1 + x^2]*Sqrt[1 + 2*x + x^2])/(1 + x) + (Sqrt[1 + 2*x + x^2]*ArcSinh[
x])/(1 + x)
```

---

**Rubi [A]** time = 0.0153268, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {970, 641, 215}

$$\frac{\sqrt{x^2+1}\sqrt{x^2+2x+1}}{x+1}+\frac{\sqrt{x^2+2x+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[1 + x^2]*Sqrt[1 + 2*x + x^2])/(1 + x) + (Sqrt[1 + 2*x + x^2]*ArcSinh[
x])/(1 + x)
```

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
 b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} \, dx &= \frac{\sqrt{1+2x+x^2}\int \frac{2+2x}{\sqrt{1+x^2}} \, dx}{2+2x} \\
&= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x}+\frac{\left(2\sqrt{1+2x+x^2}\right)\int \frac{1}{\sqrt{1+x^2}} \, dx}{2+2x} \\
&= \frac{\sqrt{1+x^2}\sqrt{1+2x+x^2}}{1+x}+\frac{\sqrt{1+2x+x^2}\sinh^{-1}(x)}{1+x}
\end{aligned}$$

**Mathematica [A]** time = 0.0176484, size = 27, normalized size = 0.56

$$\frac{\sqrt{(x+1)^2}\left(\sqrt{x^2+1}+\sinh^{-1}(x)\right)}{x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x + x^2]/Sqrt[1 + x^2],x]
```

```
[Out] (Sqrt[(1 + x)^2]*(Sqrt[1 + x^2] + ArcSinh[x]))/(1 + x)
```

---

**Maple [C]** time = 0.168, size = 16, normalized size = 0.3

$$\mathrm{csgn}\,(1+x)\left(\mathrm{Arcsinh}\,(x)+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x)
```

```
[Out] csgn(1+x)*(arcsinh(x)+(x^2+1)^(1/2))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((x + 1)^2)/sqrt(x^2 + 1), x)
```

---

**Fricas [A]** time = 0.866251, size = 55, normalized size = 1.15

$$\sqrt{x^2+1}-\log\left(-x+\sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(x^2 + 1) - log(-x + sqrt(x^2 + 1))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(x+1)^2}}{\sqrt{x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)**2)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt((x + 1)**2)/sqrt(x**2 + 1), x)
```

---

**Giac [A]** time = 1.18228, size = 66, normalized size = 1.38

$$-\left(\sqrt{2}-\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}\left(x+1\right)-\log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}\left(x+1\right)+\sqrt{x^2+1}\operatorname{sgn}\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+x)^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x +
1) + sqrt(x^2 + 1)*sgn(x + 1)
```

## 3.12 $\int \frac{1}{\left(-1+x^2\right)^2\sqrt{-1+x+x^2}}\,dx$

**Optimal**. Leaf size=70

$$\frac{\sqrt{x^2+x-1}}{2\left(1-x^2\right)}-\frac{1}{8}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

```
[Out] Sqrt[-1 + x + x^2]/(2*(1 - x^2)) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/8
 - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8
```

---

**Rubi [A]** time = 0.0514388, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {976, 1033, 724, 206, 204}

$$\frac{\sqrt{x^2+x-1}}{2\left(1-x^2\right)}-\frac{1}{8}\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] Sqrt[-1 + x + x^2]/(2*(1 - x^2)) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])]/8
 - (5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8
```

Rule 976

```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[((2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p +
1)*(d + e*x + f*x^2)^(q + 1))/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)),
 x] - Dist[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)), Int[(a + c*x^2)^
(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - (-(a*e))*(c*e))*(p +
1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
+ q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*(-(c*e*(2*p
 + q + 4))))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
/; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ
[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[
q, 0]
```

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q
), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q
), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f
, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(-1+x^2\right)^2\sqrt{-1+x+x^2}}\,dx &= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)}-\frac{1}{4}\int \frac{3+2x}{\left(-1+x^2\right)\sqrt{-1+x+x^2}}\,dx \\
&= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)}+\frac{1}{8}\int \frac{1}{(1+x)\sqrt{-1+x+x^2}}\,dx-\frac{5}{8}\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}}\,dx \\
&= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)}-\frac{1}{4}\text{Subst}\left(\int \frac{1}{-4-x^2}\,dx, x, \frac{-3-x}{\sqrt{-1+x+x^2}}\right)+\frac{5}{4}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x\right. \\
&= \frac{\sqrt{-1+x+x^2}}{2\left(1-x^2\right)}-\frac{1}{8}\tan^{-1}\left(\frac{3+x}{2\sqrt{-1+x+x^2}}\right)-\frac{5}{8}\tanh^{-1}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0834212, size = 66, normalized size = 0.94

$$\frac{1}{8}\left(-\frac{4\sqrt{x^2+x-1}}{x^2-1}-\tan^{-1}\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)-5\tanh^{-1}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((-1 + x^2)^2*Sqrt[-1 + x + x^2]),x]
```

```
[Out] ((-4*Sqrt[-1 + x + x^2])/(-1 + x^2) - ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])
] - 5*ArcTanh[(1 - 3*x)/(2*Sqrt[-1 + x + x^2])])/8
```

---

**Maple [A]** time = 0.059, size = 84, normalized size = 1.2

$$\frac{1}{8}\arctan\left(\frac{-3-x}{2}\frac{1}{\sqrt{(1+x)^2-2-x}}\right)-\frac{1}{-4+4x}\sqrt{(-1+x)^2-2+3x}+\frac{5}{8}\text{Artanh}\left(\frac{3x-1}{2}\frac{1}{\sqrt{(-1+x)^2-2+3x}}\right)+\frac{1}{4+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2-1)^2/(x^2+x-1)^(1/2),x)
```

```
[Out] 1/8*arctan(1/2*(-3-x)/((1+x)^2-2-x)^(1/2))-1/4/(-1+x)*((-1+x)^2-2+3*x)^(1/2
)+5/8*arctanh(1/2*(3*x-1)/((-1+x)^2-2+3*x)^(1/2))+1/4/(1+x)*((1+x)^2-2-x)^(
1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2+x-1}\left(x^2-1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(x^2 + x - 1)*(x^2 - 1)^2), x)
```

---

**Fricas [A]** time = 0.808353, size = 235, normalized size = 3.36

$$\frac{2\left(x^2-1\right)\arctan\left(-x+\sqrt{x^2+x-1}-1\right)+5\left(x^2-1\right)\log\left(-x+\sqrt{x^2+x-1}+2\right)-5\left(x^2-1\right)\log\left(-x+\sqrt{x^2+x-1}\right)}{8\left(x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(2*(x^2 - 1)*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5*(x^2 - 1)*log(-x +
sqrt(x^2 + x - 1) + 2) - 5*(x^2 - 1)*log(-x + sqrt(x^2 + x - 1)) - 4*sqrt(x
^2 + x - 1))/(x^2 - 1)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x-1)^2(x+1)^2\sqrt{x^2+x-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**2-1)**2/(x**2+x-1)**(1/2),x)
```

```
[Out] Integral(1/((x - 1)**2*(x + 1)**2*sqrt(x**2 + x - 1)), x)
```

---

**Giac [B]** time = 1.16836, size = 193, normalized size = 2.76

$$\frac{2\left(x-\sqrt{x^2+x-1}\right)^3+3\left(x-\sqrt{x^2+x-1}\right)^2-x+\sqrt{x^2+x-1}-1}{2\left(\left(x-\sqrt{x^2+x-1}\right)^4-2\left(x-\sqrt{x^2+x-1}\right)^2-4x+4\sqrt{x^2+x-1}\right)}+\frac{1}{4}\arctan\left(-x+\sqrt{x^2+x-1}-1\right)+\frac{5}{8}\log\left(\left|-x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2-1)^2/(x^2+x-1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(x - sqrt(x^2 + x - 1))^3 + 3*(x - sqrt(x^2 + x - 1))^2 - x + sqrt(x
^2 + x - 1) - 1)/((x - sqrt(x^2 + x - 1))^4 - 2*(x - sqrt(x^2 + x - 1))^2 -
 4*x + 4*sqrt(x^2 + x - 1)) + 1/4*arctan(-x + sqrt(x^2 + x - 1) - 1) + 5/8*
log(abs(-x + sqrt(x^2 + x - 1) + 2)) - 5/8*log(abs(-x + sqrt(x^2 + x - 1)))
```

## 3.13 $\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx$

**Optimal**. Leaf size=1077

result too large to display

```
[Out] -(((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*(b + Sqrt[b^2 -
4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c -
(b + Sqrt[b^2 - 4*a*c])^2)^2*(d + f*x^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4
*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - 2*a*c*f +
b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d
 + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
))*Sqrt[(1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt[b^2 -
4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2
- 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*
f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d -
 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[(
(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[2*a + (b + Sqr
t[b^2 - 4*a*c])*x])/((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4
)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (1 + ((b + Sqrt[b^2 - 4*a*c])*(c*d
 + a*f))/(Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*Sqrt[b^2*d
+ b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]))/2])/((4*a*c - (b + Sqrt[b^2 -
4*a*c])^2)*(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[a +
 b*x + c*x^2]*Sqrt[d + f*x^2]*Sqrt[1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*
f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2
*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^
2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4
*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]))
```

---

**Rubi [A]** time = 3.09899, antiderivative size = 1077, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {993, 936, 1103}

$$-\frac{\sqrt[4]{db^2+\sqrt{b^2-4ac}\,db-2a(cd-af)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2-4ac}\right)^2\right)^2(fx^2+d)}{\left(4fa^2+\left(b+\sqrt{b^2-4ac}\right)^2d\right)\left(b+2cx+\sqrt{b^2-4\right.}}}}{\left(4ac-\left(b+\sqrt{b^2-4ac}\right)^2\right)\sqrt[4]{2c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]
```

```
[Out] -(((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4)*(b + Sqrt[b^2 -
4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[b^2 - 4*a*c])*x]*Sqrt[((4*a*c -
(b + Sqrt[b^2 - 4*a*c])^2)^2*(d + f*x^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4
*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - 2*a*c*f +
b*(b + Sqrt[b^2 - 4*a*c])*f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d
 + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)
))*Sqrt[(1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*f)*(2*a + (b + Sqrt[b^2 -
```

```
4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c]
 + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2
- 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2*f)*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*
f]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*d + b*Sqrt[b^2 - 4*a*c]*d -
 2*a*(c*d - a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[(
(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[2*a + (b + Sqr
t[b^2 - 4*a*c])*x])/((b^2*d + b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f))^(1/4
)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])], (1 + ((b + Sqrt[b^2 - 4*a*c])*(c*d
 + a*f))/(Sqrt[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f]*Sqrt[b^2*d
+ b*Sqrt[b^2 - 4*a*c]*d - 2*a*(c*d - a*f)]))/2])/((4*a*c - (b + Sqrt[b^2 -
4*a*c])^2)*(2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f)^(1/4)*Sqrt[a +
 b*x + c*x^2]*Sqrt[d + f*x^2]*Sqrt[1 - (4*(b + Sqrt[b^2 - 4*a*c])*(c*d + a*
f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4*a^2
*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d + (b + Sqrt[b^2 - 4*a*c])^
2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d + 4
*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]))
```

Rule 993

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (f_.)*(x_)^2]), x
_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r + 2*c*x]*Sqrt[2
*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*c*x]*Sqrt[2*a
 + (b + r)*x]*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ
[b^2 - 4*a*c, 0]
```

Rule 936

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*
(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2*(a + c*x^2))/
((c*f^2 + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sqrt[a + c*x^2]), Subst[Int[1/
Sqrt[1 - ((2*c*d*f + 2*a*e*g)*x^2)/(c*f^2 + a*g^2) + ((c*d^2 + a*e^2)*x^4)/
(c*f^2 + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, c, d
, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+fx^2}}\,dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\right)\int \frac{1}{\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{d+f}}}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2-4ac}\right)^2\right)^2(d+fx^2)}{\left(\left(b+\sqrt{b^2-4ac}\right)^2d+4a^2f\right)\left(b+\sqrt{b^2-4ac}+\right.}}\right.}{\left(4ac-\left(b+\sqrt{b^2-4ac}\right)^{}\right.}$$

$$= -\frac{\sqrt[4]{b^2d+b\sqrt{b^2-4ac}d-2a(cd-af)}\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}}{}$$

**Mathematica [C]** time = 1.52761, size = 600, normalized size = 0.56

$$-\frac{2\sqrt{2}\left(\sqrt{f}x-i\sqrt{d}\right)\left(\sqrt{b^2-4ac}-b-2cx\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(\sqrt{f}x+i\sqrt{d}\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(\sqrt{f}\left(\sqrt{b^2-4ac}+b\right)-2ic\sqrt{d}\right)}}\sqrt{\frac{c\left(-i\sqrt{d}\left(\sqrt{b^2-4ac}+2cx\right)+\sqrt{f}\left(x\sqrt{b^2-4ac}-2a\right)+b\left(-\right.\right.}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(\sqrt{f}\left(\sqrt{b^2-4ac}+b\right)+2ic\right.}}}{\sqrt{d+fx^2}\sqrt{a+x(b+cx)}\left(\sqrt{f}\left(\sqrt{b^2-4ac}-b\right)-2ic\sqrt{d}\right)\sqrt{\left(\sqrt{b^2\right.}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + f*x^2]),x]
```

```
[Out] (-2*Sqrt[2]*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*((-I)*Sqrt[d] + Sqrt[f]*x)*Sqr
t[-((c*Sqrt[b^2 - 4*a*c]*(I*Sqrt[d] + Sqrt[f]*x))/(((-2*I)*c*Sqrt[d] + (b +
 Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*Sqrt[(c*((
-I)*Sqrt[d]*(Sqrt[b^2 - 4*a*c] + 2*c*x) + Sqrt[f]*(-2*a + Sqrt[b^2 - 4*a*c]
*x) + b*((-I)*Sqrt[d] - Sqrt[f]*x)))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*
a*c])*Sqrt[f])*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*EllipticF[ArcSin[Sqrt[(((
-2*I)*c*Sqrt[d] + (-b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x))]], (c*d - I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)/
(c*d + I*Sqrt[b^2 - 4*a*c]*Sqrt[d]*Sqrt[f] + a*f)])/(((-2*I)*c*Sqrt[d] + (-
b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*Sqrt[(I*c*Sqrt[b^2 - 4*a*c]*(Sqrt[d] + I*Sq
rt[f]*x))/(((2*I)*c*Sqrt[d] + (b + Sqrt[b^2 - 4*a*c])*Sqrt[f])*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x))]*Sqrt[d + f*x^2]*Sqrt[a + x*(b + c*x)])
```

---

**Maple [A]** time = 0.618, size = 661, normalized size = 0.6

$$4\,\frac{\left(bf^2x^2+2\,x^2cf\sqrt{-df}+\sqrt{-4\,ac+b^2}f^2x^2+2\,xbf\sqrt{-df}-4\,cxfd+2\,xf\sqrt{-4\,ac+b^2}\sqrt{-df}-bdf-2\,cd\sqrt{-df}-\sqrt{-4\,ac}\right.}{\sqrt{-df}\left(f\sqrt{-4\,ac+b^2}-2\,\sqrt{-df}c+bf\right)\sqrt{cfx^4+bfx^3+afx^2+cdx^2+bdx+ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x)
```

```
[Out] 4*(b*f^2*x^2+2*x^2*c*f*(-d*f)^(1/2)+(-4*a*c+b^2)^(1/2)*f^2*x^2+2*x*b*f*(-d*
f)^(1/2)-4*c*x*f*d+2*x*f*(-4*a*c+b^2)^(1/2)*(-d*f)^(1/2)-b*d*f-2*c*d*(-d*f)
^(1/2)-(-4*a*c+b^2)^(1/2)*d*f)*EllipticF((-(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(
1/2)*c+b*f)*(-f*x+(-d*f)^(1/2))/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c+b*f)
/(f*x+(-d*f)^(1/2)))^(1/2),((f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c-b*f)*(f*
(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c+b*f)/(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/
2)*c-b*f)/(f*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f))^(1/2))*((-d*f)^(1/2)
*(b+2*c*x+(-4*a*c+b^2)^(1/2))*f/(f*(-4*a*c+b^2)^(1/2)+2*(-d*f)^(1/2)*c+b*f)
/(f*x+(-d*f)^(1/2)))^(1/2)*((-d*f)^(1/2)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))*f/(f
*(-4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c-b*f)/(f*x+(-d*f)^(1/2)))^(1/2)*(-(f*(-
4*a*c+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f)*(-f*x+(-d*f)^(1/2))/(f*(-4*a*c+b^2)^
(1/2)+2*(-d*f)^(1/2)*c+b*f)/(f*x+(-d*f)^(1/2)))^(1/2)*(c*x^2+b*x+a)^(1/2)*(
f*x^2+d)^(1/2)/(1/c/f*(-f*x+(-d*f)^(1/2))*(f*x+(-d*f)^(1/2))*(-b-2*c*x+(-4*
a*c+b^2)^(1/2))*(b+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)/(-d*f)^(1/2)/(f*(-4*a*c
+b^2)^(1/2)-2*(-d*f)^(1/2)*c+b*f)/(c*f*x^4+b*f*x^3+a*f*x^2+c*d*x^2+b*d*x+a*
d)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{fx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{cx^2+bx+a}\sqrt{fx^2+d}}{cfx^4+bfx^3+bdx+\left(cd+af\right)x^2+ad},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)/(c*f*x^4 + b*f*x^3 + b*d*x +
 (c*d + a*f)*x^2 + a*d), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d+fx^2}\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(d + f*x**2)*sqrt(a + b*x + c*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + d)), x)
```

## 3.14 $\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2} \, dx$

**Optimal**. Leaf size=98

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{1}{2}\sin^{-1}(x+2)$$

```
[Out] -ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[
2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x
/Sqrt[-3 - 4*x - x^2]]/2
```

---

**Rubi [A]** time = 0.209648, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 12, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {989, 619, 216, 1028, 986, 12, 1026, 1161, 618, 204, 1027, 206}

$$-\frac{\tan^{-1}\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{1}{2}\sin^{-1}(x+2)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] -ArcSin[2 + x]/2 - ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[
2] + ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/Sqrt[2] - ArcTanh[x
/Sqrt[-3 - 4*x - x^2]]/2
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 1028

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> -Dist[(2*h*d - g*e)/e, Int[1/((a + b
*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/e, Int[(2*d + e*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g
, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

```
&& NeQ[2*h*d - g*e, 0]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1026

```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Dist[-2*e, Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2*
c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + ((e + Sqrt[e^2 -
 4*d*f])*x)/(2*d))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

Rule 1161

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[(2*d)/e - b/c, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2
, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && (
GtQ[(2*d)/e - b/c, 0] || ( !LtQ[(2*d)/e - b/c, 0] && EqQ[d - e*Rt[a/c, 2],
0]))
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1027

```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[g, Subst[Int[1/(a + (c*d - a*f)
*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h
}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &&
 EqQ[2*h*d - g*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{-3-4x-x^2}}{3+4x+2x^2}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{-3-4x-x^2}}\,dx\right) - \frac{1}{2}\int \frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx$$

$$= \frac{1}{4}\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx, x, -4-2x\right) + \frac{1}{2}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx + \frac{3}{2}\int \frac{}{\sqrt{-3-}}$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{4}\int \frac{-6-4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx + \frac{1}{4}\int -\frac{4x}{\sqrt{-3-4x-x^2}\left(3+4x+2\right.}$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{3-3x^2}\,dx, x, \frac{x}{\sqrt{-3-4x-x^2}}\right) -$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - 8\,\text{Subst}\left(\int \frac{1+3x^2}{-4-8x^2-36x^4}\,dx, x, \frac{1+}{\sqrt{-3-4x}}\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{\frac{1}{3}-\frac{2x}{3}+x^2}\,dx, x, \frac{1+\frac{x}{3}}{\sqrt{-3-4x-x}}\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right) - \frac{2}{3}\text{Subst}\left(\int \frac{1}{-\frac{8}{9}-x^2}\,dx, x, \frac{2}{3}\left(-1+\frac{3+}{\sqrt{-3-4}}\right.\right.$$

$$= -\frac{1}{2}\sin^{-1}(2+x) - \frac{\tan^{-1}\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\tan^{-1}\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}} - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

**Mathematica [C]** time = 0.400879, size = 159, normalized size = 1.62

$$\frac{1}{4}\left(-i\sqrt{1-2i\sqrt{2}}\tanh^{-1}\left(\frac{i\sqrt{2}x+2x+2i\sqrt{2}+2}{\sqrt{2-4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) + i\sqrt{1+2i\sqrt{2}}\tanh^{-1}\left(\frac{\left(2-i\sqrt{2}\right)x-2i\sqrt{2}+2}{\sqrt{2+4i\sqrt{2}}\sqrt{-x^2-4x-3}}\right) - 2\sin^{-1}(x+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-3 - 4*x - x^2]/(3 + 4*x + 2*x^2),x]
```

```
[Out] (-2*ArcSin[2 + x] - I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] +
2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])] + I*Sqrt
[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2
 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])])/4
```

---

**Maple [B]** time = 0.112, size = 341, normalized size = 3.5

$$-\frac{\arcsin(2+x)}{2} + \frac{\sqrt{4}\sqrt{3}}{12}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{6}\sqrt{3\frac{x^2}{(-3/2-x)^2}-12}\right) - \text{Artanh}\left(3\frac{x}{-3/2-x}\frac{1}{\sqrt{3\frac{x^2}{(-3/2}}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x)
```

```
[Out] -1/2*arcsin(2+x)+1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*
arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2
/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-
3/2-x))-1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+
x/(-3/2-x))*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^
2-12)^(1/2)*2^(1/2))+1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/
2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*
x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x
/(-3/2-x))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-x^2 - 4x - 3}}{2x^2 + 4x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 - 4*x - 3)/(2*x^2 + 4*x + 3), x)
```

---

**Fricas [A]** time = 0.974786, size = 441, normalized size = 4.5

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x + 3\sqrt{2}\sqrt{-x^2 - 4x - 3}}{2(2x+3)}\right) - \frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x - 3\sqrt{2}\sqrt{-x^2 - 4x - 3}}{2(2x+3)}\right) + \frac{1}{2}\arctan\left(\frac{\sqrt{-x^2 - 4x - 3}}{x^2 + 4x + \cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))
/(2*x + 3)) + 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/
8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2 - 4
*x - 3)*x - 4*x - 3)/x^2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x+1)(x+3)}}{2x^2 + 4x + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-4*x-3)**(1/2)/(2*x**2+4*x+3),x)
```

```
[Out] Integral(sqrt(-(x + 1)*(x + 3))/(2*x**2 + 4*x + 3), x)
```

---

**Giac [B]** time = 1.17704, size = 231, normalized size = 2.36

$$-\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2 - 4x - 3} - 1\right)}{x+2} + 1\right)\right) - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2 - 4x - 3} - 1}{x+2} + 1\right)\right) - \frac{1}{2}\arcsin(x+2) - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-4*x-3)^(1/2)/(2*x^2+4*x+3),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 - 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sq
rt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3)
- 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

## 3.15 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^4\,dx$

**Optimal**. Leaf size=68

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

---

**Rubi [A]** time = 0.0491625, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^4\,dx = \int \left(48+272x+1064x^2+2624x^3+5099x^4+7131x^5+8232x^6+6830x^7+5075\right.$$

$$= 48x+136x^2+\frac{1064x^3}{3}+656x^4+\frac{5099x^5}{5}+\frac{2377x^6}{2}+1176x^7+\frac{3415x^8}{4}+\frac{5075x}{9}$$

**Mathematica [A]** time = 0.0026852, size = 68, normalized size = 1.

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^9}{9}+\frac{3415x^8}{4}+1176x^7+\frac{2377x^6}{2}+\frac{5099x^5}{5}+656x^4+\frac{1064x^3}{3}+136x^2+48x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 48*x + 136*x^2 + (1064*x^3)/3 + 656*x^4 + (5099*x^5)/5 + (2377*x^6)/2 + 117
6*x^7 + (3415*x^8)/4 + (5075*x^9)/9 + (475*x^10)/2 + (1250*x^11)/11
```

---

**Maple [A]** time = 0.044, size = 55, normalized size = 0.8

$$48\,x+136\,x^2+\frac{1064\,x^3}{3}+656\,x^4+\frac{5099\,x^5}{5}+\frac{2377\,x^6}{2}+1176\,x^7+\frac{3415\,x^8}{4}+\frac{5075\,x^9}{9}+\frac{475\,x^{10}}{2}+\frac{1250\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 48*x+136*x^2+1064/3*x^3+656*x^4+5099/5*x^5+2377/2*x^6+1176*x^7+3415/4*x^8+5
075/9*x^9+475/2*x^10+1250/11*x^11
```

---

**Maxima [A]** time = 0.972346, size = 73, normalized size = 1.07

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^{9}+\frac{3415}{4}\,x^{8}+1176\,x^{7}+\frac{2377}{2}\,x^{6}+\frac{5099}{5}\,x^{5}+656\,x^{4}+\frac{1064}{3}\,x^{3}+136\,x^{2}+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

---

**Fricas [A]** time = 0.847187, size = 176, normalized size = 2.59

$$\frac{1250}{11}x^{11}+\frac{475}{2}x^{10}+\frac{5075}{9}x^{9}+\frac{3415}{4}x^{8}+1176x^{7}+\frac{2377}{2}x^{6}+\frac{5099}{5}x^{5}+656x^{4}+\frac{1064}{3}x^{3}+136x^{2}+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

---

**Sympy [A]** time = 0.084379, size = 65, normalized size = 0.96

$$\frac{1250x^{11}}{11}+\frac{475x^{10}}{2}+\frac{5075x^{9}}{9}+\frac{3415x^{8}}{4}+1176x^{7}+\frac{2377x^{6}}{2}+\frac{5099x^{5}}{5}+656x^{4}+\frac{1064x^{3}}{3}+136x^{2}+48x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**4,x)
```

```
[Out] 1250*x**11/11 + 475*x**10/2 + 5075*x**9/9 + 3415*x**8/4 + 1176*x**7 + 2377*
x**6/2 + 5099*x**5/5 + 656*x**4 + 1064*x**3/3 + 136*x**2 + 48*x
```

---

**Giac [A]** time = 1.15386, size = 73, normalized size = 1.07

$$\frac{1250}{11}\,x^{11}+\frac{475}{2}\,x^{10}+\frac{5075}{9}\,x^{9}+\frac{3415}{4}\,x^{8}+1176\,x^{7}+\frac{2377}{2}\,x^{6}+\frac{5099}{5}\,x^{5}+656\,x^{4}+\frac{1064}{3}\,x^{3}+136\,x^{2}+48\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1250/11*x^11 + 475/2*x^10 + 5075/9*x^9 + 3415/4*x^8 + 1176*x^7 + 2377/2*x^6
 + 5099/5*x^5 + 656*x^4 + 1064/3*x^3 + 136*x^2 + 48*x
```

## 3.16 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3\,dx$

**Optimal**. Leaf size=56

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

---

**Rubi [A]** time = 0.0377753, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^3\,dx &= \int \left(24+100x+322x^2+579x^3+876x^4+804x^5+720x^6+325x^7+250x^8\right)\,dx\\ &= 24x+50x^2+\frac{322x^3}{3}+\frac{579x^4}{4}+\frac{876x^5}{5}+134x^6+\frac{720x^7}{7}+\frac{325x^8}{8}+\frac{250x^9}{9}\end{aligned}$$

**Mathematica [A]** time = 0.0017287, size = 56, normalized size = 1.

$$\frac{250x^9}{9}+\frac{325x^8}{8}+\frac{720x^7}{7}+134x^6+\frac{876x^5}{5}+\frac{579x^4}{4}+\frac{322x^3}{3}+50x^2+24x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 24*x + 50*x^2 + (322*x^3)/3 + (579*x^4)/4 + (876*x^5)/5 + 134*x^6 + (720*x^
7)/7 + (325*x^8)/8 + (250*x^9)/9
```

---

**Maple [A]** time = 0.044, size = 45, normalized size = 0.8

$$24\,x+50\,x^2+\frac{322\,x^3}{3}+\frac{579\,x^4}{4}+\frac{876\,x^5}{5}+134\,x^6+\frac{720\,x^7}{7}+\frac{325\,x^8}{8}+\frac{250\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^3,x)
```

```
[Out] 24*x+50*x^2+322/3*x^3+579/4*x^4+876/5*x^5+134*x^6+720/7*x^7+325/8*x^8+250/9
*x^9
```

---

**Maxima [A]** time = 0.983348, size = 59, normalized size = 1.05

$$\frac{250}{9}\,x^{9}+\frac{325}{8}\,x^{8}+\frac{720}{7}\,x^{7}+134\,x^{6}+\frac{876}{5}\,x^{5}+\frac{579}{4}\,x^{4}+\frac{322}{3}\,x^{3}+50\,x^{2}+24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

---

**Fricas [A]** time = 0.716507, size = 131, normalized size = 2.34

$$\frac{250}{9}x^{9}+\frac{325}{8}x^{8}+\frac{720}{7}x^{7}+134x^{6}+\frac{876}{5}x^{5}+\frac{579}{4}x^{4}+\frac{322}{3}x^{3}+50x^{2}+24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

---

**Sympy [A]** time = 0.084274, size = 53, normalized size = 0.95

$$\frac{250x^{9}}{9}+\frac{325x^{8}}{8}+\frac{720x^{7}}{7}+134x^{6}+\frac{876x^{5}}{5}+\frac{579x^{4}}{4}+\frac{322x^{3}}{3}+50x^{2}+24x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**3,x)
```

```
[Out] 250*x**9/9 + 325*x**8/8 + 720*x**7/7 + 134*x**6 + 876*x**5/5 + 579*x**4/4 +
 322*x**3/3 + 50*x**2 + 24*x
```

---

**Giac [A]** time = 1.17455, size = 59, normalized size = 1.05

$$\frac{250}{9}\,x^{9}+\frac{325}{8}\,x^{8}+\frac{720}{7}\,x^{7}+134\,x^{6}+\frac{876}{5}\,x^{5}+\frac{579}{4}\,x^{4}+\frac{322}{3}\,x^{3}+50\,x^{2}+24\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 250/9*x^9 + 325/8*x^8 + 720/7*x^7 + 134*x^6 + 876/5*x^5 + 579/4*x^4 + 322/3
*x^3 + 50*x^2 + 24*x
```

## 3.17 $\int \left(3-x+2x^2\right) \left(2+3x+5x^2\right)^2 \, dx$

**Optimal**. Leaf size=44

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

---

**Rubi [A]** time = 0.0285789, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2 \, dx &= \int \left(12+32x+83x^2+85x^3+103x^4+35x^5+50x^6\right) dx\\ &= 12x+16x^2+\frac{83x^3}{3}+\frac{85x^4}{4}+\frac{103x^5}{5}+\frac{35x^6}{6}+\frac{50x^7}{7}\end{aligned}$$

**Mathematica [A]** time = 0.001421, size = 44, normalized size = 1.

$$\frac{50x^7}{7}+\frac{35x^6}{6}+\frac{103x^5}{5}+\frac{85x^4}{4}+\frac{83x^3}{3}+16x^2+12x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 12*x + 16*x^2 + (83*x^3)/3 + (85*x^4)/4 + (103*x^5)/5 + (35*x^6)/6 + (50*x^
7)/7
```

---

**Maple [A]** time = 0.042, size = 35, normalized size = 0.8

$$12\,x+16\,x^2+\frac{83\,x^3}{3}+\frac{85\,x^4}{4}+\frac{103\,x^5}{5}+\frac{35\,x^6}{6}+\frac{50\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 12*x+16*x^2+83/3*x^3+85/4*x^4+103/5*x^5+35/6*x^6+50/7*x^7
```

---

**Maxima [A]** time = 0.960582, size = 46, normalized size = 1.05

$$\frac{50}{7}\,x^{7}+\frac{35}{6}\,x^{6}+\frac{103}{5}\,x^{5}+\frac{85}{4}\,x^{4}+\frac{83}{3}\,x^{3}+16\,x^{2}+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

---

**Fricas [A]** time = 0.711843, size = 96, normalized size = 2.18

$$\frac{50}{7}x^{7}+\frac{35}{6}x^{6}+\frac{103}{5}x^{5}+\frac{85}{4}x^{4}+\frac{83}{3}x^{3}+16x^{2}+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

---

**Sympy [A]** time = 0.079116, size = 41, normalized size = 0.93

$$\frac{50x^{7}}{7}+\frac{35x^{6}}{6}+\frac{103x^{5}}{5}+\frac{85x^{4}}{4}+\frac{83x^{3}}{3}+16x^{2}+12x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2)**2,x)
```

```
[Out] 50*x**7/7 + 35*x**6/6 + 103*x**5/5 + 85*x**4/4 + 83*x**3/3 + 16*x**2 + 12*x
```

---

**Giac [A]** time = 1.18458, size = 46, normalized size = 1.05

$$\frac{50}{7}\,x^{7}+\frac{35}{6}\,x^{6}+\frac{103}{5}\,x^{5}+\frac{85}{4}\,x^{4}+\frac{83}{3}\,x^{3}+16\,x^{2}+12\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 50/7*x^7 + 35/6*x^6 + 103/5*x^5 + 85/4*x^4 + 83/3*x^3 + 16*x^2 + 12*x
```

## 3.18 $\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx$

**Optimal**. Leaf size=30

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

---

**Rubi [A]** time = 0.0161242, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {1657}

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)\left(2+3x+5x^2\right)\,dx &= \int \left(6+7x+16x^2+x^3+10x^4\right)\,dx\\ &= 6x+\frac{7x^2}{2}+\frac{16x^3}{3}+\frac{x^4}{4}+2x^5\end{aligned}$$

**Mathematica [A]** time = 0.0009675, size = 30, normalized size = 1.

$$2x^5+\frac{x^4}{4}+\frac{16x^3}{3}+\frac{7x^2}{2}+6x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] 6*x + (7*x^2)/2 + (16*x^3)/3 + x^4/4 + 2*x^5
```

---

**Maple [A]** time = 0.043, size = 25, normalized size = 0.8

$$6\,x+\frac{7\,x^2}{2}+\frac{16\,x^3}{3}+\frac{x^4}{4}+2\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)*(5*x^2+3*x+2),x)
```

```
[Out] 6*x+7/2*x^2+16/3*x^3+1/4*x^4+2*x^5
```

---

**Maxima [A]** time = 0.99311, size = 32, normalized size = 1.07

$$2\,x^5 + \frac{1}{4}\,x^4 + \frac{16}{3}\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

---

**Fricas [A]** time = 0.71146, size = 59, normalized size = 1.97

$$2x^5 + \frac{1}{4}x^4 + \frac{16}{3}x^3 + \frac{7}{2}x^2 + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

---

**Sympy [A]** time = 0.062864, size = 26, normalized size = 0.87

$$2x^5 + \frac{x^4}{4} + \frac{16x^3}{3} + \frac{7x^2}{2} + 6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)*(5*x**2+3*x+2),x)
```

```
[Out] 2*x**5 + x**4/4 + 16*x**3/3 + 7*x**2/2 + 6*x
```

---

**Giac [A]** time = 1.17373, size = 32, normalized size = 1.07

$$2\,x^5 + \frac{1}{4}\,x^4 + \frac{16}{3}\,x^3 + \frac{7}{2}\,x^2 + 6\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 2*x^5 + 1/4*x^4 + 16/3*x^3 + 7/2*x^2 + 6*x
```

# 3.19 $\int \frac{3-x+2x^2}{2+3x+5x^2} \, dx$

**Optimal**. Leaf size=42

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

---

**Rubi [A]** time = 0.0402874, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3-x+2x^2}{2+3x+5x^2}\,dx &= \int \left(\frac{2}{5}+\frac{11(1-x)}{5\left(2+3x+5x^2\right)}\right)dx\\ &= \frac{2x}{5}+\frac{11}{5}\int \frac{1-x}{2+3x+5x^2}\,dx\\ &= \frac{2x}{5}-\frac{11}{50}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{143}{50}\int \frac{1}{2+3x+5x^2}\,dx\\ &= \frac{2x}{5}-\frac{11}{50}\log\left(2+3x+5x^2\right)-\frac{143}{25}\,\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right)\\ &= \frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{25\sqrt{31}}-\frac{11}{50}\log\left(2+3x+5x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.0154453, size = 42, normalized size = 1.

$$-\frac{11}{50}\log\left(5x^2+3x+2\right)+\frac{2x}{5}+\frac{143\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{25\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (2*x)/5 + (143*ArcTan[(3 + 10*x)/Sqrt[31]])/(25*Sqrt[31]) - (11*Log[2 + 3*x
 + 5*x^2])/50
```

---

**Maple [A]** time = 0.049, size = 34, normalized size = 0.8

$$\frac{2\,x}{5}-\frac{11\,\ln\left(5\,x^2+3\,x+2\right)}{50}+\frac{143\,\sqrt{31}}{775}\arctan\left(\frac{(3+10\,x)\,\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2),x)
```

```
[Out] 2/5*x-11/50*ln(5*x^2+3*x+2)+143/775*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

---

**Maxima [A]** time = 1.4844, size = 45, normalized size = 1.07

$$\frac{143}{775}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

---

**Fricas [A]** time = 0.83428, size = 119, normalized size = 2.83

$$\frac{143}{775}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

---

**Sympy [A]** time = 0.15425, size = 49, normalized size = 1.17

$$\frac{2x}{5}-\frac{11\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{50}+\frac{143\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{775}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2),x)
```

```
[Out] 2*x/5 - 11*log(x**2 + 3*x/5 + 2/5)/50 + 143*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/775
```

---

**Giac [A]** time = 1.16842, size = 45, normalized size = 1.07

$$\frac{143}{775}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{2}{5}\,x-\frac{11}{50}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 143/775*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2/5*x - 11/50*log(5*x^2
 + 3*x + 2)
```

## 3.20 $\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2} dx$

Optimal. Leaf size=43

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}+\frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

---

Rubi [A] time = 0.0267706, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 204}

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}+\frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^2}\,dx = \frac{11(7+13x)}{155\left(2+3x+5x^2\right)} + \frac{1}{31}\int \frac{41}{2+3x+5x^2}\,dx$$
$$= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)} + \frac{41}{31}\int \frac{1}{2+3x+5x^2}\,dx$$
$$= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)} - \frac{82}{31}\operatorname{Subst}\left(\int \frac{1}{-31-x^2}\,dx, x, 3+10x\right)$$
$$= \frac{11(7+13x)}{155\left(2+3x+5x^2\right)} + \frac{82\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{31\sqrt{31}}$$

**Mathematica [A]** time = 0.0150616, size = 43, normalized size = 1.

$$\frac{11(13x+7)}{155\left(5x^2+3x+2\right)} + \frac{82\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{31\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (11*(7 + 13*x))/(155*(2 + 3*x + 5*x^2)) + (82*ArcTan[(3 + 10*x)/Sqrt[31]])/
(31*Sqrt[31])
```

---

**Maple [A]** time = 0.046, size = 34, normalized size = 0.8

$$\left(\frac{143\,x}{775} + \frac{77}{775}\right)\left(x^2 + \frac{3\,x}{5} + \frac{2}{5}\right)^{-1} + \frac{82\,\sqrt{31}}{961}\arctan\left(\frac{(3+10\,x)\,\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^2,x)
```

```
[Out] (143/775*x+77/775)/(x^2+3/5*x+2/5)+82/961*arctan(1/31*(3+10*x)*31^(1/2))*31
^(1/2)
```

---

**Maxima [A]** time = 1.46585, size = 49, normalized size = 1.14

$$\frac{82}{961}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{11\,(13\,x+7)}{155\left(5\,x^2+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2
 + 3*x + 2)
```

---

**Fricas [A]** time = 0.85103, size = 146, normalized size = 3.4

$$\frac{410\sqrt{31}\left(5x^2+3x+2\right)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+4433x+2387}{4805\left(5x^2+3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4805*(410*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4
433*x + 2387)/(5*x^2 + 3*x + 2)
```

---

**Sympy [A]** time = 0.213093, size = 42, normalized size = 0.98

$$\frac{143x+77}{775x^2+465x+310}+\frac{82\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{961}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

```
[Out] (143*x + 77)/(775*x**2 + 465*x + 310) + 82*sqrt(31)*atan(10*sqrt(31)*x/31 +
 3*sqrt(31)/31)/961
```

---

**Giac [A]** time = 1.22253, size = 49, normalized size = 1.14

$$\frac{82}{961}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{11(13x+7)}{155\left(5x^2+3x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 82/961*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/155*(13*x + 7)/(5*x^2
 + 3*x + 2)
```

## 3.21 $\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^3} \, dx$

**Optimal**. Leaf size=64

$$\frac{553(10x+3)}{9610\left(5x^2+3x+2\right)}+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}+\frac{1106\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

```
[Out] (11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*(3 + 10*x))/(9610*(2 + 3*x
 + 5*x^2)) + (1106*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

---

**Rubi [A]** time = 0.0370305, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1660, 12, 614, 618, 204}

$$\frac{553(10x+3)}{9610\left(5x^2+3x+2\right)}+\frac{11(13x+7)}{310\left(5x^2+3x+2\right)^2}+\frac{1106\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(7 + 13*x))/(310*(2 + 3*x + 5*x^2)^2) + (553*(3 + 10*x))/(9610*(2 + 3*x
 + 5*x^2)) + (1106*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{3-x+2x^2}{\left(2+3x+5x^2\right)^3}\,dx &= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{1}{62}\int \frac{553}{5\left(2+3x+5x^2\right)^2}\,dx \\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553}{310}\int \frac{1}{\left(2+3x+5x^2\right)^2}\,dx \\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{553}{961}\int \frac{1}{2+3x+5x^2}\,dx \\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}-\frac{1106}{961}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right) \\
&= \frac{11(7+13x)}{310\left(2+3x+5x^2\right)^2}+\frac{553(3+10x)}{9610\left(2+3x+5x^2\right)}+\frac{1106\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}
\end{aligned}$$

**Mathematica [A]** time = 0.0249325, size = 53, normalized size = 0.83

$$\frac{\frac{31\left(5530x^3+4977x^2+4094x+1141\right)}{\left(5x^2+3x+2\right)^2}+2212\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{59582}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((31*(1141 + 4094*x + 4977*x^2 + 5530*x^3))/(2 + 3*x + 5*x^2)^2 + 2212*Sqrt
[31]*ArcTan[(3 + 10*x)/Sqrt[31]])/59582
```

---

**Maple [A]** time = 0.044, size = 47, normalized size = 0.7

$$25\,\frac{1}{\left(5\,x^2+3\,x+2\right)^2}\left(\frac{553\,x^3}{4805}+\frac{4977\,x^2}{48050}+\frac{2047\,x}{24025}+\frac{1141}{48050}\right)+\frac{1106\,\sqrt{31}}{29791}\arctan\left(\frac{(3+10\,x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25*(553/4805*x^3+4977/48050*x^2+2047/24025*x+1141/48050)/(5*x^2+3*x+2)^2+11
06/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

---

**Maxima [A]** time = 1.47233, size = 76, normalized size = 1.19

$$\frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{5530x^3+4977x^2+4094x+1141}{1922(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4
977*x^2 + 4094*x + 1141)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Fricas [A]** time = 0.801471, size = 242, normalized size = 3.78

$$\frac{171430x^3+2212\sqrt{31}(25x^4+30x^3+29x^2+12x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+154287x^2+126914x+35371}{59582(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/59582*(171430*x^3 + 2212*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*a
rctan(1/31*sqrt(31)*(10*x + 3)) + 154287*x^2 + 126914*x + 35371)/(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Sympy [A]** time = 0.280638, size = 63, normalized size = 0.98

$$\frac{5530x^3+4977x^2+4094x+1141}{48050x^4+57660x^3+55738x^2+23064x+7688}+\frac{1106\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (5530*x**3 + 4977*x**2 + 4094*x + 1141)/(48050*x**4 + 57660*x**3 + 55738*x*
*2 + 23064*x + 7688) + 1106*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)
/29791
```

---

**Giac [A]** time = 1.25107, size = 62, normalized size = 0.97

$$\frac{1106}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{5530x^3+4977x^2+4094x+1141}{1922(5x^2+3x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1106/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/1922*(5530*x^3 + 4
977*x^2 + 4094*x + 1141)/(5*x^2 + 3*x + 2)^2
```

## 3.22 $\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx$

**Optimal**. Leaf size=80

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}$$

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

---

**Rubi [A]** time = 0.0599792, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1657}

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
 (27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (3-x+2x^2)^2 (2+3x+5x^2)^4 \, dx = \int (144+768x+3016x^2+7352x^3+14801x^4+21542x^5+27763x^6+26520$$
$$=144x+384x^2+\frac{3016x^3}{3}+1838x^4+\frac{14801x^5}{5}+\frac{10771x^6}{3}+\frac{27763x^7}{7}+3315x^8$$

**Mathematica [A]** time = 0.0029768, size = 80, normalized size = 1.

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^9}{9}+3315x^8+\frac{27763x^7}{7}+\frac{10771x^6}{3}+\frac{14801x^5}{5}+1838x^4+\frac{3016x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 144*x + 384*x^2 + (3016*x^3)/3 + 1838*x^4 + (14801*x^5)/5 + (10771*x^6)/3 +
(27763*x^7)/7 + 3315*x^8 + (24859*x^9)/9 + 1571*x^10 + (11525*x^11)/11 + (
875*x^12)/3 + (2500*x^13)/13
```

---

**Maple [A]** time = 0.043, size = 65, normalized size = 0.8

$$144x+384x^{2}+\frac{3016x^{3}}{3}+1838x^{4}+\frac{14801x^{5}}{5}+\frac{10771x^{6}}{3}+\frac{27763x^{7}}{7}+3315x^{8}+\frac{24859x^{9}}{9}+1571x^{10}+\frac{11525x^{11}}{11}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x)
```

```
[Out] 144*x+384*x^2+3016/3*x^3+1838*x^4+14801/5*x^5+10771/3*x^6+27763/7*x^7+3315*
x^8+24859/9*x^9+1571*x^10+11525/11*x^11+875/3*x^12+2500/13*x^13
```

---

**Maxima [A]** time = 0.956925, size = 86, normalized size = 1.08

$$\frac{2500}{13}x^{13}+\frac{875}{3}x^{12}+\frac{11525}{11}x^{11}+1571x^{10}+\frac{24859}{9}x^{9}+3315x^{8}+\frac{27763}{7}x^{7}+\frac{10771}{3}x^{6}+\frac{14801}{5}x^{5}+1838x^{4}+\frac{3016}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

---

**Fricas [A]** time = 0.62232, size = 221, normalized size = 2.76

$$\frac{2500}{13}x^{13}+\frac{875}{3}x^{12}+\frac{11525}{11}x^{11}+1571x^{10}+\frac{24859}{9}x^{9}+3315x^{8}+\frac{27763}{7}x^{7}+\frac{10771}{3}x^{6}+\frac{14801}{5}x^{5}+1838x^{4}+\frac{3016}{3}x^{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

---

**Sympy [A]** time = 0.133479, size = 76, normalized size = 0.95

$$\frac{2500x^{13}}{13}+\frac{875x^{12}}{3}+\frac{11525x^{11}}{11}+1571x^{10}+\frac{24859x^{9}}{9}+3315x^{8}+\frac{27763x^{7}}{7}+\frac{10771x^{6}}{3}+\frac{14801x^{5}}{5}+1838x^{4}+\frac{3016x^{3}}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**4,x)
```

```
[Out] 2500*x**13/13 + 875*x**12/3 + 11525*x**11/11 + 1571*x**10 + 24859*x**9/9 +
3315*x**8 + 27763*x**7/7 + 10771*x**6/3 + 14801*x**5/5 + 1838*x**4 + 3016*x
**3/3 + 384*x**2 + 144*x
```

---

**Giac [A]** time = 1.19878, size = 86, normalized size = 1.08

$$\frac{2500}{13}x^{13} + \frac{875}{3}x^{12} + \frac{11525}{11}x^{11} + 1571\,x^{10} + \frac{24859}{9}x^{9} + 3315\,x^{8} + \frac{27763}{7}x^{7} + \frac{10771}{3}x^{6} + \frac{14801}{5}x^{5} + 1838\,x^{4} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 2500/13*x^13 + 875/3*x^12 + 11525/11*x^11 + 1571*x^10 + 24859/9*x^9 + 3315*
x^8 + 27763/7*x^7 + 10771/3*x^6 + 14801/5*x^5 + 1838*x^4 + 3016/3*x^3 + 384
*x^2 + 144*x
```

## 3.23 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx$

**Optimal**. Leaf size=66

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

---

**Rubi [A]** time = 0.0481998, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1657}

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^3 \, dx = \int \left(72+276x+914x^2+1615x^3+2693x^4+2694x^5+3108x^6+1863x^7+1865x\right.$$

$$= 72x+138x^2+\frac{914x^3}{3}+\frac{1615x^4}{4}+\frac{2693x^5}{5}+449x^6+444x^7+\frac{1863x^8}{8}+\frac{1865x^9}{9}$$

**Mathematica [A]** time = 0.0021084, size = 66, normalized size = 1.

$$\frac{500x^{11}}{11}+40x^{10}+\frac{1865x^9}{9}+\frac{1863x^8}{8}+444x^7+449x^6+\frac{2693x^5}{5}+\frac{1615x^4}{4}+\frac{914x^3}{3}+138x^2+72x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 72*x + 138*x^2 + (914*x^3)/3 + (1615*x^4)/4 + (2693*x^5)/5 + 449*x^6 + 444*
x^7 + (1863*x^8)/8 + (1865*x^9)/9 + 40*x^10 + (500*x^11)/11
```

---

**Maple [A]** time = 0.042, size = 55, normalized size = 0.8

$$72\,x+138\,x^2+\frac{914\,x^3}{3}+\frac{1615\,x^4}{4}+\frac{2693\,x^5}{5}+449\,x^6+444\,x^7+\frac{1863\,x^8}{8}+\frac{1865\,x^9}{9}+40\,x^{10}+\frac{500\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x)
```

```
[Out] 72*x+138*x^2+914/3*x^3+1615/4*x^4+2693/5*x^5+449*x^6+444*x^7+1863/8*x^8+186
5/9*x^9+40*x^10+500/11*x^11
```

---

**Maxima [A]** time = 0.964319, size = 73, normalized size = 1.11

$$\frac{500}{11}\,x^{11} + 40\,x^{10} + \frac{1865}{9}\,x^{9} + \frac{1863}{8}\,x^{8} + 444\,x^{7} + 449\,x^{6} + \frac{2693}{5}\,x^{5} + \frac{1615}{4}\,x^{4} + \frac{914}{3}\,x^{3} + 138\,x^{2} + 72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

---

**Fricas [A]** time = 0.734116, size = 167, normalized size = 2.53

$$\frac{500}{11}x^{11} + 40x^{10} + \frac{1865}{9}x^{9} + \frac{1863}{8}x^{8} + 444x^{7} + 449x^{6} + \frac{2693}{5}x^{5} + \frac{1615}{4}x^{4} + \frac{914}{3}x^{3} + 138x^{2} + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

---

**Sympy [A]** time = 0.137197, size = 63, normalized size = 0.95

$$\frac{500x^{11}}{11} + 40x^{10} + \frac{1865x^{9}}{9} + \frac{1863x^{8}}{8} + 444x^{7} + 449x^{6} + \frac{2693x^{5}}{5} + \frac{1615x^{4}}{4} + \frac{914x^{3}}{3} + 138x^{2} + 72x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**3,x)
```

```
[Out] 500*x**11/11 + 40*x**10 + 1865*x**9/9 + 1863*x**8/8 + 444*x**7 + 449*x**6 +
 2693*x**5/5 + 1615*x**4/4 + 914*x**3/3 + 138*x**2 + 72*x
```

---

**Giac [A]** time = 1.16031, size = 73, normalized size = 1.11

$$\frac{500}{11}\,x^{11} + 40\,x^{10} + \frac{1865}{9}\,x^{9} + \frac{1863}{8}\,x^{8} + 444\,x^{7} + 449\,x^{6} + \frac{2693}{5}\,x^{5} + \frac{1615}{4}\,x^{4} + \frac{914}{3}\,x^{3} + 138\,x^{2} + 72\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 500/11*x^11 + 40*x^10 + 1865/9*x^9 + 1863/8*x^8 + 444*x^7 + 449*x^6 + 2693/
5*x^5 + 1615/4*x^4 + 914/3*x^3 + 138*x^2 + 72*x
```

## 3.24 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx$

**Optimal**. Leaf size=54

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

---

**Rubi [A]** time = 0.042492, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1657}

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right)^2 \, dx &= \int \left(36+84x+241x^2+236x^3+390x^4+172x^5+321x^6+20x^7+100x^8\right) dx\\ &= 36x+42x^2+\frac{241x^3}{3}+59x^4+78x^5+\frac{86x^6}{3}+\frac{321x^7}{7}+\frac{5x^8}{2}+\frac{100x^9}{9}\end{aligned}$$

**Mathematica [A]** time = 0.0017383, size = 54, normalized size = 1.

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 36*x + 42*x^2 + (241*x^3)/3 + 59*x^4 + 78*x^5 + (86*x^6)/3 + (321*x^7)/7 +
(5*x^8)/2 + (100*x^9)/9
```

---

**Maple [A]** time = 0.044, size = 45, normalized size = 0.8

$$36\,x+42\,x^2+\frac{241\,x^3}{3}+59\,x^4+78\,x^5+\frac{86\,x^6}{3}+\frac{321\,x^7}{7}+\frac{5\,x^8}{2}+\frac{100\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x)
```

```
[Out] 36*x+42*x^2+241/3*x^3+59*x^4+78*x^5+86/3*x^6+321/7*x^7+5/2*x^8+100/9*x^9
```

---

**Maxima [A]** time = 0.967055, size = 59, normalized size = 1.09

$$\frac{100}{9}\,x^9+\frac{5}{2}\,x^8+\frac{321}{7}\,x^7+\frac{86}{3}\,x^6+78\,x^5+59\,x^4+\frac{241}{3}\,x^3+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

---

**Fricas [A]** time = 0.904191, size = 122, normalized size = 2.26

$$\frac{100}{9}x^9+\frac{5}{2}x^8+\frac{321}{7}x^7+\frac{86}{3}x^6+78x^5+59x^4+\frac{241}{3}x^3+42x^2+36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

---

**Sympy [A]** time = 0.108465, size = 51, normalized size = 0.94

$$\frac{100x^9}{9}+\frac{5x^8}{2}+\frac{321x^7}{7}+\frac{86x^6}{3}+78x^5+59x^4+\frac{241x^3}{3}+42x^2+36x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2)**2,x)
```

```
[Out] 100*x**9/9 + 5*x**8/2 + 321*x**7/7 + 86*x**6/3 + 78*x**5 + 59*x**4 + 241*x*
*3/3 + 42*x**2 + 36*x
```

---

**Giac [A]** time = 1.19494, size = 59, normalized size = 1.09

$$\frac{100}{9}\,x^9+\frac{5}{2}\,x^8+\frac{321}{7}\,x^7+\frac{86}{3}\,x^6+78\,x^5+59\,x^4+\frac{241}{3}\,x^3+42\,x^2+36\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 100/9*x^9 + 5/2*x^8 + 321/7*x^7 + 86/3*x^6 + 78*x^5 + 59*x^4 + 241/3*x^3 +
42*x^2 + 36*x
```

## 3.25 $\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right) \, dx$

**Optimal**. Leaf size=46

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

---

**Rubi [A]** time = 0.0296107, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^2 \left(2+3x+5x^2\right) \, dx &= \int \left(18+15x+53x^2+x^3+61x^4-8x^5+20x^6\right) \, dx\\ &= 18x+\frac{15x^2}{2}+\frac{53x^3}{3}+\frac{x^4}{4}+\frac{61x^5}{5}-\frac{4x^6}{3}+\frac{20x^7}{7}\end{aligned}$$

**Mathematica [A]** time = 0.000904, size = 46, normalized size = 1.

$$\frac{20x^7}{7}-\frac{4x^6}{3}+\frac{61x^5}{5}+\frac{x^4}{4}+\frac{53x^3}{3}+\frac{15x^2}{2}+18x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2),x]
```

```
[Out] 18*x + (15*x^2)/2 + (53*x^3)/3 + x^4/4 + (61*x^5)/5 - (4*x^6)/3 + (20*x^7)/
7
```

---

**Maple [A]** time = 0.045, size = 35, normalized size = 0.8

$$18\,x+\frac{15\,x^2}{2}+\frac{53\,x^3}{3}+\frac{x^4}{4}+\frac{61\,x^5}{5}-\frac{4\,x^6}{3}+\frac{20\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2*(5*x^2+3*x+2),x)
```

```
[Out] 18*x+15/2*x^2+53/3*x^3+1/4*x^4+61/5*x^5-4/3*x^6+20/7*x^7
```

---

**Maxima [A]** time = 0.964242, size = 46, normalized size = 1.

$$\frac{20}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{61}{5}\,x^5 + \frac{1}{4}\,x^4 + \frac{53}{3}\,x^3 + \frac{15}{2}\,x^2 + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

---

**Fricas [A]** time = 0.82888, size = 95, normalized size = 2.07

$$\frac{20}{7}x^7 - \frac{4}{3}x^6 + \frac{61}{5}x^5 + \frac{1}{4}x^4 + \frac{53}{3}x^3 + \frac{15}{2}x^2 + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

---

**Sympy [A]** time = 0.097347, size = 41, normalized size = 0.89

$$\frac{20x^7}{7} - \frac{4x^6}{3} + \frac{61x^5}{5} + \frac{x^4}{4} + \frac{53x^3}{3} + \frac{15x^2}{2} + 18x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2*(5*x**2+3*x+2),x)
```

```
[Out] 20*x**7/7 - 4*x**6/3 + 61*x**5/5 + x**4/4 + 53*x**3/3 + 15*x**2/2 + 18*x
```

---

**Giac [A]** time = 1.17085, size = 46, normalized size = 1.

$$\frac{20}{7}\,x^7 - \frac{4}{3}\,x^6 + \frac{61}{5}\,x^5 + \frac{1}{4}\,x^4 + \frac{53}{3}\,x^3 + \frac{15}{2}\,x^2 + 18\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 20/7*x^7 - 4/3*x^6 + 61/5*x^5 + 1/4*x^4 + 53/3*x^3 + 15/2*x^2 + 18*x
```

# 3.26 $\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx$

**Optimal**. Leaf size=56

$$\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log\left(5x^2+3x+2\right)}{1250}+\frac{381x}{125}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

```
[Out] (381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]])
/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250
```

---

**Rubi [A]** time = 0.0513899, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1657, 634, 618, 204, 628}

$$\frac{4x^3}{15}-\frac{16x^2}{25}-\frac{1573\log\left(5x^2+3x+2\right)}{1250}+\frac{381x}{125}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] (381*x)/125 - (16*x^2)/25 + (4*x^3)/15 + (8349*ArcTan[(3 + 10*x)/Sqrt[31]])
/(625*Sqrt[31]) - (1573*Log[2 + 3*x + 5*x^2])/1250
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^2}{2+3x+5x^2}\,dx = \int \left(\frac{381}{125}-\frac{32x}{25}+\frac{4x^2}{5}+\frac{121(3-13x)}{125(2+3x+5x^2)}\right)dx$$
$$=\frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{121}{125}\int \frac{3-13x}{2+3x+5x^2}\,dx$$
$$=\frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\int \frac{3+10x}{2+3x+5x^2}\,dx}{1250}+\frac{8349\int \frac{1}{2+3x+5x^2}\,dx}{1250}$$
$$=\frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}-\frac{1573\log(2+3x+5x^2)}{1250}-\frac{8349}{625}\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right)$$
$$=\frac{381x}{125}-\frac{16x^2}{25}+\frac{4x^3}{15}+\frac{8349\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{625\sqrt{31}}-\frac{1573\log(2+3x+5x^2)}{1250}$$

**Mathematica [A]** time = 0.01929, size = 53, normalized size = 0.95

$$\frac{10x(100x^2-240x+1143)-4719\log(5x^2+3x+2)}{3750}+\frac{8349\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2),x]
```

```
[Out] (8349*ArcTan[(3 + 10*x)/Sqrt[31]])/(625*Sqrt[31]) + (10*x*(1143 - 240*x + 1
00*x^2) - 4719*Log[2 + 3*x + 5*x^2])/3750
```

---

**Maple [A]** time = 0.044, size = 44, normalized size = 0.8

$$\frac{381\,x}{125}-\frac{16\,x^2}{25}+\frac{4\,x^3}{15}-\frac{1573\,\ln(5\,x^2+3\,x+2)}{1250}+\frac{8349\,\sqrt{31}}{19375}\arctan\left(\frac{(3+10\,x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2),x)
```

```
[Out] 381/125*x-16/25*x^2+4/15*x^3-1573/1250*ln(5*x^2+3*x+2)+8349/19375*arctan(1/
31*(3+10*x)*31^(1/2))*31^(1/2)
```

---

**Maxima [A]** time = 1.48807, size = 58, normalized size = 1.04

$$\frac{4}{15}x^3-\frac{16}{25}x^2+\frac{8349}{19375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{381}{125}x-\frac{1573}{1250}\log(5\,x^2+3\,x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

---

**Fricas [A]** time = 0.993161, size = 165, normalized size = 2.95

$$\frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

---

**Sympy [A]** time = 0.188934, size = 63, normalized size = 1.12

$$\frac{4x^3}{15} - \frac{16x^2}{25} + \frac{381x}{125} - \frac{1573\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{1250} + \frac{8349\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{19375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

```
[Out] 4*x**3/15 - 16*x**2/25 + 381*x/125 - 1573*log(x**2 + 3*x/5 + 2/5)/1250 + 83
49*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19375
```

---

**Giac [A]** time = 1.16036, size = 58, normalized size = 1.04

$$\frac{4}{15}\,x^3 - \frac{16}{25}\,x^2 + \frac{8349}{19375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{381}{125}\,x - \frac{1573}{1250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 4/15*x^3 - 16/25*x^2 + 8349/19375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3))
 + 381/125*x - 1573/1250*log(5*x^2 + 3*x + 2)
```

## 3.27 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2} dx$

**Optimal**. Leaf size=63

$$\frac{121(69x+61)}{3875(5x^2+3x+2)}-\frac{22}{125}\log\left(5x^2+3x+2\right)+\frac{4x}{25}+\frac{41932\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

```
[Out] (4*x)/25 + (121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (41932*ArcTan[(3 +
10*x)/Sqrt[31]])/(3875*Sqrt[31]) - (22*Log[2 + 3*x + 5*x^2])/125
```

---

**Rubi [A]** time = 0.0608414, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(69x+61)}{3875(5x^2+3x+2)}-\frac{22}{125}\log\left(5x^2+3x+2\right)+\frac{4x}{25}+\frac{41932\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{3875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*x)/25 + (121*(61 + 69*x))/(3875*(2 + 3*x + 5*x^2)) + (41932*ArcTan[(3 +
10*x)/Sqrt[31]])/(3875*Sqrt[31]) - (22*Log[2 + 3*x + 5*x^2])/125
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^2}\,dx &= \frac{121(61+69x)}{3875(2+3x+5x^2)}+\frac{1}{31}\int \frac{\frac{4032}{25}-\frac{992x}{25}+\frac{124x^2}{5}}{2+3x+5x^2}\,dx\\
&= \frac{121(61+69x)}{3875(2+3x+5x^2)}+\frac{1}{31}\int \left(\frac{124}{25}+\frac{44(86-31x)}{25(2+3x+5x^2)}\right)dx\\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875(2+3x+5x^2)}+\frac{44}{775}\int \frac{86-31x}{2+3x+5x^2}\,dx\\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875(2+3x+5x^2)}-\frac{22}{125}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{20966\int \frac{1}{2+3x+5x^2}\,dx}{3875}\\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875(2+3x+5x^2)}-\frac{22}{125}\log\left(2+3x+5x^2\right)-\frac{41932\,\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx,x,3+10x\right)}{3875}\\
&= \frac{4x}{25}+\frac{121(61+69x)}{3875(2+3x+5x^2)}+\frac{41932\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{3875\sqrt{31}}-\frac{22}{125}\log\left(2+3x+5x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0308515, size = 59, normalized size = 0.94

$$\frac{\frac{3751(69x+61)}{5x^2+3x+2}-21142\log\left(5x^2+3x+2\right)+19220x+41932\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{120125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (19220*x + (3751*(61 + 69*x))/(2 + 3*x + 5*x^2) + 41932*Sqrt[31]*ArcTan[(3
+ 10*x)/Sqrt[31]] - 21142*Log[2 + 3*x + 5*x^2])/120125
```

---

**Maple [A]** time = 0.046, size = 51, normalized size = 0.8

$$\frac{4\,x}{25}-\frac{11}{25}\left(-\frac{759\,x}{775}-\frac{671}{775}\right)\left(x^2+\frac{3\,x}{5}+\frac{2}{5}\right)^{-1}-\frac{22\,\ln\left(5\,x^2+3\,x+2\right)}{125}+\frac{41932\,\sqrt{31}}{120125}\arctan\left(\frac{(3+10\,x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)
```

```
[Out] 4/25*x-11/25*(-759/775*x-671/775)/(x^2+3/5*x+2/5)-22/125*ln(5*x^2+3*x+2)+41
932/120125*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

---

**Maxima [A]** time = 1.45998, size = 70, normalized size = 1.11

$$\frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{4}{25}\,x+\frac{121\,(69\,x+61)}{3875\left(5\,x^2+3\,x+2\right)}-\frac{22}{125}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875*
(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

---

**Fricas [A]** time = 0.997507, size = 252, normalized size = 4.

$$\frac{96100\,x^3+41932\sqrt{31}\left(5\,x^2+3\,x+2\right)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+57660\,x^2-21142\left(5\,x^2+3\,x+2\right)\log\left(5\,x^2+3\,x+\right.}{120125\left(5\,x^2+3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/120125*(96100*x^3 + 41932*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)
*(10*x + 3)) + 57660*x^2 - 21142*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 2
97259*x + 228811)/(5*x^2 + 3*x + 2)
```

---

**Sympy [A]** time = 0.213602, size = 65, normalized size = 1.03

$$\frac{4x}{25}+\frac{8349x+7381}{19375x^2+11625x+7750}-\frac{22\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{125}+\frac{41932\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{120125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] 4*x/25 + (8349*x + 7381)/(19375*x**2 + 11625*x + 7750) - 22*log(x**2 + 3*x/
5 + 2/5)/125 + 41932*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/120125
```

---

**Giac [A]** time = 1.18491, size = 70, normalized size = 1.11

$$\frac{41932}{120125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{4}{25}\,x+\frac{121\,(69\,x+61)}{3875\left(5\,x^2+3\,x+2\right)}-\frac{22}{125}\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 41932/120125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 4/25*x + 121/3875*
(69*x + 61)/(5*x^2 + 3*x + 2) - 22/125*log(5*x^2 + 3*x + 2)
```

## 3.28 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3} \, dx$

**Optimal**. Leaf size=64

$$\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}+\frac{11(45710x+17557)}{240250\left(5x^2+3x+2\right)}+\frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

```
[Out] (121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + (11*(17557 + 45710*x))/(2402
50*(2 + 3*x + 5*x^2)) + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

---

**Rubi [A]** time = 0.0516227, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1660, 12, 618, 204}

$$\frac{121(69x+61)}{7750\left(5x^2+3x+2\right)^2}+\frac{11(45710x+17557)}{240250\left(5x^2+3x+2\right)}+\frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (121*(61 + 69*x))/(7750*(2 + 3*x + 5*x^2)^2) + (11*(17557 + 45710*x))/(2402
50*(2 + 3*x + 5*x^2)) + (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^3}\,dx = \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{1}{62}\int \frac{\frac{48669}{125} - \frac{1984x}{25} + \frac{248x^2}{5}}{(2+3x+5x^2)^2}\,dx$$
$$= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{\int \frac{4330}{2+3x+5x^2}\,dx}{1922}$$
$$= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{2165}{961}\int \frac{1}{2+3x+5x^2}\,dx$$
$$= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} - \frac{4330}{961}\,\text{Subst}\left(\int \frac{1}{-31-x^2}\,dx, x, 3+10x\right)$$
$$= \frac{121(61+69x)}{7750(2+3x+5x^2)^2} + \frac{11(17557+45710x)}{240250(2+3x+5x^2)} + \frac{4330\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{961\sqrt{31}}$$

**Mathematica [A]** time = 0.0233833, size = 53, normalized size = 0.83

$$\frac{11(45710x^3+44983x^2+33524x+11183)}{48050(5x^2+3x+2)^2} + \frac{4330\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{961\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11*(11183 + 33524*x + 44983*x^2 + 45710*x^3))/(48050*(2 + 3*x + 5*x^2)^2)
+ (4330*ArcTan[(3 + 10*x)/Sqrt[31]])/(961*Sqrt[31])
```

---

**Maple [A]** time = 0.046, size = 47, normalized size = 0.7

$$25\frac{1}{(5x^2+3x+2)^2}\left(\frac{50281x^3}{120125} + \frac{494813x^2}{1201250} + \frac{184382x}{600625} + \frac{123013}{1201250}\right) + \frac{4330\sqrt{31}}{29791}\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25*(50281/120125*x^3+494813/1201250*x^2+184382/600625*x+123013/1201250)/(5*
x^2+3*x+2)^2+4330/29791*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

---

**Maxima [A]** time = 1.4875, size = 76, normalized size = 1.19

$$\frac{4330}{29791}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{11(45710x^3+44983x^2+33524x+11183)}{48050(25x^4+30x^3+29x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
+ 44983*x^2 + 33524*x + 11183)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Fricas [A]** time = 0.948494, size = 258, normalized size = 4.03

$$\frac{15587110\,x^3 + 216500\,\sqrt{31}\left(25\,x^4 + 30\,x^3 + 29\,x^2 + 12\,x + 4\right)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x + 3)\right) + 15339203\,x^2 + 11431684\,x}{1489550\left(25\,x^4 + 30\,x^3 + 29\,x^2 + 12\,x + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/1489550*(15587110*x^3 + 216500*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x
+ 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 15339203*x^2 + 11431684*x + 3813403
)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Sympy [A]** time = 0.298147, size = 63, normalized size = 0.98

$$\frac{502810x^3 + 494813x^2 + 368764x + 123013}{1201250x^4 + 1441500x^3 + 1393450x^2 + 576600x + 192200} + \frac{4330\sqrt{31}\,\operatorname{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{31}}{31}\right)}{29791}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] (502810*x**3 + 494813*x**2 + 368764*x + 123013)/(1201250*x**4 + 1441500*x**
3 + 1393450*x**2 + 576600*x + 192200) + 4330*sqrt(31)*atan(10*sqrt(31)*x/31
 + 3*sqrt(31)/31)/29791
```

---

**Giac [A]** time = 1.23913, size = 62, normalized size = 0.97

$$\frac{4330}{29791}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x + 3)\right) + \frac{11\left(45710\,x^3 + 44983\,x^2 + 33524\,x + 11183\right)}{48050\left(5\,x^2 + 3\,x + 2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 4330/29791*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 11/48050*(45710*x^3
+ 44983*x^2 + 33524*x + 11183)/(5*x^2 + 3*x + 2)^2
```

## 3.29 $\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx$

**Optimal**. Leaf size=85

$$\frac{16688(10x+3)}{148955\left(5x^2+3x+2\right)}+\frac{11(12060x+4579)}{120125\left(5x^2+3x+2\right)^2}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

```
[Out] (121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (11*(4579 + 12060*x))/(1201
25*(2 + 3*x + 5*x^2)^2) + (16688*(3 + 10*x))/(148955*(2 + 3*x + 5*x^2)) + (
66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(29791*Sqrt[31])
```

---

**Rubi [A]** time = 0.0633293, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1660, 12, 614, 618, 204}

$$\frac{16688(10x+3)}{148955\left(5x^2+3x+2\right)}+\frac{11(12060x+4579)}{120125\left(5x^2+3x+2\right)^2}+\frac{121(69x+61)}{11625\left(5x^2+3x+2\right)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (121*(61 + 69*x))/(11625*(2 + 3*x + 5*x^2)^3) + (11*(4579 + 12060*x))/(1201
25*(2 + 3*x + 5*x^2)^2) + (16688*(3 + 10*x))/(148955*(2 + 3*x + 5*x^2)) + (
66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(29791*Sqrt[31])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(3-x+2x^2)^2}{(2+3x+5x^2)^4}\,dx = \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{1}{93}\int \frac{\frac{77178}{125}-\frac{2976x}{25}+\frac{372x^2}{5}}{(2+3x+5x^2)^3}\,dx$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{\int \frac{100128}{5(2+3x+5x^2)^2}\,dx}{5766}$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688\int \frac{1}{(2+3x+5x^2)^2}\,dx}{4805}$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}+\frac{33376\int \frac{1}{2+3x+}}{29791}$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}-\frac{66752\,\mathrm{Subst}(}{}$$

$$= \frac{121(61+69x)}{11625(2+3x+5x^2)^3}+\frac{11(4579+12060x)}{120125(2+3x+5x^2)^2}+\frac{16688(3+10x)}{148955(2+3x+5x^2)}+\frac{66752\tan^{-1}(}{29791\sqrt{3}}$$

**Mathematica [A]** time = 0.0449281, size = 63, normalized size = 0.74

$$\frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+1259239}{446865(5x^2+3x+2)^3}+\frac{66752\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{29791\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^2/(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (1259239 + 5674908*x + 12780597*x^2 + 21491796*x^3 + 18774000*x^4 + 1251600
0*x^5)/(446865*(2 + 3*x + 5*x^2)^3) + (66752*ArcTan[(3 + 10*x)/Sqrt[31]])/(
29791*Sqrt[31])
```

---

**Maple [A]** time = 0.05, size = 57, normalized size = 0.7

$$125\frac{1}{(5x^2+3x+2)^3}\left(\frac{33376x^5}{148955}+\frac{50064x^4}{148955}+\frac{7163932x^3}{18619375}+\frac{4260199x^2}{18619375}+\frac{1891636x}{18619375}+\frac{1259239}{55858125}\right)+\frac{66752\sqrt{31}}{923521}\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x)
```

```
[Out] 125*(33376/148955*x^5+50064/148955*x^4+7163932/18619375*x^3+4260199/1861937
5*x^2+1891636/18619375*x+1259239/55858125)/(5*x^2+3*x+2)^3+66752/923521*arc
tan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

---

**Maxima [A]** time = 1.44632, size = 103, normalized size = 1.21

$$\frac{66752}{923521}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+125}{446865\left(125x^6+225x^5+285x^4+207x^3+114x^2+36x+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000
*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(1
25*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

---

**Fricas [A]** time = 0.963056, size = 371, normalized size = 4.36

$$\frac{387996000x^5+581994000x^4+666245676x^3+1001280\sqrt{31}\left(125x^6+225x^5+285x^4+207x^3+114x^2+36x+8\right)a}{13852815\left(125x^6+225x^5+285x^4+207x^3+114x^2+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/13852815*(387996000*x^5 + 581994000*x^4 + 666245676*x^3 + 1001280*sqrt(31
)*(125*x^6 + 225*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)*arctan(1/31*
sqrt(31)*(10*x + 3)) + 396198507*x^2 + 175922148*x + 39036409)/(125*x^6 + 2
25*x^5 + 285*x^4 + 207*x^3 + 114*x^2 + 36*x + 8)
```

---

**Sympy [A]** time = 0.251072, size = 83, normalized size = 0.98

$$\frac{12516000x^5+18774000x^4+21491796x^3+12780597x^2+5674908x+1259239}{55858125x^6+100544625x^5+127356525x^4+92501055x^3+50942610x^2+16087140x+3574920}+\frac{66752\sqrt{31}\operatorname{atan}\left(\right.}{9235}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**2/(5*x**2+3*x+2)**4,x)
```

```
[Out] (12516000*x**5 + 18774000*x**4 + 21491796*x**3 + 12780597*x**2 + 5674908*x
+ 1259239)/(55858125*x**6 + 100544625*x**5 + 127356525*x**4 + 92501055*x**3
 + 50942610*x**2 + 16087140*x + 3574920) + 66752*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/923521
```

---

**Giac [A]** time = 1.18208, size = 76, normalized size = 0.89

$$\frac{66752}{923521}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right)+\frac{12516000\,x^5+18774000\,x^4+21491796\,x^3+12780597\,x^2+5674908\,x+1}{446865\,\left(5\,x^2+3\,x+2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^2/(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 66752/923521*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1/446865*(12516000
*x^5 + 18774000*x^4 + 21491796*x^3 + 12780597*x^2 + 5674908*x + 1259239)/(5
*x^2 + 3*x + 2)^3
```

## 3.30 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx$

**Optimal**. Leaf size=96

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+$$

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

---

**Rubi [A]** time = 0.0705267, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1657}

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^4 \, dx = \int \left(432+2160x+8568x^2+20576x^3+43083x^4+64529x^5+91349x^6+94881x\right.$$
$$= 432x+1080x^2+2856x^3+5144x^4+\frac{43083x^5}{5}+\frac{64529x^6}{6}+\frac{91349x^7}{7}+\frac{94881x^8}{8}$$

**Mathematica [A]** time = 0.0029899, size = 96, normalized size = 1.

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^9}{9}+\frac{94881x^8}{8}+\frac{91349x^7}{7}+\frac{64529x^6}{6}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] 432*x + 1080*x^2 + 2856*x^3 + 5144*x^4 + (43083*x^5)/5 + (64529*x^6)/6 + (9
1349*x^7)/7 + (94881*x^8)/8 + (103583*x^9)/9 + (75311*x^10)/10 + (68583*x^1
1)/11 + (30395*x^12)/12 + (27050*x^13)/13 + (2250*x^14)/7 + (1000*x^15)/3
```

---

**Maple [A]** time = 0.044, size = 75, normalized size = 0.8

$$432\,x+1080\,x^{2}+2856\,x^{3}+5144\,x^{4}+\frac{43083\,x^{5}}{5}+\frac{64529\,x^{6}}{6}+\frac{91349\,x^{7}}{7}+\frac{94881\,x^{8}}{8}+\frac{103583\,x^{9}}{9}+\frac{75311\,x^{10}}{10}+\frac{685}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x)
```

```
[Out] 432*x+1080*x^2+2856*x^3+5144*x^4+43083/5*x^5+64529/6*x^6+91349/7*x^7+94881/
8*x^8+103583/9*x^9+75311/10*x^10+68583/11*x^11+30395/12*x^12+27050/13*x^13+
2250/7*x^14+1000/3*x^15
```

---

**Maxima [A]** time = 0.983455, size = 100, normalized size = 1.04

$$\frac{1000}{3}\,x^{15}+\frac{2250}{7}\,x^{14}+\frac{27050}{13}\,x^{13}+\frac{30395}{12}\,x^{12}+\frac{68583}{11}\,x^{11}+\frac{75311}{10}\,x^{10}+\frac{103583}{9}\,x^{9}+\frac{94881}{8}\,x^{8}+\frac{91349}{7}\,x^{7}+\frac{645}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

---

**Fricas [A]** time = 0.823481, size = 274, normalized size = 2.85

$$\frac{1000}{3}x^{15}+\frac{2250}{7}x^{14}+\frac{27050}{13}x^{13}+\frac{30395}{12}x^{12}+\frac{68583}{11}x^{11}+\frac{75311}{10}x^{10}+\frac{103583}{9}x^{9}+\frac{94881}{8}x^{8}+\frac{91349}{7}x^{7}+\frac{64529}{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

---

**Sympy [A]** time = 0.126187, size = 92, normalized size = 0.96

$$\frac{1000x^{15}}{3}+\frac{2250x^{14}}{7}+\frac{27050x^{13}}{13}+\frac{30395x^{12}}{12}+\frac{68583x^{11}}{11}+\frac{75311x^{10}}{10}+\frac{103583x^{9}}{9}+\frac{94881x^{8}}{8}+\frac{91349x^{7}}{7}+\frac{64529x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**4,x)
```

```
[Out] 1000*x**15/3 + 2250*x**14/7 + 27050*x**13/13 + 30395*x**12/12 + 68583*x**11
/11 + 75311*x**10/10 + 103583*x**9/9 + 94881*x**8/8 + 91349*x**7/7 + 64529*
x**6/6 + 43083*x**5/5 + 5144*x**4 + 2856*x**3 + 1080*x**2 + 432*x
```

**Giac [A]** time = 1.17693, size = 100, normalized size = 1.04

$$\frac{1000}{3}x^{15} + \frac{2250}{7}x^{14} + \frac{27050}{13}x^{13} + \frac{30395}{12}x^{12} + \frac{68583}{11}x^{11} + \frac{75311}{10}x^{10} + \frac{103583}{9}x^{9} + \frac{94881}{8}x^{8} + \frac{91349}{7}x^{7} + \frac{64529}{6}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1000/3*x^15 + 2250/7*x^14 + 27050/13*x^13 + 30395/12*x^12 + 68583/11*x^11 +
 75311/10*x^10 + 103583/9*x^9 + 94881/8*x^8 + 91349/7*x^7 + 64529/6*x^6 + 4
3083/5*x^5 + 5144*x^4 + 2856*x^3 + 1080*x^2 + 432*x
```

## 3.31 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx$

**Optimal**. Leaf size=82

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+37$$

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

---

**Rubi [A]** time = 0.0555126, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1657}

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+37$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^3 \, dx = \int \left(216+756x+2610x^2+4483x^3+8292x^4+8619x^5+12016x^6+7869x^7+\right.$$
$$= 216x+378x^2+870x^3+\frac{4483x^4}{4}+\frac{8292x^5}{5}+\frac{2873x^6}{2}+\frac{12016x^7}{7}+\frac{7869x^8}{8}+$$

**Mathematica [A]** time = 0.0018334, size = 82, normalized size = 1.

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^9}{3}+\frac{7869x^8}{8}+\frac{12016x^7}{7}+\frac{2873x^6}{2}+\frac{8292x^5}{5}+\frac{4483x^4}{4}+870x^3+37$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] 216*x + 378*x^2 + 870*x^3 + (4483*x^4)/4 + (8292*x^5)/5 + (2873*x^6)/2 + (1
2016*x^7)/7 + (7869*x^8)/8 + (3316*x^9)/3 + (3061*x^10)/10 + (4830*x^11)/11
 + 25*x^12 + (1000*x^13)/13
```

---

**Maple [A]** time = 0.045, size = 65, normalized size = 0.8

$$216\,x+378\,x^{2}+870\,x^{3}+\frac{4483\,x^{4}}{4}+\frac{8292\,x^{5}}{5}+\frac{2873\,x^{6}}{2}+\frac{12016\,x^{7}}{7}+\frac{7869\,x^{8}}{8}+\frac{3316\,x^{9}}{3}+\frac{3061\,x^{10}}{10}+\frac{4830\,x^{11}}{11}+25\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x)
```

```
[Out] 216*x+378*x^2+870*x^3+4483/4*x^4+8292/5*x^5+2873/2*x^6+12016/7*x^7+7869/8*x
^8+3316/3*x^9+3061/10*x^10+4830/11*x^11+25*x^12+1000/13*x^13
```

---

**Maxima [A]** time = 0.969923, size = 86, normalized size = 1.05

$$\frac{1000}{13}\,x^{13}+25\,x^{12}+\frac{4830}{11}\,x^{11}+\frac{3061}{10}\,x^{10}+\frac{3316}{3}\,x^{9}+\frac{7869}{8}\,x^{8}+\frac{12016}{7}\,x^{7}+\frac{2873}{2}\,x^{6}+\frac{8292}{5}\,x^{5}+\frac{4483}{4}\,x^{4}+870\,x^{3}+3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

---

**Fricas [A]** time = 0.83181, size = 217, normalized size = 2.65

$$\frac{1000}{13}x^{13}+25x^{12}+\frac{4830}{11}x^{11}+\frac{3061}{10}x^{10}+\frac{3316}{3}x^{9}+\frac{7869}{8}x^{8}+\frac{12016}{7}x^{7}+\frac{2873}{2}x^{6}+\frac{8292}{5}x^{5}+\frac{4483}{4}x^{4}+870x^{3}+378x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

---

**Sympy [A]** time = 0.084503, size = 78, normalized size = 0.95

$$\frac{1000x^{13}}{13}+25x^{12}+\frac{4830x^{11}}{11}+\frac{3061x^{10}}{10}+\frac{3316x^{9}}{3}+\frac{7869x^{8}}{8}+\frac{12016x^{7}}{7}+\frac{2873x^{6}}{2}+\frac{8292x^{5}}{5}+\frac{4483x^{4}}{4}+870x^{3}+378x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**3,x)
```

```
[Out] 1000*x**13/13 + 25*x**12 + 4830*x**11/11 + 3061*x**10/10 + 3316*x**9/3 + 78
69*x**8/8 + 12016*x**7/7 + 2873*x**6/2 + 8292*x**5/5 + 4483*x**4/4 + 870*x*
*3 + 378*x**2 + 216*x
```

---

**Giac [A]** time = 1.13975, size = 86, normalized size = 1.05

$$\frac{1000}{13}\,x^{13} + 25\,x^{12} + \frac{4830}{11}\,x^{11} + \frac{3061}{10}\,x^{10} + \frac{3316}{3}\,x^{9} + \frac{7869}{8}\,x^{8} + \frac{12016}{7}\,x^{7} + \frac{2873}{2}\,x^{6} + \frac{8292}{5}\,x^{5} + \frac{4483}{4}\,x^{4} + 870\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1000/13*x^13 + 25*x^12 + 4830/11*x^11 + 3061/10*x^10 + 3316/3*x^9 + 7869/8*
x^8 + 12016/7*x^7 + 2873/2*x^6 + 8292/5*x^5 + 4483/4*x^4 + 870*x^3 + 378*x^
2 + 216*x
```

## 3.32 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx$

**Optimal**. Leaf size=68

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

---

**Rubi [A]** time = 0.0505835, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.04, Rules used = {1657}

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right)^2 \, dx = \int \left(108+216x+711x^2+635x^3+1416x^4+598x^5+1571x^6+83x^7+922x^8-6\right.$$
$$= 108x+108x^2+237x^3+\frac{635x^4}{4}+\frac{1416x^5}{5}+\frac{299x^6}{3}+\frac{1571x^7}{7}+\frac{83x^8}{8}+\frac{922x^9}{9}-$$

**Mathematica [A]** time = 0.0022274, size = 68, normalized size = 1.

$$\frac{200x^{11}}{11}-6x^{10}+\frac{922x^9}{9}+\frac{83x^8}{8}+\frac{1571x^7}{7}+\frac{299x^6}{3}+\frac{1416x^5}{5}+\frac{635x^4}{4}+237x^3+108x^2+108x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] 108*x + 108*x^2 + 237*x^3 + (635*x^4)/4 + (1416*x^5)/5 + (299*x^6)/3 + (157
1*x^7)/7 + (83*x^8)/8 + (922*x^9)/9 - 6*x^10 + (200*x^11)/11
```

---

**Maple [A]** time = 0.043, size = 55, normalized size = 0.8

$$108\,x+108\,x^2+237\,x^3+\frac{635\,x^4}{4}+\frac{1416\,x^5}{5}+\frac{299\,x^6}{3}+\frac{1571\,x^7}{7}+\frac{83\,x^8}{8}+\frac{922\,x^9}{9}-6\,x^{10}+\frac{200\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x)
```

```
[Out] 108*x+108*x^2+237*x^3+635/4*x^4+1416/5*x^5+299/3*x^6+1571/7*x^7+83/8*x^8+92
2/9*x^9-6*x^10+200/11*x^11
```

---

**Maxima [A]** time = 0.956996, size = 73, normalized size = 1.07

$$\frac{200}{11}\,x^{11} - 6\,x^{10} + \frac{922}{9}\,x^{9} + \frac{83}{8}\,x^{8} + \frac{1571}{7}\,x^{7} + \frac{299}{3}\,x^{6} + \frac{1416}{5}\,x^{5} + \frac{635}{4}\,x^{4} + 237\,x^{3} + 108\,x^{2} + 108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

---

**Fricas [A]** time = 0.850299, size = 166, normalized size = 2.44

$$\frac{200}{11}x^{11} - 6x^{10} + \frac{922}{9}x^{9} + \frac{83}{8}x^{8} + \frac{1571}{7}x^{7} + \frac{299}{3}x^{6} + \frac{1416}{5}x^{5} + \frac{635}{4}x^{4} + 237x^{3} + 108x^{2} + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

---

**Sympy [A]** time = 0.085544, size = 65, normalized size = 0.96

$$\frac{200x^{11}}{11} - 6x^{10} + \frac{922x^{9}}{9} + \frac{83x^{8}}{8} + \frac{1571x^{7}}{7} + \frac{299x^{6}}{3} + \frac{1416x^{5}}{5} + \frac{635x^{4}}{4} + 237x^{3} + 108x^{2} + 108x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2)**2,x)
```

```
[Out] 200*x**11/11 - 6*x**10 + 922*x**9/9 + 83*x**8/8 + 1571*x**7/7 + 299*x**6/3
+ 1416*x**5/5 + 635*x**4/4 + 237*x**3 + 108*x**2 + 108*x
```

---

**Giac [A]** time = 1.19404, size = 73, normalized size = 1.07

$$\frac{200}{11}\,x^{11} - 6\,x^{10} + \frac{922}{9}\,x^{9} + \frac{83}{8}\,x^{8} + \frac{1571}{7}\,x^{7} + \frac{299}{3}\,x^{6} + \frac{1416}{5}\,x^{5} + \frac{635}{4}\,x^{4} + 237\,x^{3} + 108\,x^{2} + 108\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 200/11*x^11 - 6*x^10 + 922/9*x^9 + 83/8*x^8 + 1571/7*x^7 + 299/3*x^6 + 1416
/5*x^5 + 635/4*x^4 + 237*x^3 + 108*x^2 + 108*x
```

## 3.33 $\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right) dx$

**Optimal**. Leaf size=56

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

---

**Rubi [A]** time = 0.0318655, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \left(3-x+2x^2\right)^3 \left(2+3x+5x^2\right) dx &= \int \left(54+27x+180x^2-20x^3+288x^4-83x^5+190x^6-36x^7+40x^8\right) dx\\ &= 54x+\frac{27x^2}{2}+60x^3-5x^4+\frac{288x^5}{5}-\frac{83x^6}{6}+\frac{190x^7}{7}-\frac{9x^8}{2}+\frac{40x^9}{9}\end{aligned}$$

**Mathematica [A]** time = 0.0012394, size = 56, normalized size = 1.

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2),x]
```

```
[Out] 54*x + (27*x^2)/2 + 60*x^3 - 5*x^4 + (288*x^5)/5 - (83*x^6)/6 + (190*x^7)/7
 - (9*x^8)/2 + (40*x^9)/9
```

---

**Maple [A]** time = 0.043, size = 45, normalized size = 0.8

$$54\,x+\frac{27\,x^2}{2}+60\,x^3-5\,x^4+\frac{288\,x^5}{5}-\frac{83\,x^6}{6}+\frac{190\,x^7}{7}-\frac{9\,x^8}{2}+\frac{40\,x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3*(5*x^2+3*x+2),x)
```

```
[Out] 54*x+27/2*x^2+60*x^3-5*x^4+288/5*x^5-83/6*x^6+190/7*x^7-9/2*x^8+40/9*x^9
```

---

**Maxima [A]** time = 0.967082, size = 59, normalized size = 1.05

$$\frac{40}{9}\,x^9-\frac{9}{2}\,x^8+\frac{190}{7}\,x^7-\frac{83}{6}\,x^6+\frac{288}{5}\,x^5-5\,x^4+60\,x^3+\frac{27}{2}\,x^2+54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

---

**Fricas [A]** time = 0.882358, size = 122, normalized size = 2.18

$$\frac{40}{9}x^9-\frac{9}{2}x^8+\frac{190}{7}x^7-\frac{83}{6}x^6+\frac{288}{5}x^5-5x^4+60x^3+\frac{27}{2}x^2+54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

---

**Sympy [A]** time = 0.107796, size = 53, normalized size = 0.95

$$\frac{40x^9}{9}-\frac{9x^8}{2}+\frac{190x^7}{7}-\frac{83x^6}{6}+\frac{288x^5}{5}-5x^4+60x^3+\frac{27x^2}{2}+54x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3*(5*x**2+3*x+2),x)
```

```
[Out] 40*x**9/9 - 9*x**8/2 + 190*x**7/7 - 83*x**6/6 + 288*x**5/5 - 5*x**4 + 60*x*
*3 + 27*x**2/2 + 54*x
```

---

**Giac [A]** time = 1.20353, size = 59, normalized size = 1.05

$$\frac{40}{9}\,x^9-\frac{9}{2}\,x^8+\frac{190}{7}\,x^7-\frac{83}{6}\,x^6+\frac{288}{5}\,x^5-5\,x^4+60\,x^3+\frac{27}{2}\,x^2+54\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 40/9*x^9 - 9/2*x^8 + 190/7*x^7 - 83/6*x^6 + 288/5*x^5 - 5*x^4 + 60*x^3 + 27
/2*x^2 + 54*x
```

## 3.34 $\int \frac{(3-x+2x^2)^3}{2+3x+5x^2} dx$

**Optimal**. Leaf size=70

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}+\frac{328757\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

```
[Out] (49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/2
5 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2 +
 3*x + 5*x^2])/31250
```

---

**Rubi [A]** time = 0.0529201, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1657, 634, 618, 204, 628}

$$\frac{8x^5}{25}-\frac{21x^4}{25}+\frac{1222x^3}{375}-\frac{7451x^2}{1250}-\frac{158389\log\left(5x^2+3x+2\right)}{31250}+\frac{49508x}{3125}+\frac{328757\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] (49508*x)/3125 - (7451*x^2)/1250 + (1222*x^3)/375 - (21*x^4)/25 + (8*x^5)/2
5 + (328757*ArcTan[(3 + 10*x)/Sqrt[31]])/(15625*Sqrt[31]) - (158389*Log[2 +
 3*x + 5*x^2])/31250
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(3-x+2x^2)^3}{2+3x+5x^2}\,dx = \int \left(\frac{49508}{3125}-\frac{7451x}{625}+\frac{1222x^2}{125}-\frac{84x^3}{25}+\frac{8x^4}{5}-\frac{1331(11+119x)}{3125(2+3x+5x^2)}\right)dx$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{1331\int \frac{11+119x}{2+3x+5x^2}\,dx}{3125}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\int \frac{3+10x}{2+3x+5x^2}\,dx}{31250}+\frac{328757\int \frac{1}{2+3x+5x^2}\,dx}{31250}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}-\frac{158389\log(2+3x+5x^2)}{31250}-\frac{328757\,\text{Subst}\left(\int\right.}{15}$$

$$= \frac{49508x}{3125}-\frac{7451x^2}{1250}+\frac{1222x^3}{375}-\frac{21x^4}{25}+\frac{8x^5}{25}+\frac{328757\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15625\sqrt{31}}-\frac{158389\log(2+3x+5}{31250}$$

**Mathematica [A]** time = 0.0224648, size = 63, normalized size = 0.9

$$\frac{31\left(5x\left(6000x^4-15750x^3+61100x^2-111765x+297048\right)-475167\log\left(5x^2+3x+2\right)\right)+1972542\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right.}{2906250}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2),x]
```

```
[Out] (1972542*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[31]] + 31*(5*x*(297048 - 111765*x
+ 61100*x^2 - 15750*x^3 + 6000*x^4) - 475167*Log[2 + 3*x + 5*x^2]))/2906250
```

---

**Maple [A]** time = 0.046, size = 54, normalized size = 0.8

$$\frac{49508\,x}{3125}-\frac{7451\,x^2}{1250}+\frac{1222\,x^3}{375}-\frac{21\,x^4}{25}+\frac{8\,x^5}{25}-\frac{158389\,\ln\left(5\,x^2+3\,x+2\right)}{31250}+\frac{328757\,\sqrt{31}}{484375}\arctan\left(\frac{(3+10\,x)\sqrt{31}}{31}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2),x)
```

```
[Out] 49508/3125*x-7451/1250*x^2+1222/375*x^3-21/25*x^4+8/25*x^5-158389/31250*ln(
5*x^2+3*x+2)+328757/484375*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)
```

---

**Maxima [A]** time = 1.46638, size = 72, normalized size = 1.03

$$\frac{8}{25}x^5-\frac{21}{25}x^4+\frac{1222}{375}x^3-\frac{7451}{1250}x^2+\frac{328757}{484375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)+\frac{49508}{3125}x-\frac{158389}{31250}\log\left(5\,x^2+3\,x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

---

**Fricas [A]** time = 0.991387, size = 219, normalized size = 3.13

$$\frac{8}{25}\,x^5 - \frac{21}{25}\,x^4 + \frac{1222}{375}\,x^3 - \frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{49508}{3125}\,x - \frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

---

**Sympy [A]** time = 0.149317, size = 76, normalized size = 1.09

$$\frac{8x^5}{25} - \frac{21x^4}{25} + \frac{1222x^3}{375} - \frac{7451x^2}{1250} + \frac{49508x}{3125} - \frac{158389\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{31250} + \frac{328757\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{484375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] 8*x**5/25 - 21*x**4/25 + 1222*x**3/375 - 7451*x**2/1250 + 49508*x/3125 - 15
8389*log(x**2 + 3*x/5 + 2/5)/31250 + 328757*sqrt(31)*atan(10*sqrt(31)*x/31
+ 3*sqrt(31)/31)/484375
```

---

**Giac [A]** time = 1.187, size = 72, normalized size = 1.03

$$\frac{8}{25}\,x^5 - \frac{21}{25}\,x^4 + \frac{1222}{375}\,x^3 - \frac{7451}{1250}\,x^2 + \frac{328757}{484375}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x+3)\right) + \frac{49508}{3125}\,x - \frac{158389}{31250}\,\log\left(5\,x^2+3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 8/25*x^5 - 21/25*x^4 + 1222/375*x^3 - 7451/1250*x^2 + 328757/484375*sqrt(31
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 49508/3125*x - 158389/31250*log(5*x^2
+ 3*x + 2)
```

## 3.35 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2} dx$

**Optimal**. Leaf size=77

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}-\frac{10769\log(5x^2+3x+2)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

---

**Rubi [A]** time = 0.0722482, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}-\frac{10769\log(5x^2+3x+2)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^2}\,dx &= \frac{1331(443+247x)}{96875(2+3x+5x^2)}+\frac{1}{31}\int \frac{\frac{372701}{625}-\frac{230981x}{625}+\frac{37882x^2}{125}-\frac{2604x^3}{25}+\frac{248x^4}{5}}{2+3x+5x^2}\,dx \\
&= \frac{1331(443+247x)}{96875(2+3x+5x^2)}+\frac{1}{31}\int \left(\frac{45446}{625}-\frac{3348x}{125}+\frac{248x^2}{25}+\frac{121(2329-2759x)}{625(2+3x+5x^2)}\right)dx \\
&= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875(2+3x+5x^2)}+\frac{121\int \frac{2329-2759x}{2+3x+5x^2}\,dx}{19375} \\
&= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875(2+3x+5x^2)}-\frac{10769\int \frac{3+10x}{2+3x+5x^2}\,dx}{6250}+\frac{3819607\int \frac{1}{2+3x+5x^2}\,dx}{193750} \\
&= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875(2+3x+5x^2)}-\frac{10769\log(2+3x+5x^2)}{6250}-\frac{3819607\,\mathrm{Subst}\left(\int \right.}{9} \\
&= \frac{1466x}{625}-\frac{54x^2}{125}+\frac{8x^3}{75}+\frac{1331(443+247x)}{96875(2+3x+5x^2)}+\frac{3819607\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{96875\sqrt{31}}-\frac{10769\log(2+3x+}{6250}
\end{aligned}$$

**Mathematica [A]** time = 0.026893, size = 77, normalized size = 1.

$$\frac{8x^3}{75}-\frac{54x^2}{125}+\frac{1331(247x+443)}{96875(5x^2+3x+2)}-\frac{10769\log(5x^2+3x+2)}{6250}+\frac{1466x}{625}+\frac{3819607\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{96875\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (1466*x)/625 - (54*x^2)/125 + (8*x^3)/75 + (1331*(443 + 247*x))/(96875*(2 +
 3*x + 5*x^2)) + (3819607*ArcTan[(3 + 10*x)/Sqrt[31]])/(96875*Sqrt[31]) - (
10769*Log[2 + 3*x + 5*x^2])/6250
```

---

**Maple [A]** time = 0.046, size = 61, normalized size = 0.8

$$\frac{8\,x^3}{75}-\frac{54\,x^2}{125}+\frac{1466\,x}{625}-\frac{121}{625}\left(-\frac{2717\,x}{775}-\frac{4873}{775}\right)\left(x^2+\frac{3\,x}{5}+\frac{2}{5}\right)^{-1}-\frac{10769\,\ln(5\,x^2+3\,x+2)}{6250}+\frac{3819607\,\sqrt{31}}{3003125}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)
```

```
[Out] 8/75*x^3-54/125*x^2+1466/625*x-121/625*(-2717/775*x-4873/775)/(x^2+3/5*x+2/
5)-10769/6250*ln(5*x^2+3*x+2)+3819607/3003125*arctan(1/31*(3+10*x)*31^(1/2)
)*31^(1/2)
```

---

**Maxima [A]** time = 1.47863, size = 84, normalized size = 1.09

$$\frac{8}{75}x^3 - \frac{54}{125}x^2 + \frac{3819607}{3003125}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{1466}{625}x + \frac{1331(247x+443)}{96875(5x^2+3x+2)} - \frac{10769}{6250}\log\left(5x^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x
 + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/62
50*log(5*x^2 + 3*x + 2)
```

---

**Fricas [A]** time = 1.03024, size = 321, normalized size = 4.17

$$\frac{9610000x^5 - 33154500x^4 + 191815600x^3 + 22917642\sqrt{31}(5x^2+3x+2)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + 111226140x}{18018750(5x^2+3x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/18018750*(9610000*x^5 - 33154500*x^4 + 191815600*x^3 + 22917642*sqrt(31)*
(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x + 3)) + 111226140*x^2 - 310470
27*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 145678362*x + 109671738)/(5*x^2
 + 3*x + 2)
```

---

**Sympy [A]** time = 0.17165, size = 78, normalized size = 1.01

$$\frac{8x^3}{75} - \frac{54x^2}{125} + \frac{1466x}{625} + \frac{328757x + 589633}{484375x^2 + 290625x + 193750} - \frac{10769\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{6250} + \frac{3819607\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}x}{31} + \frac{3\sqrt{}}{3}\right.}{3003125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

```
[Out] 8*x**3/75 - 54*x**2/125 + 1466*x/625 + (328757*x + 589633)/(484375*x**2 + 2
90625*x + 193750) - 10769*log(x**2 + 3*x/5 + 2/5)/6250 + 3819607*sqrt(31)*a
tan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3003125
```

---

**Giac [A]** time = 1.20751, size = 84, normalized size = 1.09

$$\frac{8}{75}\,x^{3} - \frac{54}{125}\,x^{2} + \frac{3819607}{3003125}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x + 3)\right) + \frac{1466}{625}\,x + \frac{1331\,(247\,x + 443)}{96875\,\left(5\,x^{2} + 3\,x + 2\right)} - \frac{10769}{6250}\,\log\left(5\,x^{2} + 3\,x +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 8/75*x^3 - 54/125*x^2 + 3819607/3003125*sqrt(31)*arctan(1/31*sqrt(31)*(10*x
 + 3)) + 1466/625*x + 1331/96875*(247*x + 443)/(5*x^2 + 3*x + 2) - 10769/62
50*log(5*x^2 + 3*x + 2)
```

## 3.36 $\int \frac{(3-x+2x^2)^3}{(2+3x+5x^2)^3}\,dx$

**Optimal**. Leaf size=84

$$\frac{121(342840x+188381)}{6006250\left(5x^2+3x+2\right)}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}-\frac{66}{625}\log\left(5x^2+3x+2\right)+\frac{8x}{125}+\frac{11341176\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

```
[Out] (8*x)/125 + (1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + (121*(18838
1 + 342840*x))/(6006250*(2 + 3*x + 5*x^2)) + (11341176*ArcTan[(3 + 10*x)/Sq
rt[31]])/(600625*Sqrt[31]) - (66*Log[2 + 3*x + 5*x^2])/625
```

---

**Rubi [A]** time = 0.0865293, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(342840x+188381)}{6006250\left(5x^2+3x+2\right)}+\frac{1331(247x+443)}{193750\left(5x^2+3x+2\right)^2}-\frac{66}{625}\log\left(5x^2+3x+2\right)+\frac{8x}{125}+\frac{11341176\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{600625\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (8*x)/125 + (1331*(443 + 247*x))/(193750*(2 + 3*x + 5*x^2)^2) + (121*(18838
1 + 342840*x))/(6006250*(2 + 3*x + 5*x^2)) + (11341176*ArcTan[(3 + 10*x)/Sq
rt[31]])/(600625*Sqrt[31]) - (66*Log[2 + 3*x + 5*x^2])/625
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^3}{\left(2+3x+5x^2\right)^3}\,dx = \frac{1331(443+247x)}{193750\left(2+3x+5x^2\right)^2}+\frac{1}{62}\int \frac{\frac{4055767}{3125}-\frac{461962x}{625}+\frac{75764x^2}{125}-\frac{5208x^3}{25}+\frac{496x^4}{5}}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{1331(443+247x)}{193750\left(2+3x+5x^2\right)^2}+\frac{121(188381+342840x)}{6006250\left(2+3x+5x^2\right)}+\frac{\int \frac{\frac{2222876}{125}-\frac{207576x}{125}+\frac{15376x^2}{25}}{2+3x+5x^2}\,dx}{1922}$$

$$= \frac{1331(443+247x)}{193750\left(2+3x+5x^2\right)^2}+\frac{121(188381+342840x)}{6006250\left(2+3x+5x^2\right)}+\frac{\int \left(\frac{15376}{125}+\frac{132(16607-1922x)}{125\left(2+3x+5x^2\right)}\right)dx}{1922}$$

$$= \frac{8x}{125}+\frac{1331(443+247x)}{193750\left(2+3x+5x^2\right)^2}+\frac{121(188381+342840x)}{6006250\left(2+3x+5x^2\right)}+\frac{66\int \frac{16607-1922x}{2+3x+5x^2}\,dx}{120125}$$

$$= \frac{8x}{125}+\frac{1331(443+247x)}{193750\left(2+3x+5x^2\right)^2}+\frac{121(188381+342840x)}{6006250\left(2+3x+5x^2\right)}-\frac{66}{625}\int \frac{3+10x}{2+3x+5x^2}\,dx+\frac{5670588}{}$$

$$= \frac{8x}{125}+\frac{1331(443+247x)}{193750\left(2+3x+5x^2\right)^2}+\frac{121(188381+342840x)}{6006250\left(2+3x+5x^2\right)}-\frac{66}{625}\log\left(2+3x+5x^2\right)-\frac{1134117}{}$$

$$= \frac{8x}{125}+\frac{1331(443+247x)}{193750\left(2+3x+5x^2\right)^2}+\frac{121(188381+342840x)}{6006250\left(2+3x+5x^2\right)}+\frac{11341176\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{600625\sqrt{31}}-\frac{66}{625}\log$$

**Mathematica [A]** time = 0.037298, size = 78, normalized size = 0.93

$$\frac{\frac{3751(342840x+188381)}{5x^2+3x+2}+\frac{1279091(247x+443)}{\left(5x^2+3x+2\right)^2}-19662060\log\left(5x^2+3x+2\right)+11916400x+113411760\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{186193750}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^3/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (11916400*x + (1279091*(443 + 247*x))/(2 + 3*x + 5*x^2)^2 + (3751*(188381 +
 342840*x))/(2 + 3*x + 5*x^2) + 113411760*Sqrt[31]*ArcTan[(3 + 10*x)/Sqrt[3
1]] - 19662060*Log[2 + 3*x + 5*x^2])/186193750
```

---

**Maple [A]** time = 0.048, size = 63, normalized size = 0.8

$$\frac{8x}{125} - \frac{11}{5\left(5x^2+3x+2\right)^2}\left(-\frac{377124x^3}{24025} - \frac{866987x^2}{48050} - \frac{293711x}{24025} - \frac{232243}{48050}\right) - \frac{66\ln\left(5x^2+3x+2\right)}{625} + \frac{11341176\sqrt{3}}{18619375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)
```

```
[Out] 8/125*x-11/5*(-377124/24025*x^3-866987/48050*x^2-293711/24025*x-232243/4805
0)/(5*x^2+3*x+2)^2-66/625*ln(5*x^2+3*x+2)+11341176/18619375*arctan(1/31*(3+
10*x)*31^(1/2))*31^(1/2)
```

---

**Maxima [A]** time = 1.44198, size = 97, normalized size = 1.15

$$\frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) + \frac{8}{125}x + \frac{121\left(68568x^3+78817x^2+53402x+21113\right)}{240250\left(25x^4+30x^3+29x^2+12x+4\right)} - \frac{66}{625}\log\left(5x^2 +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121
/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(25*x^4 + 30*x^3 + 29*x^2
 + 12*x + 4) - 66/625*log(5*x^2 + 3*x + 2)
```

---

**Fricas [A]** time = 1.01191, size = 406, normalized size = 4.83

$$\frac{59582000x^5 + 71498400x^4 + 1355107960x^3 + 22682352\sqrt{31}\left(25x^4+30x^3+29x^2+12x+4\right)\arctan\left(\frac{1}{31}\sqrt{31}(10x\right.}{37238750\left(25x^4+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/37238750*(59582000*x^5 + 71498400*x^4 + 1355107960*x^3 + 22682352*sqrt(31
)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) +
1506812195*x^2 - 3932412*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 +
3*x + 2) + 1011087630*x + 395974315)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Sympy [A]** time = 0.216081, size = 85, normalized size = 1.01

$$\frac{8x}{125} + \frac{8296728x^3 + 9536857x^2 + 6461642x + 2554673}{6006250x^4 + 7207500x^3 + 6967250x^2 + 2883000x + 961000} - \frac{66\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{625} + \frac{11341176\sqrt{31}\,\mathrm{atan}\left(1\right.}{18619375}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

```
[Out] 8*x/125 + (8296728*x**3 + 9536857*x**2 + 6461642*x + 2554673)/(6006250*x**4
 + 7207500*x**3 + 6967250*x**2 + 2883000*x + 961000) - 66*log(x**2 + 3*x/5
+ 2/5)/625 + 11341176*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/18619
375
```

---

**Giac [A]** time = 1.21183, size = 84, normalized size = 1.

$$\frac{11341176}{18619375}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{8}{125}x+\frac{121\left(68568x^3+78817x^2+53402x+21113\right)}{240250\left(5x^2+3x+2\right)^2}-\frac{66}{625}\log\left(5x^2+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 11341176/18619375*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 8/125*x + 121
/240250*(68568*x^3 + 78817*x^2 + 53402*x + 21113)/(5*x^2 + 3*x + 2)^2 - 66/
625*log(5*x^2 + 3*x + 2)
```

## 3.37 $\int \frac{(2+3x+5x^2)^4}{3-x+2x^2} dx$

**Optimal**. Leaf size=84

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}+\frac{1156639\tan^{-1}}{256\sqrt{2}}$$

```
[Out] (122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*x
^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512
```

---

**Rubi [A]** time = 0.056764, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1657, 634, 618, 204, 628}

$$\frac{625x^7}{14}+\frac{3625x^6}{24}+\frac{1855x^5}{8}+\frac{6245x^4}{64}-\frac{21229x^3}{96}-\frac{28747x^2}{128}+\frac{307461}{512}\log\left(2x^2-x+3\right)+\frac{122691x}{128}+\frac{1156639\tan^{-1}}{256\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

```
[Out] (122691*x)/128 - (28747*x^2)/128 - (21229*x^3)/96 + (6245*x^4)/64 + (1855*x
^5)/8 + (3625*x^6)/24 + (625*x^7)/14 + (1156639*ArcTan[(1 - 4*x)/Sqrt[23]])
/(256*Sqrt[23]) + (307461*Log[3 - x + 2*x^2])/512
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^4}{3-x+2x^2}\,dx = \int \left(\frac{122691}{128} - \frac{28747x}{64} - \frac{21229x^2}{32} + \frac{6245x^3}{16} + \frac{9275x^4}{8} + \frac{3625x^5}{4} + \frac{625x^6}{2} - \frac{14641(25-21x)}{128(3-x+2x^2)}\right.$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} - \frac{14641}{128}\int \frac{25-21x}{3-x+2x}$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512}\int \frac{-1+4}{3-x+}$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{307461}{512}\log(3-x+$$

$$= \frac{122691x}{128} - \frac{28747x^2}{128} - \frac{21229x^3}{96} + \frac{6245x^4}{64} + \frac{1855x^5}{8} + \frac{3625x^6}{24} + \frac{625x^7}{14} + \frac{1156639\tan^{-1}\left(\frac{1-4}{\sqrt{23}}\right.}{256\sqrt{23}}$$

**Mathematica [A]** time = 0.0268885, size = 72, normalized size = 0.86

$$\frac{x(120000x^6+406000x^5+623280x^4+262290x^3-594412x^2-603687x+2576511)}{2688} + \frac{307461}{512}\log(2x^2-x+3) - \frac{115}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2),x]
```

```
[Out] (x*(2576511 - 603687*x - 594412*x^2 + 262290*x^3 + 623280*x^4 + 406000*x^5
+ 120000*x^6))/2688 - (1156639*ArcTan[(-1 + 4*x)/Sqrt[23]])/(256*Sqrt[23])
+ (307461*Log[3 - x + 2*x^2])/512
```

---

**Maple [A]** time = 0.046, size = 64, normalized size = 0.8

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} + \frac{307461\ln(2x^2-x+3)}{512} - \frac{1156639\sqrt{23}}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3),x)
```

```
[Out] 625/14*x^7+3625/24*x^6+1855/8*x^5+6245/64*x^4-21229/96*x^3-28747/128*x^2+12
2691/128*x+307461/512*ln(2*x^2-x+3)-1156639/5888*23^(1/2)*arctan(1/23*(-1+4
*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.43462, size = 85, normalized size = 1.01

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}x +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/
128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12
8*x + 307461/512*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 0.975225, size = 252, normalized size = 3.

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/
128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12
8*x + 307461/512*log(2*x^2 - x + 3)
```

---

**Sympy [A]** time = 0.156462, size = 87, normalized size = 1.04

$$\frac{625x^7}{14} + \frac{3625x^6}{24} + \frac{1855x^5}{8} + \frac{6245x^4}{64} - \frac{21229x^3}{96} - \frac{28747x^2}{128} + \frac{122691x}{128} + \frac{307461\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{512} - \frac{1156639\sqrt{23}\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3),x)
```

```
[Out] 625*x**7/14 + 3625*x**6/24 + 1855*x**5/8 + 6245*x**4/64 - 21229*x**3/96 - 2
8747*x**2/128 + 122691*x/128 + 307461*log(x**2 - x/2 + 3/2)/512 - 1156639*s
qrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/5888
```

---

**Giac [A]** time = 1.25004, size = 85, normalized size = 1.01

$$\frac{625}{14}x^7 + \frac{3625}{24}x^6 + \frac{1855}{8}x^5 + \frac{6245}{64}x^4 - \frac{21229}{96}x^3 - \frac{28747}{128}x^2 - \frac{1156639}{5888}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{122691}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 625/14*x^7 + 3625/24*x^6 + 1855/8*x^5 + 6245/64*x^4 - 21229/96*x^3 - 28747/
128*x^2 - 1156639/5888*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 122691/12
8*x + 307461/512*log(2*x^2 - x + 3)
```

## 3.38 $\int \frac{(2+3x+5x^2)^3}{3-x+2x^2} dx$

**Optimal**. Leaf size=70

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

```
[Out] (-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (5
9895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/
128
```

---

**Rubi [A]** time = 0.0560196, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}+\frac{1331}{128}\log\left(2x^2-x+3\right)-\frac{4795x}{32}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

```
[Out] (-4795*x)/32 - (829*x^2)/32 + (965*x^3)/24 + (575*x^4)/16 + (25*x^5)/2 - (5
9895*ArcTan[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (1331*Log[3 - x + 2*x^2])/
128
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^3}{3-x+2x^2}\,dx = \int \left(-\frac{4795}{32}-\frac{829x}{16}+\frac{965x^2}{8}+\frac{575x^3}{4}+\frac{125x^4}{2}+\frac{1331(11+x)}{32(3-x+2x^2)}\right)dx$$
$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{32}\int \frac{11+x}{3-x+2x^2}\,dx$$
$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{128}\int \frac{-1+4x}{3-x+2x^2}\,dx+\frac{59895}{128}\int \frac{1}{3-x+2}$$
$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}+\frac{1331}{128}\log(3-x+2x^2)-\frac{59895}{64}\text{Subst}\left(\int -\right.$$
$$= -\frac{4795x}{32}-\frac{829x^2}{32}+\frac{965x^3}{24}+\frac{575x^4}{16}+\frac{25x^5}{2}-\frac{59895\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{23}}+\frac{1331}{128}\log(3-x+2x^2)$$

**Mathematica [A]** time = 0.0210563, size = 63, normalized size = 0.9

$$\frac{1}{384}\left(4x\left(1200x^4+3450x^3+3860x^2-2487x-14385\right)+3993\log\left(2x^2-x+3\right)\right)+\frac{59895\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2),x]
```

```
[Out] (59895*ArcTan[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[23]) + (4*x*(-14385 - 2487*x +
 3860*x^2 + 3450*x^3 + 1200*x^4) + 3993*Log[3 - x + 2*x^2])/384
```

---

**Maple [A]** time = 0.047, size = 54, normalized size = 0.8

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\ln\left(2x^2-x+3\right)}{128}+\frac{59895\sqrt{23}}{1472}\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3),x)
```

```
[Out] 25/2*x^5+575/16*x^4+965/24*x^3-829/32*x^2-4795/32*x+1331/128*ln(2*x^2-x+3)+
59895/1472*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.47194, size = 72, normalized size = 1.03

$$\frac{25}{2}x^5+\frac{575}{16}x^4+\frac{965}{24}x^3-\frac{829}{32}x^2+\frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4795}{32}x+\frac{1331}{128}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 0.935331, size = 196, normalized size = 2.8

$$\frac{25}{2}x^5+\frac{575}{16}x^4+\frac{965}{24}x^3-\frac{829}{32}x^2+\frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4795}{32}x+\frac{1331}{128}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

---

**Sympy [A]** time = 0.143687, size = 73, normalized size = 1.04

$$\frac{25x^5}{2}+\frac{575x^4}{16}+\frac{965x^3}{24}-\frac{829x^2}{32}-\frac{4795x}{32}+\frac{1331\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{128}+\frac{59895\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{1472}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3),x)
```

```
[Out] 25*x**5/2 + 575*x**4/16 + 965*x**3/24 - 829*x**2/32 - 4795*x/32 + 1331*log(
x**2 - x/2 + 3/2)/128 + 59895*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/
1472
```

---

**Giac [A]** time = 1.16731, size = 72, normalized size = 1.03

$$\frac{25}{2}x^5+\frac{575}{16}x^4+\frac{965}{24}x^3-\frac{829}{32}x^2+\frac{59895}{1472}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{4795}{32}x+\frac{1331}{128}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 25/2*x^5 + 575/16*x^4 + 965/24*x^3 - 829/32*x^2 + 59895/1472*sqrt(23)*arcta
n(1/23*sqrt(23)*(4*x - 1)) - 4795/32*x + 1331/128*log(2*x^2 - x + 3)
```

## 3.39 $\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx$

**Optimal**. Leaf size=56

$$\frac{25x^3}{6}+\frac{85x^2}{8}-\frac{363}{32}\log\left(2x^2-x+3\right)+\frac{51x}{8}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

```
[Out] (51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*S
qrt[23]) - (363*Log[3 - x + 2*x^2])/32
```

---

**Rubi [A]** time = 0.0503026, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1657, 634, 618, 204, 628}

$$\frac{25x^3}{6}+\frac{85x^2}{8}-\frac{363}{32}\log\left(2x^2-x+3\right)+\frac{51x}{8}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] (51*x)/8 + (85*x^2)/8 + (25*x^3)/6 + (847*ArcTan[(1 - 4*x)/Sqrt[23]])/(16*S
qrt[23]) - (363*Log[3 - x + 2*x^2])/32
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x+5x^2)^2}{3-x+2x^2}\,dx &= \int \left(\frac{51}{8}+\frac{85x}{4}+\frac{25x^2}{2}-\frac{121(1+3x)}{8(3-x+2x^2)}\right)dx\\ &= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{121}{8}\int \frac{1+3x}{3-x+2x^2}\,dx\\ &= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{363}{32}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{847}{32}\int \frac{1}{3-x+2x^2}\,dx\\ &= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}-\frac{363}{32}\log\left(3-x+2x^2\right)+\frac{847}{16}\,\mathrm{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right)\\ &= \frac{51x}{8}+\frac{85x^2}{8}+\frac{25x^3}{6}+\frac{847\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{16\sqrt{23}}-\frac{363}{32}\log\left(3-x+2x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.0168558, size = 52, normalized size = 0.93

$$\frac{1}{24}x\left(100x^2+255x+153\right)-\frac{363}{32}\log\left(2x^2-x+3\right)-\frac{847\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{16\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2),x]
```

```
[Out] (x*(153 + 255*x + 100*x^2))/24 - (847*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16*Sqrt
[23]) - (363*Log[3 - x + 2*x^2])/32
```

---

**Maple [A]** time = 0.049, size = 44, normalized size = 0.8

$$\frac{25\,x^3}{6}+\frac{85\,x^2}{8}+\frac{51\,x}{8}-\frac{363\,\ln\left(2\,x^2-x+3\right)}{32}-\frac{847\,\sqrt{23}}{368}\arctan\left(\frac{(-1+4\,x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3),x)
```

```
[Out] 25/6*x^3+85/8*x^2+51/8*x-363/32*ln(2*x^2-x+3)-847/368*23^(1/2)*arctan(1/23*
(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.43979, size = 58, normalized size = 1.04

$$\frac{25}{6}\,x^3+\frac{85}{8}\,x^2-\frac{847}{368}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{51}{8}\,x-\frac{363}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 0.993869, size = 147, normalized size = 2.62

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32}\log\left(2x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

---

**Sympy [A]** time = 0.197072, size = 60, normalized size = 1.07

$$\frac{25x^3}{6} + \frac{85x^2}{8} + \frac{51x}{8} - \frac{363\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{32} - \frac{847\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{368}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3),x)
```

```
[Out] 25*x**3/6 + 85*x**2/8 + 51*x/8 - 363*log(x**2 - x/2 + 3/2)/32 - 847*sqrt(23
)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/368
```

---

**Giac [A]** time = 1.12605, size = 58, normalized size = 1.04

$$\frac{25}{6}x^3 + \frac{85}{8}x^2 - \frac{847}{368}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) + \frac{51}{8}x - \frac{363}{32}\log\left(2x^2 - x + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] 25/6*x^3 + 85/8*x^2 - 847/368*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 51
/8*x - 363/32*log(2*x^2 - x + 3)
```

# 3.40 $\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx$

**Optimal**. Leaf size=42

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

```
[Out] (5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2*
x^2])/8
```

---

**Rubi [A]** time = 0.0348584, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1657, 634, 618, 204, 628}

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

```
[Out] (5*x)/2 + (33*ArcTan[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2*
x^2])/8
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x+5x^2}{3-x+2x^2}\,dx &= \int \left(\frac{5}{2}-\frac{11(1-x)}{2\left(3-x+2x^2\right)}\right)dx\\
&= \frac{5x}{2}-\frac{11}{2}\int \frac{1-x}{3-x+2x^2}\,dx\\
&= \frac{5x}{2}+\frac{11}{8}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{33}{8}\int \frac{1}{3-x+2x^2}\,dx\\
&= \frac{5x}{2}+\frac{11}{8}\log\left(3-x+2x^2\right)+\frac{33}{4}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right)\\
&= \frac{5x}{2}+\frac{33\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{23}}+\frac{11}{8}\log\left(3-x+2x^2\right)
\end{aligned}
$$

**Mathematica [A]** time = 0.009824, size = 42, normalized size = 1.

$$\frac{11}{8}\log\left(2x^2-x+3\right)+\frac{5x}{2}-\frac{33\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2),x]
```

```
[Out] (5*x)/2 - (33*ArcTan[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[23]) + (11*Log[3 - x + 2
*x^2])/8
```

---

**Maple [A]** time = 0.044, size = 34, normalized size = 0.8

$$\frac{5\,x}{2}+\frac{11\,\ln\left(2\,x^2-x+3\right)}{8}-\frac{33\,\sqrt{23}}{92}\arctan\left(\frac{(-1+4\,x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3),x)
```

```
[Out] 5/2*x+11/8*ln(2*x^2-x+3)-33/92*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.43148, size = 45, normalized size = 1.07

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)+\frac{5}{2}\,x+\frac{11}{8}\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="maxima")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

---

**Fricas [A]** time = 1.05056, size = 112, normalized size = 2.67

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{5}{2}x+\frac{11}{8}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="fricas")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

---

**Sympy [A]** time = 0.123831, size = 46, normalized size = 1.1

$$\frac{5x}{2}+\frac{11\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{33\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{92}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3),x)
```

```
[Out] 5*x/2 + 11*log(x**2 - x/2 + 3/2)/8 - 33*sqrt(23)*atan(4*sqrt(23)*x/23 - sqr
t(23)/23)/92
```

---

**Giac [A]** time = 1.19791, size = 45, normalized size = 1.07

$$-\frac{33}{92}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{5}{2}x+\frac{11}{8}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3),x, algorithm="giac")
```

```
[Out] -33/92*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 5/2*x + 11/8*log(2*x^2 -
x + 3)
```

## 3.41 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)} dx$

**Optimal**. Leaf size=73

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)+\frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

```
[Out] (3*ArcTan[(1 - 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

---

**Rubi [A]** time = 0.0533335, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {980, 634, 618, 204, 628}

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)+\frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```


```
[Out] (3*ArcTan[(1 - 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```


Rule 980

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2
)), x_Symbol] :> With[{q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*
f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(c^2*d - b*c*e + b^2*f - a*c*f - (c^2
*e - b*c*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*e^2 - c*d*f - b
*e*f + a*f^2 + (c*e*f - b*f^2)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0
]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)}\,dx &= \frac{1}{242}\int \frac{-11-22x}{3-x+2x^2}\,dx + \frac{1}{242}\int \frac{88+55x}{2+3x+5x^2}\,dx\\ &= -\left(\frac{1}{44}\int \frac{-1+4x}{3-x+2x^2}\,dx\right) + \frac{1}{44}\int \frac{3+10x}{2+3x+5x^2}\,dx - \frac{3}{44}\int \frac{1}{3-x+2x^2}\,dx + \frac{13}{44}\int\\ &= -\frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x+5x^2\right) + \frac{3}{22}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx, x, -1\right.\\ &= \frac{3\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{22\sqrt{23}} + \frac{13\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{22\sqrt{31}} - \frac{1}{44}\log\left(3-x+2x^2\right) + \frac{1}{44}\log\left(2+3x+5x^2\right)\end{aligned}$$

**Mathematica [A]** time = 0.0307099, size = 73, normalized size = 1.

$$-\frac{1}{44}\log\left(2x^2-x+3\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{3\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{22\sqrt{23}}+\frac{13\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{22\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (-3*ArcTan[(-1 + 4*x)/Sqrt[23]])/(22*Sqrt[23]) + (13*ArcTan[(3 + 10*x)/Sqrt
[31]])/(22*Sqrt[31]) - Log[3 - x + 2*x^2]/44 + Log[2 + 3*x + 5*x^2]/44
```

---

**Maple [A]** time = 0.048, size = 60, normalized size = 0.8

$$\frac{\ln\left(5x^2+3x+2\right)}{44}+\frac{13\sqrt{31}}{682}\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)-\frac{\ln\left(2x^2-x+3\right)}{44}-\frac{3\sqrt{23}}{506}\arctan\left(\frac{(-1+4x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2),x)
```

```
[Out] 1/44*ln(5*x^2+3*x+2)+13/682*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)-1/44*ln
(2*x^2-x+3)-3/506*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.42659, size = 80, normalized size = 1.1

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{1}{44}\log\left(2x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

---

**Fricas [A]** time = 1.00017, size = 207, normalized size = 2.84

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{1}{44}\log\left(2x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

---

**Sympy [A]** time = 0.267898, size = 83, normalized size = 1.14

$$-\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{44}+\frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{44}-\frac{3\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{506}+\frac{13\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{31}x}{31}+\frac{3\sqrt{31}}{31}\right)}{682}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2),x)
```

```
[Out] -log(x**2 - x/2 + 3/2)/44 + log(x**2 + 3*x/5 + 2/5)/44 - 3*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/506 + 13*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sq
rt(31)/31)/682
```

---

**Giac [A]** time = 1.26467, size = 80, normalized size = 1.1

$$\frac{13}{682}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{3}{506}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{1}{44}\log\left(5x^2+3x+2\right)-\frac{1}{44}\log\left(2x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 13/682*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3/506*sqrt(23)*arctan(1/
23*sqrt(23)*(4*x - 1)) + 1/44*log(5*x^2 + 3*x + 2) - 1/44*log(2*x^2 - x + 3
)
```

## 3.42 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx$

**Optimal**. Leaf size=94

$$\frac{65x+4}{682\left(5x^2+3x+2\right)}+\frac{3}{968}\log\left(2x^2-x+3\right)-\frac{3}{968}\log\left(5x^2+3x+2\right)+\frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + (7*ArcTan[(1 - 4*x)/Sqrt[23]])/(484*Sq
rt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

---

**Rubi [A]** time = 0.0886149, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{682\left(5x^2+3x+2\right)}+\frac{3}{968}\log\left(2x^2-x+3\right)-\frac{3}{968}\log\left(5x^2+3x+2\right)+\frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) + (7*ArcTan[(1 - 4*x)/Sqrt[23]])/(484*Sq
rt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2}\,dx &= \frac{4+65x}{682\left(2+3x+5x^2\right)}-\frac{\int \frac{-1804+1397x-1430x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}\,dx}{7502}\\
&= \frac{4+65x}{682\left(2+3x+5x^2\right)}-\frac{\int \frac{18755-22506x}{3-x+2x^2}\,dx}{1815484}-\frac{\int \frac{-158026+56265x}{2+3x+5x^2}\,dx}{1815484}\\
&= \frac{4+65x}{682\left(2+3x+5x^2\right)}+\frac{3}{968}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{3}{968}\int \frac{3+10x}{2+3x+5x^2}\,dx-\frac{7}{968}\\
&= \frac{4+65x}{682\left(2+3x+5x^2\right)}+\frac{3}{968}\log\left(3-x+2x^2\right)-\frac{3}{968}\log\left(2+3x+5x^2\right)+\frac{7}{484}\,\mathrm{S}\\
&= \frac{4+65x}{682\left(2+3x+5x^2\right)}+\frac{7\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{15004\sqrt{31}}+\frac{3}{968}\log\left(3-x+\right.
\end{aligned}$$

**Mathematica [A]** time = 0.0779902, size = 94, normalized size = 1.

$$\frac{65x+4}{682\left(5x^2+3x+2\right)}+\frac{3}{968}\log\left(2x^2-x+3\right)-\frac{3}{968}\log\left(5x^2+3x+2\right)-\frac{7\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{484\sqrt{23}}+\frac{2891\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{15004\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (4 + 65*x)/(682*(2 + 3*x + 5*x^2)) - (7*ArcTan[(-1 + 4*x)/Sqrt[23]])/(484*S
qrt[23]) + (2891*ArcTan[(3 + 10*x)/Sqrt[31]])/(15004*Sqrt[31]) + (3*Log[3 -
 x + 2*x^2])/968 - (3*Log[2 + 3*x + 5*x^2])/968
```

---

**Maple [A]** time = 0.049, size = 77, normalized size = 0.8

$$-\frac{1}{484}\left(-\frac{286\,x}{31}-\frac{88}{155}\right)\left(x^{2}+\frac{3\,x}{5}+\frac{2}{5}\right)^{-1}-\frac{3\,\ln\left(5\,x^{2}+3\,x+2\right)}{968}+\frac{2891\,\sqrt{31}}{465124}\arctan\left(\frac{(3+10\,x)\,\sqrt{31}}{31}\right)+\frac{3\,\ln\left(2\,x^{2}-x+\right.}{968}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/484*(-286/31*x-88/155)/(x^2+3/5*x+2/5)-3/968*ln(5*x^2+3*x+2)+2891/465124
*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+3/968*ln(2*x^2-x+3)-7/11132*23^(1/
2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.48301, size = 105, normalized size = 1.12

$$\frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)-\frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)+\frac{65\,x+4}{682\left(5\,x^{2}+3\,x+2\right)}-\frac{3}{968}\log\left(5\,x^{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*
log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 0.984673, size = 377, normalized size = 4.01

$$\frac{132986\,\sqrt{31}\left(5\,x^{2}+3\,x+2\right)\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right)-13454\,\sqrt{23}\left(5\,x^{2}+3\,x+2\right)\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-66309\,(5}{21395704\left(5\,x^{2}+3\,x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/21395704*(132986*sqrt(31)*(5*x^2 + 3*x + 2)*arctan(1/31*sqrt(31)*(10*x +
3)) - 13454*sqrt(23)*(5*x^2 + 3*x + 2)*arctan(1/23*sqrt(23)*(4*x - 1)) - 66
309*(5*x^2 + 3*x + 2)*log(5*x^2 + 3*x + 2) + 66309*(5*x^2 + 3*x + 2)*log(2*
x^2 - x + 3) + 2039180*x + 125488)/(5*x^2 + 3*x + 2)
```

---

**Sympy [A]** time = 0.297665, size = 102, normalized size = 1.09

$$\frac{65x+4}{3410x^{2}+2046x+1364}+\frac{3\log\left(x^{2}-\frac{x}{2}+\frac{3}{2}\right)}{968}-\frac{3\log\left(x^{2}+\frac{3x}{5}+\frac{2}{5}\right)}{968}-\frac{7\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{11132}+\frac{2891\sqrt{31}\,\mathrm{atan}\left(\frac{10\sqrt{31}}{31}\right.}{465124}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**2,x)
```

```
[Out] (65*x + 4)/(3410*x**2 + 2046*x + 1364) + 3*log(x**2 - x/2 + 3/2)/968 - 3*lo
g(x**2 + 3*x/5 + 2/5)/968 - 7*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/
11132 + 2891*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/465124
```

---

**Giac [A]** time = 1.22338, size = 105, normalized size = 1.12

$$\frac{2891}{465124}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{7}{11132}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{65x+4}{682\left(5x^2+3x+2\right)}-\frac{3}{968}\log\left(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 2891/465124*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 7/11132*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1/682*(65*x + 4)/(5*x^2 + 3*x + 2) - 3/968*
log(5*x^2 + 3*x + 2) + 3/968*log(2*x^2 - x + 3)
```

## 3.43 $\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3} \, dx$

**Optimal**. Leaf size=115

$$\frac{65x+4}{1364(5x^2+3x+2)^2}+\frac{21605x+7923}{465124(5x^2+3x+2)}-\frac{\log(2x^2-x+3)}{21296}+\frac{\log(5x^2+3x+2)}{21296}-\frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{847793\,\mathrm{ta}}{10232}$$

```
[Out] (4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + (7923 + 21605*x)/(465124*(2 + 3*x +
 5*x^2)) - (45*ArcTan[(1 - 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (847793*ArcTa
n[(3 + 10*x)/Sqrt[31]])/(10232728*Sqrt[31]) - Log[3 - x + 2*x^2]/21296 + Lo
g[2 + 3*x + 5*x^2]/21296
```

---

**Rubi [A]** time = 0.124178, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{65x+4}{1364(5x^2+3x+2)^2}+\frac{21605x+7923}{465124(5x^2+3x+2)}-\frac{\log(2x^2-x+3)}{21296}+\frac{\log(5x^2+3x+2)}{21296}-\frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}}+\frac{847793\,\mathrm{ta}}{10232}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (4 + 65*x)/(1364*(2 + 3*x + 5*x^2)^2) + (7923 + 21605*x)/(465124*(2 + 3*x +
 5*x^2)) - (45*ArcTan[(1 - 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (847793*ArcTa
n[(3 + 10*x)/Sqrt[31]])/(10232728*Sqrt[31]) - Log[3 - x + 2*x^2]/21296 + Lo
g[2 + 3*x + 5*x^2]/21296
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
```

```
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx = \frac{4+65x}{1364(2+3x+5x^2)^2} - \frac{\int \frac{-5753+3509x-4290x^2}{(3-x+2x^2)(2+3x+5x^2)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-14522420+3833038x-10456820x^2}{(3-x+2x^2)(2+3x+5x^2)}\,d}{112560008}$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-58838186+5116364x}{3-x+2x^2}\,dx}{27239521936} - \frac{\int \frac{-113}{}}{}$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\int \frac{-1+4x}{3-x+2x^2}\,dx}{21296} + \frac{\int \frac{3+10x}{2+3x+5x^2}\,dx}{21296} +$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{\log(3-x+2x^2)}{21296} + \frac{\log(2+3x}{2129}$$

$$= \frac{4+65x}{1364(2+3x+5x^2)^2} + \frac{7923+21605x}{465124(2+3x+5x^2)} - \frac{45\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{10648\sqrt{23}} + \frac{847793\tan^{-}}{1023272}$$

**Mathematica [A]** time = 0.143564, size = 104, normalized size = 0.9

$$\frac{31\left(\frac{44(108025x^3+104430x^2+89144x+17210)}{(5x^2+3x+2)^2} - 961\log(2x^2-x+3) + 961\log(5x^2+3x+2)\right) + 1695586\sqrt{31}\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{634429136} + \frac{45}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (45*ArcTan[(-1 + 4*x)/Sqrt[23]])/(10648*Sqrt[23]) + (1695586*Sqrt[31]*ArcTa
n[(3 + 10*x)/Sqrt[31]] + 31*((44*(17210 + 89144*x + 104430*x^2 + 108025*x^3
))/(2 + 3*x + 5*x^2)^2 - 961*Log[3 - x + 2*x^2] + 961*Log[2 + 3*x + 5*x^2])
)/634429136
```

---

**Maple [A]** time = 0.049, size = 89, normalized size = 0.8

$$\frac{25}{10648(5x^2+3x+2)^2}\left(\frac{95062x^3}{961} + \frac{459492x^2}{4805} + \frac{1961168x}{24025} + \frac{75724}{4805}\right) + \frac{\ln(5x^2+3x+2)}{21296} + \frac{847793\sqrt{31}}{317214568}\arctan\Big((3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x)
```

```
[Out] 25/10648*(95062/961*x^3+459492/4805*x^2+1961168/24025*x+75724/4805)/(5*x^2+
3*x+2)^2+1/21296*ln(5*x^2+3*x+2)+847793/317214568*arctan(1/31*(3+10*x)*31^(
1/2))*31^(1/2)-1/21296*ln(2*x^2-x+3)+45/244904*23^(1/2)*arctan(1/23*(-1+4*x
)*23^(1/2))
```

---

**Maxima [A]** time = 1.43887, size = 132, normalized size = 1.15

$$\frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{108025x^3+104430x^2+891}{465124(25x^4+30x^3+29x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 +
89144*x + 17210)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1/21296*log(5*x^2
+ 3*x + 2) - 1/21296*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.0267, size = 578, normalized size = 5.03

$$\frac{3388960300x^3+38998478\sqrt{31}(25x^4+30x^3+29x^2+12x+4)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+2681190\sqrt{23}(25x^4+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/14591870128*(3388960300*x^3 + 38998478*sqrt(31)*(25*x^4 + 30*x^3 + 29*x^2
 + 12*x + 4)*arctan(1/31*sqrt(31)*(10*x + 3)) + 2681190*sqrt(23)*(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3276177960*x^
2 + 685193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(5*x^2 + 3*x + 2) - 685
193*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*log(2*x^2 - x + 3) + 2796625568*x
 + 539912120)/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Sympy [A]** time = 0.399983, size = 119, normalized size = 1.03

$$\frac{108025x^3+104430x^2+89144x+17210}{11628100x^4+13953720x^3+13488596x^2+5581488x+1860496}-\frac{\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296}+\frac{\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296}+\frac{45\sqrt{23}\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)/(5*x**2+3*x+2)**3,x)
```

```
[Out] (108025*x**3 + 104430*x**2 + 89144*x + 17210)/(11628100*x**4 + 13953720*x**
3 + 13488596*x**2 + 5581488*x + 1860496) - log(x**2 - x/2 + 3/2)/21296 + lo
g(x**2 + 3*x/5 + 2/5)/21296 + 45*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/2
3)/244904 + 847793*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/31721456
8
```

---

**Giac [A]** time = 1.25983, size = 119, normalized size = 1.03

$$\frac{847793}{317214568}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{45}{244904}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{108025x^3+104430x^2+891}{465124(5x^2+3x+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 847793/317214568*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 45/244904*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/465124*(108025*x^3 + 104430*x^2 +
89144*x + 17210)/(5*x^2 + 3*x + 2)^2 + 1/21296*log(5*x^2 + 3*x + 2) - 1/212
96*log(2*x^2 - x + 3)
```

## 3.44 $\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^2} \, dx$

**Optimal**. Leaf size=91

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{14641(79x+101)}{2944\left(2x^2-x+3\right)}-\frac{30613}{128}\log\left(2x^2-x+3\right)-\frac{89359x}{64}-\frac{13292697\tan^{-1}\left(\right.}{1472\sqrt{23}}$$

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) - (13292697*ArcTan[(1 - 4*x)/
Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

---

**Rubi [A]** time = 0.0863451, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^5}{4}+\frac{2125x^4}{16}+\frac{9775x^3}{48}-\frac{1185x^2}{8}-\frac{14641(79x+101)}{2944\left(2x^2-x+3\right)}-\frac{30613}{128}\log\left(2x^2-x+3\right)-\frac{89359x}{64}-\frac{13292697\tan^{-1}\left(\right.}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) - (13292697*ArcTan[(1 - 4*x)/
Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^2}\,dx = -\frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1}{23}\int \frac{\frac{832627}{64} - \frac{661181x}{64} - \frac{488267x^2}{32} + \frac{143635x^3}{16} + \frac{213325x^4}{8} + \frac{83375x^5}{4} + \frac{14375x^6}{2}}{3-x+2x^2}$$

$$= -\frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1}{23}\int \left(-\frac{2055257}{64} - \frac{27255x}{4} + \frac{224825x^2}{16} + \frac{48875x^3}{4} + \frac{14375x^4}{4} + \frac{133}{32}\right.$$

$$= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} + \frac{1331}{736}\int \frac{2629-529x}{3-x+2x^2}$$

$$= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{30613}{128}\int \frac{-1+4x}{3-x+2x^2}$$

$$= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{30613}{128}\log(3-x+2$$

$$= -\frac{89359x}{64} - \frac{1185x^2}{8} + \frac{9775x^3}{48} + \frac{2125x^4}{16} + \frac{125x^5}{4} - \frac{14641(101+79x)}{2944(3-x+2x^2)} - \frac{13292697\tan^{-1}\left(\frac{1-4}{\sqrt{23}}\right.}{1472\sqrt{23}}$$

**Mathematica [A]** time = 0.0499269, size = 91, normalized size = 1.

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{14641(79x+101)}{2944(2x^2-x+3)} - \frac{30613}{128}\log(2x^2-x+3) - \frac{89359x}{64} + \frac{13292697\tan^{-1}\left(\frac{4x-}{\sqrt{23}}\right.}{1472\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-89359*x)/64 - (1185*x^2)/8 + (9775*x^3)/48 + (2125*x^4)/16 + (125*x^5)/4
- (14641*(101 + 79*x))/(2944*(3 - x + 2*x^2)) + (13292697*ArcTan[(-1 + 4*x)
/Sqrt[23]])/(1472*Sqrt[23]) - (30613*Log[3 - x + 2*x^2])/128
```

---

**Maple [A]** time = 0.048, size = 71, normalized size = 0.8

$$\frac{125\,x^5}{4} + \frac{2125\,x^4}{16} + \frac{9775\,x^3}{48} - \frac{1185\,x^2}{8} - \frac{89359\,x}{64} - \frac{1331}{64}\left(\frac{869\,x}{92} + \frac{1111}{92}\right)\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)^{-1} - \frac{30613\,\ln(2\,x^2-x+3)}{128} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x)
```

```
[Out] 125/4*x^5+2125/16*x^4+9775/48*x^3-1185/8*x^2-89359/64*x-1331/64*(869/92*x+1
111/92)/(x^2-1/2*x+3/2)-30613/128*ln(2*x^2-x+3)+13292697/33856*23^(1/2)*arc
tan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.42549, size = 97, normalized size = 1.07

$$\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{89359}{64}x - \frac{14641\,(79x+101)}{2944\,(2x^2-x+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(23
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2
*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 0.9736, size = 348, normalized size = 3.82

$$\frac{12696000\,x^7 + 47610000\,x^6 + 74800600\,x^5 - 20609840\,x^4 - 413058012\,x^3 + 79756182\sqrt{23}\left(2x^2-x+3\right)\arctan\left(\frac{1}{23}\right.}{203136\left(2x^2-x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/203136*(12696000*x^7 + 47610000*x^6 + 74800600*x^5 - 20609840*x^4 - 41305
8012*x^3 + 79756182*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)
) + 193356906*x^2 - 48582831*(2*x^2 - x + 3)*log(2*x^2 - x + 3) - 930684489
*x - 102033129)/(2*x^2 - x + 3)
```

---

**Sympy [A]** time = 0.245003, size = 88, normalized size = 0.97

$$\frac{125x^5}{4} + \frac{2125x^4}{16} + \frac{9775x^3}{48} - \frac{1185x^2}{8} - \frac{89359x}{64} - \frac{1156639x + 1478741}{5888x^2 - 2944x + 8832} - \frac{30613\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{128} + \frac{13292697\sqrt{23}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**5/4 + 2125*x**4/16 + 9775*x**3/48 - 1185*x**2/8 - 89359*x/64 - (1156
639*x + 1478741)/(5888*x**2 - 2944*x + 8832) - 30613*log(x**2 - x/2 + 3/2)/
128 + 13292697*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/33856
```

---

**Giac [A]** time = 1.1539, size = 97, normalized size = 1.07

$$\frac{125}{4}x^5 + \frac{2125}{16}x^4 + \frac{9775}{48}x^3 - \frac{1185}{8}x^2 + \frac{13292697}{33856}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{89359}{64}x - \frac{14641\,(79x+101)}{2944\,(2x^2-x+3)} - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 125/4*x^5 + 2125/16*x^4 + 9775/48*x^3 - 1185/8*x^2 + 13292697/33856*sqrt(23
)*arctan(1/23*sqrt(23)*(4*x - 1)) - 89359/64*x - 14641/2944*(79*x + 101)/(2
*x^2 - x + 3) - 30613/128*log(2*x^2 - x + 3)
```

## 3.45 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2} \, dx$

**Optimal**. Leaf size=77

$$\frac{125x^3}{12}+\frac{175x^2}{4}-\frac{1331(17-45x)}{736(2x^2-x+3)}-\frac{2057}{32}\log\left(2x^2-x+3\right)+\frac{915x}{16}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 - (1331*(17 - 45*x))/(736*(3 - x +
2*x^2)) + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[3
- x + 2*x^2])/32
```

---

**Rubi [A]** time = 0.0725653, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{125x^3}{12}+\frac{175x^2}{4}-\frac{1331(17-45x)}{736(2x^2-x+3)}-\frac{2057}{32}\log\left(2x^2-x+3\right)+\frac{915x}{16}+\frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```


```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 - (1331*(17 - 45*x))/(736*(3 - x +
2*x^2)) + (223971*ArcTan[(1 - 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[3
- x + 2*x^2])/32
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^2}\,dx &= -\frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{1}{23}\int \frac{-\frac{25195}{16} - \frac{19067x}{16} + \frac{22195x^2}{8} + \frac{13225x^3}{4} + \frac{2875x^4}{2}}{3-x+2x^2}\,dx \\ &= -\frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{1}{23}\int \left(\frac{21045}{16} + \frac{4025x}{2} + \frac{2875x^2}{4} - \frac{121(365+391x)}{8(3-x+2x^2)}\right) dx \\ &= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{121}{184}\int \frac{365+391x}{3-x+2x^2}\,dx \\ &= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{2057}{32}\int \frac{-1+4x}{3-x+2x^2}\,dx - \frac{223971}{736}\int \frac{1}{3-x+2x^2} \\ &= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} - \frac{2057}{32}\log(3-x+2x^2) + \frac{223971}{368}\,\mathrm{Subst}\left(\int \frac{}{-23} \right. \\ &= \frac{915x}{16} + \frac{175x^2}{4} + \frac{125x^3}{12} - \frac{1331(17-45x)}{736(3-x+2x^2)} + \frac{223971\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{368\sqrt{23}} - \frac{2057}{32}\log(3-x+2x^2)\end{aligned}$$

**Mathematica [A]** time = 0.0274481, size = 77, normalized size = 1.

$$\frac{125x^3}{12} + \frac{175x^2}{4} + \frac{1331(45x-17)}{736(2x^2-x+3)} - \frac{2057}{32}\log(2x^2-x+3) + \frac{915x}{16} - \frac{223971\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{368\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^2,x]
```

```
[Out] (915*x)/16 + (175*x^2)/4 + (125*x^3)/12 + (1331*(-17 + 45*x))/(736*(3 - x +
 2*x^2)) - (223971*ArcTan[(-1 + 4*x)/Sqrt[23]])/(368*Sqrt[23]) - (2057*Log[
3 - x + 2*x^2])/32
```

---

**Maple [A]** time = 0.047, size = 61, normalized size = 0.8

$$\frac{125\,x^3}{12} + \frac{175\,x^2}{4} + \frac{915\,x}{16} - \frac{121}{16}\left(-\frac{495\,x}{92} + \frac{187}{92}\right)\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)^{-1} - \frac{2057\,\ln\left(2\,x^2 - x + 3\right)}{32} - \frac{223971\,\sqrt{23}}{8464}\arctan\left(\frac{(-1+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x)
```

```
[Out] 125/12*x^3+175/4*x^2+915/16*x-121/16*(-495/92*x+187/92)/(x^2-1/2*x+3/2)-205
7/32*ln(2*x^2-x+3)-223971/8464*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.43452, size = 84, normalized size = 1.09

$$\frac{125}{12}\,x^3+\frac{175}{4}\,x^2-\frac{223971}{8464}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{915}{16}\,x+\frac{1331\,(45\,x-17)}{736\left(2\,x^2-x+3\right)}-\frac{2057}{32}\,\log\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 -
x + 3)
```

---

**Fricas [A]** time = 1.03317, size = 292, normalized size = 3.79

$$\frac{1058000\,x^5+3914600\,x^4+5173620\,x^3-1343826\,\sqrt{23}\left(2\,x^2-x+3\right)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+3761190\,x^2-32644}{50784\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/50784*(1058000*x^5 + 3914600*x^4 + 5173620*x^3 - 1343826*sqrt(23)*(2*x^2
- x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 3761190*x^2 - 3264459*(2*x^2 - x
 + 3)*log(2*x^2 - x + 3) + 12845385*x - 1561263)/(2*x^2 - x + 3)
```

---

**Sympy [A]** time = 0.254475, size = 75, normalized size = 0.97

$$\frac{125x^3}{12}+\frac{175x^2}{4}+\frac{915x}{16}+\frac{59895x-22627}{1472x^2-736x+2208}-\frac{2057\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{223971\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{8464}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**2,x)
```

```
[Out] 125*x**3/12 + 175*x**2/4 + 915*x/16 + (59895*x - 22627)/(1472*x**2 - 736*x
+ 2208) - 2057*log(x**2 - x/2 + 3/2)/32 - 223971*sqrt(23)*atan(4*sqrt(23)*x
/23 - sqrt(23)/23)/8464
```

---

**Giac [A]** time = 1.18254, size = 84, normalized size = 1.09

$$\frac{125}{12}\,x^{3} + \frac{175}{4}\,x^{2} - \frac{223971}{8464}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right) + \frac{915}{16}\,x + \frac{1331\,(45\,x-17)}{736\,\left(2\,x^{2}-x+3\right)} - \frac{2057}{32}\,\log\left(2\,x^{2}-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 125/12*x^3 + 175/4*x^2 - 223971/8464*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1
)) + 915/16*x + 1331/736*(45*x - 17)/(2*x^2 - x + 3) - 2057/32*log(2*x^2 -
x + 3)
```

## 3.46 $\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^2}\,dx$

**Optimal**. Leaf size=63

$$\frac{121(19-7x)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)+\frac{25x}{4}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

```
[Out] (25*x)/4 + (121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (1859*ArcTan[(1 - 4*x)/
Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

---

**Rubi [A]** time = 0.0629492, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(19-7x)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)+\frac{25x}{4}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] (25*x)/4 + (121*(19 - 7*x))/(184*(3 - x + 2*x^2)) + (1859*ArcTan[(1 - 4*x)/
Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^2}\,dx &= \frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1}{23}\int \frac{\frac{163}{4}+\frac{1955x}{4}+\frac{575x^2}{2}}{3-x+2x^2}\,dx\\
&= \frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1}{23}\int \left(\frac{575}{4}-\frac{11(71-115x)}{2\left(3-x+2x^2\right)}\right)dx\\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}-\frac{11}{46}\int \frac{71-115x}{3-x+2x^2}\,dx\\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{55}{8}\int \frac{-1+4x}{3-x+2x^2}\,dx-\frac{1859}{184}\int \frac{1}{3-x+2x^2}\,dx\\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{55}{8}\log\left(3-x+2x^2\right)+\frac{1859}{92}\operatorname{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right)\\
&= \frac{25x}{4}+\frac{121(19-7x)}{184\left(3-x+2x^2\right)}+\frac{1859\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{92\sqrt{23}}+\frac{55}{8}\log\left(3-x+2x^2\right)
\end{aligned}$$

**Mathematica [A]** time = 0.0301396, size = 63, normalized size = 1.

$$-\frac{121(7x-19)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)+\frac{25x}{4}-\frac{1859\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{92\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^2,x]
```

```
[Out] (25*x)/4 - (121*(-19 + 7*x))/(184*(3 - x + 2*x^2)) - (1859*ArcTan[(-1 + 4*x
)/Sqrt[23]])/(92*Sqrt[23]) + (55*Log[3 - x + 2*x^2])/8
```

---

**Maple [A]** time = 0.045, size = 51, normalized size = 0.8

$$\frac{25\,x}{4}+\frac{11}{4}\left(-\frac{77\,x}{92}+\frac{209}{92}\right)\left(x^2-\frac{x}{2}+\frac{3}{2}\right)^{-1}+\frac{55\,\ln\left(2\,x^2-x+3\right)}{8}-\frac{1859\,\sqrt{23}}{2116}\arctan\left(\frac{(-1+4\,x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x)
```

```
[Out] 25/4*x+11/4*(-77/92*x+209/92)/(x^2-1/2*x+3/2)+55/8*ln(2*x^2-x+3)-1859/2116*
23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.42417, size = 70, normalized size = 1.11

$$-\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{25}{4}x-\frac{121\,(7x-19)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] -1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*x
 - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.02616, size = 234, normalized size = 3.71

$$\frac{52900\,x^3-3718\sqrt{23}\left(2x^2-x+3\right)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-26450\,x^2+29095\left(2x^2-x+3\right)\log\left(2x^2-x+3\right)+5}{4232\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/4232*(52900*x^3 - 3718*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x
 - 1)) - 26450*x^2 + 29095*(2*x^2 - x + 3)*log(2*x^2 - x + 3) + 59869*x + 5
2877)/(2*x^2 - x + 3)
```

---

**Sympy [A]** time = 0.170856, size = 61, normalized size = 0.97

$$\frac{25x}{4}-\frac{847x-2299}{368x^2-184x+552}+\frac{55\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{8}-\frac{1859\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{2116}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**2,x)
```

```
[Out] 25*x/4 - (847*x - 2299)/(368*x**2 - 184*x + 552) + 55*log(x**2 - x/2 + 3/2)
/8 - 1859*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/2116
```

---

**Giac [A]** time = 1.18982, size = 70, normalized size = 1.11

$$-\frac{1859}{2116}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{25}{4}x-\frac{121\,(7x-19)}{184\left(2x^2-x+3\right)}+\frac{55}{8}\log\left(2x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] -1859/2116*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 25/4*x - 121/184*(7*x
 - 19)/(2*x^2 - x + 3) + 55/8*log(2*x^2 - x + 3)
```

## 3.47 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2}\,dx$

**Optimal**. Leaf size=43

$$-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*
Sqrt[23])
```

---

**Rubi [A]** time = 0.0260438, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1660, 12, 618, 204}

$$-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}-\frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) - (82*ArcTan[(1 - 4*x)/Sqrt[23]])/(23*
Sqrt[23])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^2}\,dx = -\frac{11(5+3x)}{46\left(3-x+2x^2\right)} + \frac{1}{23}\int \frac{41}{3-x+2x^2}\,dx$$
$$= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)} + \frac{41}{23}\int \frac{1}{3-x+2x^2}\,dx$$
$$= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)} - \frac{82}{23}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx, x, -1+4x\right)$$
$$= -\frac{11(5+3x)}{46\left(3-x+2x^2\right)} - \frac{82\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{23\sqrt{23}}$$

**Mathematica [A]** time = 0.0143532, size = 43, normalized size = 1.

$$\frac{82\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{23\sqrt{23}} - \frac{11(3x+5)}{46\left(2x^2-x+3\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^2,x]
```

```
[Out] (-11*(5 + 3*x))/(46*(3 - x + 2*x^2)) + (82*ArcTan[(-1 + 4*x)/Sqrt[23]])/(23
*Sqrt[23])
```

---

**Maple [A]** time = 0.047, size = 34, normalized size = 0.8

$$\left(-\frac{33\,x}{92} - \frac{55}{92}\right)\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)^{-1} + \frac{82\sqrt{23}}{529}\arctan\left(\frac{(-1+4\,x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^2,x)
```

```
[Out] (-33/92*x-55/92)/(x^2-1/2*x+3/2)+82/529*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1
/2))
```

---

**Maxima [A]** time = 1.4319, size = 49, normalized size = 1.14

$$\frac{82}{529}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) - \frac{11\,(3\,x+5)}{46\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="maxima")
```

```
[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
x + 3)
```

---

**Fricas [A]** time = 0.988498, size = 138, normalized size = 3.21

$$\frac{164\sqrt{23}\left(2x^2-x+3\right)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-759x-1265}{1058\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1058*(164*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) - 759*
x - 1265)/(2*x^2 - x + 3)
```

---

**Sympy [A]** time = 0.205915, size = 41, normalized size = 0.95

$$-\frac{33x+55}{92x^2-46x+138}+\frac{82\sqrt{23}\,\mathrm{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{529}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**2,x)
```

```
[Out] -(33*x + 55)/(92*x**2 - 46*x + 138) + 82*sqrt(23)*atan(4*sqrt(23)*x/23 - sq
rt(23)/23)/529
```

---

**Giac [A]** time = 1.13908, size = 49, normalized size = 1.14

$$\frac{82}{529}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{11(3x+5)}{46\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^2,x, algorithm="giac")
```

```
[Out] 82/529*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/46*(3*x + 5)/(2*x^2 -
x + 3)
```

## 3.48 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)}\,dx$

**Optimal**. Leaf size=94

$$\frac{13-6x}{506\left(2x^2-x+3\right)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) + (241*ArcTan[(1 - 4*x)/Sqrt[23]])/(11132*
Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 - x
 + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

---

**Rubi [A]** time = 0.0884198, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {974, 1072, 634, 618, 204, 628}

$$\frac{13-6x}{506\left(2x^2-x+3\right)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) + (241*ArcTan[(1 - 4*x)/Sqrt[23]])/(11132*
Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 - x
 + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
```

```
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{506\left(3-x+2x^2\right)}-\frac{\int \frac{-1892-1067x+330x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}\,dx}{5566}$$

$$= \frac{13-6x}{506\left(3-x+2x^2\right)}-\frac{\int \frac{-3509+72358x}{3-x+2x^2}\,dx}{1346972}-\frac{\int \frac{-150282-180895x}{2+3x+5x^2}\,dx}{1346972}$$

$$= \frac{13-6x}{506\left(3-x+2x^2\right)}-\frac{241\int \frac{1}{3-x+2x^2}\,dx}{22264}-\frac{13}{968}\int \frac{-1+4x}{3-x+2x^2}\,dx+\frac{13}{968}\int \frac{3+}{2+3}$$

$$= \frac{13-6x}{506\left(3-x+2x^2\right)}-\frac{13}{968}\log\left(3-x+2x^2\right)+\frac{13}{968}\log\left(2+3x+5x^2\right)+\frac{241\,\mathrm{Su}}{}$$

$$= \frac{13-6x}{506\left(3-x+2x^2\right)}+\frac{241\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{3+10x}{\sqrt{31}}\right)}{484\sqrt{31}}-\frac{13}{968}\log\left(3-x+\right.$$

**Mathematica [A]** time = 0.0552043, size = 94, normalized size = 1.

$$\frac{13-6x}{506\left(2x^2-x+3\right)}-\frac{13}{968}\log\left(2x^2-x+3\right)+\frac{13}{968}\log\left(5x^2+3x+2\right)-\frac{241\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{11132\sqrt{23}}+\frac{69\tan^{-1}\left(\frac{10x+3}{\sqrt{31}}\right)}{484\sqrt{31}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(506*(3 - x + 2*x^2)) - (241*ArcTan[(-1 + 4*x)/Sqrt[23]])/(11132
*Sqrt[23]) + (69*ArcTan[(3 + 10*x)/Sqrt[31]])/(484*Sqrt[31]) - (13*Log[3 -
x + 2*x^2])/968 + (13*Log[2 + 3*x + 5*x^2])/968
```

---

**Maple [A]** time = 0.049, size = 77, normalized size = 0.8

$$\frac{13\ln\left(5x^2+3x+2\right)}{968}+\frac{69\sqrt{31}}{15004}\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right)-\frac{1}{484}\left(\frac{66x}{23}-\frac{143}{23}\right)\left(x^2-\frac{x}{2}+\frac{3}{2}\right)^{-1}-\frac{13\ln\left(2x^2-x+3\right)}{968}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x)
```

```
[Out] 13/968*ln(5*x^2+3*x+2)+69/15004*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)-1/4
84*(66/23*x-143/23)/(x^2-1/2*x+3/2)-13/968*ln(2*x^2-x+3)-241/256036*23^(1/2
)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.44483, size = 105, normalized size = 1.12

$$\frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{6x-13}{506\left(2x^2-x+3\right)}+\frac{13}{968}\log\left(5x^2+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*l
og(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.00196, size = 363, normalized size = 3.86

$$\frac{73002\sqrt{31}\left(2x^2-x+3\right)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-14942\sqrt{23}\left(2x^2-x+3\right)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+213187\left(2x^2\right.}{15874232\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/15874232*(73002*sqrt(31)*(2*x^2 - x + 3)*arctan(1/31*sqrt(31)*(10*x + 3))
 - 14942*sqrt(23)*(2*x^2 - x + 3)*arctan(1/23*sqrt(23)*(4*x - 1)) + 213187*
(2*x^2 - x + 3)*log(5*x^2 + 3*x + 2) - 213187*(2*x^2 - x + 3)*log(2*x^2 - x
 + 3) - 188232*x + 407836)/(2*x^2 - x + 3)
```

---

**Sympy [A]** time = 0.306932, size = 102, normalized size = 1.09

$$-\frac{6x-13}{1012x^2-506x+1518}-\frac{13\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{968}+\frac{13\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{968}-\frac{241\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{256036}+\frac{69\sqrt{31}\operatorname{atan}\left(\frac{10\sqrt{}}{3}\right.}{1500}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2),x)
```

```
[Out] -(6*x - 13)/(1012*x**2 - 506*x + 1518) - 13*log(x**2 - x/2 + 3/2)/968 + 13*
log(x**2 + 3*x/5 + 2/5)/968 - 241*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/
23)/256036 + 69*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/15004
```

---

**Giac [A]** time = 1.13347, size = 105, normalized size = 1.12

$$\frac{69}{15004}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right) - \frac{241}{256036}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right) - \frac{6x-13}{506\left(2x^2-x+3\right)} + \frac{13}{968}\log\left(5x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 69/15004*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 241/256036*sqrt(23)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) - 1/506*(6*x - 13)/(2*x^2 - x + 3) + 13/968*l
og(5*x^2 + 3*x + 2) - 13/968*log(2*x^2 - x + 3)
```

## 3.49 $\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx$

**Optimal**. Leaf size=127

$$-\frac{25(117-137x)}{172546\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}+\frac{19\log\left(2x^2-x+3\right)}{10648}-\frac{19\log\left(5x^2+3x+2\right)}{10648}+\frac{2769\tan}{12245}$$

```
[Out] (-25*(117 - 137*x))/(172546*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(506*(3 - x + 2
*x^2)*(2 + 3*x + 5*x^2)) + (2769*ArcTan[(1 - 4*x)/Sqrt[23]])/(122452*Sqrt[2
3]) + (12643*ArcTan[(3 + 10*x)/Sqrt[31]])/(165044*Sqrt[31]) + (19*Log[3 - x
 + 2*x^2])/10648 - (19*Log[2 + 3*x + 5*x^2])/10648
```

---

**Rubi [A]** time = 0.122755, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{25(117-137x)}{172546\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}+\frac{19\log\left(2x^2-x+3\right)}{10648}-\frac{19\log\left(5x^2+3x+2\right)}{10648}+\frac{2769\tan}{12245}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```


```
[Out] (-25*(117 - 137*x))/(172546*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(506*(3 - x + 2
*x^2)*(2 + 3*x + 5*x^2)) + (2769*ArcTan[(1 - 4*x)/Sqrt[23]])/(122452*Sqrt[2
3]) + (12643*ArcTan[(3 + 10*x)/Sqrt[31]])/(165044*Sqrt[31]) + (19*Log[3 - x
 + 2*x^2])/10648 - (19*Log[2 + 3*x + 5*x^2])/10648
```


Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
```

```
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx = \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \frac{\int \frac{-2321-2299x+990x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)^2}\,dx}{5566}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \frac{\int \frac{-3034196+4654870}{\left(3-x+2x^2\right)(2+}}{417561}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} - \frac{\int \frac{132282766-721242}{3-x+2x^2}}{1010498394}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \frac{19\int \frac{-1+4x}{3-x+2x^2}\,dx}{10648} -$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \frac{19\log\left(3-x+2x\right.}{10648}$$

$$= -\frac{25(117-137x)}{172546\left(2+3x+5x^2\right)} + \frac{13-6x}{506\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} + \frac{2769\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{122452\sqrt{23}}$$

**Mathematica [A]** time = 0.057922, size = 106, normalized size = 0.83

$$\frac{\frac{31372\left(6850x^3-9275x^2+11154x-4342\right)}{10x^4+x^3+16x^2+7x+6} + 9659011\log\left(2x^2-x+3\right) - 9659011\log\left(5x^2+3x+2\right) - 5322018\sqrt{23}\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right) + 1}{5413113112}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((31372*(-4342 + 11154*x - 9275*x^2 + 6850*x^3))/(6 + 7*x + 16*x^2 + x^3 +
10*x^4) - 5322018*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 13376294*Sqrt[31]*
ArcTan[(3 + 10*x)/Sqrt[31]] + 9659011*Log[3 - x + 2*x^2] - 9659011*Log[2 +
3*x + 5*x^2])/5413113112
```

---

**Maple [A]** time = 0.052, size = 94, normalized size = 0.7

$$-\frac{1}{5324}\left(-\frac{759\,x}{31}+\frac{1078}{155}\right)\left(x^2+\frac{3\,x}{5}+\frac{2}{5}\right)^{-1} - \frac{19\,\ln\left(5\,x^2+3\,x+2\right)}{10648} + \frac{12643\,\sqrt{31}}{5116364}\arctan\left(\frac{(3+10\,x)\,\sqrt{31}}{31}\right) + \frac{1}{5324}\left(-\frac{77}{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/5324*(-759/31*x+1078/155)/(x^2+3/5*x+2/5)-19/10648*ln(5*x^2+3*x+2)+12643
/5116364*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+1/5324*(-77/23*x-341/46)/(
x^2-1/2*x+3/2)+19/10648*ln(2*x^2-x+3)-2769/2816396*23^(1/2)*arctan(1/23*(-1
+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.44659, size = 130, normalized size = 1.02

$$\frac{12643}{5116364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10\,x+3)\right) - \frac{2769}{2816396}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right) + \frac{6850\,x^3-9275\,x^2+11154\,x-43}{172546\left(10\,x^4+x^3+16\,x^2+7\,x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 1115
4*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x +
2) + 19/10648*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 0.991225, size = 548, normalized size = 4.31

$$\frac{214898200\,x^3 + 13376294\,\sqrt{31}\left(10\,x^4 + x^3 + 16\,x^2 + 7\,x + 6\right)\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x + 3)\right) - 5322018\,\sqrt{23}\left(10\,x^4 + x^3 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/5413113112*(214898200*x^3 + 13376294*sqrt(31)*(10*x^4 + x^3 + 16*x^2 + 7*
x + 6)*arctan(1/31*sqrt(31)*(10*x + 3)) - 5322018*sqrt(23)*(10*x^4 + x^3 +
16*x^2 + 7*x + 6)*arctan(1/23*sqrt(23)*(4*x - 1)) - 290975300*x^2 - 9659011
*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*log(5*x^2 + 3*x + 2) + 9659011*(10*x^4 +
 x^3 + 16*x^2 + 7*x + 6)*log(2*x^2 - x + 3) + 349923288*x - 136217224)/(10*
x^4 + x^3 + 16*x^2 + 7*x + 6)
```

---

**Sympy [A]** time = 0.449151, size = 122, normalized size = 0.96

$$\frac{6850x^3 - 9275x^2 + 11154x - 4342}{1725460x^4 + 172546x^3 + 2760736x^2 + 1207822x + 1035276} + \frac{19\log\left(x^2 - \frac{x}{2} + \frac{3}{2}\right)}{10648} - \frac{19\log\left(x^2 + \frac{3x}{5} + \frac{2}{5}\right)}{10648} - \frac{2769\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**2,x)
```

```
[Out] (6850*x**3 - 9275*x**2 + 11154*x - 4342)/(1725460*x**4 + 172546*x**3 + 2760
736*x**2 + 1207822*x + 1035276) + 19*log(x**2 - x/2 + 3/2)/10648 - 19*log(x
**2 + 3*x/5 + 2/5)/10648 - 2769*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23
)/2816396 + 12643*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/5116364
```

---

**Giac [A]** time = 1.1847, size = 130, normalized size = 1.02

$$\frac{12643}{5116364}\,\sqrt{31}\arctan\left(\frac{1}{31}\,\sqrt{31}(10\,x + 3)\right) - \frac{2769}{2816396}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x - 1)\right) + \frac{6850\,x^3 - 9275\,x^2 + 11154\,x -}{172546\left(10\,x^4 + x^3 + 16\,x^2 + 7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 12643/5116364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2769/2816396*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/172546*(6850*x^3 - 9275*x^2 + 1115
```

```
4*x - 4342)/(10*x^4 + x^3 + 16*x^2 + 7*x + 6) - 19/10648*log(5*x^2 + 3*x +
2) + 19/10648*log(2*x^2 - x + 3)
```

## 3.50 $\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx$

**Optimal**. Leaf size=148

$$-\frac{9446-5765x}{690184\left(5x^2+3x+2\right)^2}+\frac{3996965x+1765599}{235352744\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{97\log\left(2x^2-x+3\right)}{468512}-$$

```
[Out] -(9446 - 5765*x)/(690184*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(506*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (1765599 + 3996965*x)/(235352744*(2 + 3*x + 5*x
^2)) - (25557*ArcTan[(1 - 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*Arc
Tan[(3 + 10*x)/Sqrt[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/46
8512 - (97*Log[2 + 3*x + 5*x^2])/468512
```

---

**Rubi [A]** time = 0.161458, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$-\frac{9446-5765x}{690184\left(5x^2+3x+2\right)^2}+\frac{3996965x+1765599}{235352744\left(5x^2+3x+2\right)}+\frac{13-6x}{506\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{97\log\left(2x^2-x+3\right)}{468512}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -(9446 - 5765*x)/(690184*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(506*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (1765599 + 3996965*x)/(235352744*(2 + 3*x + 5*x
^2)) - (25557*ArcTan[(1 - 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*Arc
Tan[(3 + 10*x)/Sqrt[31]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/46
8512 - (97*Log[2 + 3*x + 5*x^2])/468512
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
```

```
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx = \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-2750-3531x+1650x^2}{(3-x+2x^2)(2+3x+5x^2)^3}\,dx}{5566}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} - \frac{\int \frac{-8251111+1\ldots}{(3-x+2\ldots}}{8\ldots}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599\ldots}{235352744\ldots}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599\ldots}{235352744\ldots}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599\ldots}{235352744\ldots}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599\ldots}{235352744\ldots}$$

$$= -\frac{9446-5765x}{690184(2+3x+5x^2)^2} + \frac{13-6x}{506(3-x+2x^2)(2+3x+5x^2)^2} + \frac{1765599\ldots}{235352744\ldots}$$

**Mathematica [A]** time = 0.0642083, size = 136, normalized size = 0.92

$$\frac{90x-11}{244904(2x^2-x+3)} + \frac{164380x+67573}{10232728(5x^2+3x+2)} + \frac{345x-98}{30008(5x^2+3x+2)^2} + \frac{97\log(2x^2-x+3)}{468512} - \frac{97\log(5x^2+3\ldots}{468512}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-11 + 90*x)/(244904*(3 - x + 2*x^2)) + (-98 + 345*x)/(30008*(2 + 3*x + 5*x
^2)^2) + (67573 + 164380*x)/(10232728*(2 + 3*x + 5*x^2)) + (25557*ArcTan[(-
1 + 4*x)/Sqrt[23]])/(5387888*Sqrt[23]) + (4464079*ArcTan[(3 + 10*x)/Sqrt[31
]])/(225120016*Sqrt[31]) + (97*Log[3 - x + 2*x^2])/468512 - (97*Log[2 + 3*x
 + 5*x^2])/468512
```

---

**Maple [A]** time = 0.053, size = 106, normalized size = 0.7

$$-\frac{25}{234256(5x^2+3x+2)^2}\left(-\frac{723272x^3}{961} - \frac{3656422x^2}{4805} - \frac{14280728x}{24025} - \frac{2238016}{24025}\right) - \frac{97\ln(5x^2+3x+2)}{468512} + \frac{446407\ldots}{697872\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x)
```

```
[Out] -25/234256*(-723272/961*x^3-3656422/4805*x^2-14280728/24025*x-2238016/24025
)/(5*x^2+3*x+2)^2-97/468512*ln(5*x^2+3*x+2)+4464079/6978720496*arctan(1/31*
(3+10*x)*31^(1/2))*31^(1/2)+1/234256*(990/23*x-121/23)/(x^2-1/2*x+3/2)+97/4
```

```
68512*ln(2*x^2-x+3)+25557/123921424*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.49092, size = 159, normalized size = 1.07

$$\frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{39969650x^5+21652955x}{235352744\left(50x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921
424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 +
21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) - 97/468512*log(5*x^2 + 3
*x + 2) + 97/468512*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.04071, size = 802, normalized size = 5.42

$$\frac{1253927859800x^5+679296504260x^4+2185021181068x^3+4722995582\sqrt{31}\left(50x^6+35x^5+103x^4+85x^3+83x^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/7383486284768*(1253927859800*x^5 + 679296504260*x^4 + 2185021181068*x^3 +
 4722995582*sqrt(31)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x +
12)*arctan(1/31*sqrt(31)*(10*x + 3)) + 1522737174*sqrt(23)*(50*x^6 + 35*x^5
 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)*arctan(1/23*sqrt(23)*(4*x - 1)) +
 1500218514344*x^2 - 1528665583*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^
2 + 32*x + 12)*log(5*x^2 + 3*x + 2) + 1528665583*(50*x^6 + 35*x^5 + 103*x^4
 + 85*x^3 + 83*x^2 + 32*x + 12)*log(2*x^2 - x + 3) + 1338609358240*x + 2188
80812656)/(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

---

**Sympy [A]** time = 0.466039, size = 143, normalized size = 0.97

$$\frac{39969650x^5+21652955x^4+69648769x^3+47820302x^2+42668920x+6976948}{11767637200x^6+8237346040x^5+24241332632x^4+20004983240x^3+19534277752x^2+7531287808x+2824232928}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**2/(5*x**2+3*x+2)**3,x)
```

```
[Out] (39969650*x**5 + 21652955*x**4 + 69648769*x**3 + 47820302*x**2 + 42668920*x
 + 6976948)/(11767637200*x**6 + 8237346040*x**5 + 24241332632*x**4 + 200049
83240*x**3 + 19534277752*x**2 + 7531287808*x + 2824232928) + 97*log(x**2 -
x/2 + 3/2)/468512 - 97*log(x**2 + 3*x/5 + 2/5)/468512 + 25557*sqrt(23)*atan
(4*sqrt(23)*x/23 - sqrt(23)/23)/123921424 + 4464079*sqrt(31)*atan(10*sqrt(3
1)*x/31 + 3*sqrt(31)/31)/6978720496
```

**Giac [A]** time = 1.13189, size = 149, normalized size = 1.01

$$\frac{4464079}{6978720496}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{25557}{123921424}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{39969650x^5+2165295}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^2/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 4464079/6978720496*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 25557/123921
424*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/235352744*(39969650*x^5 +
21652955*x^4 + 69648769*x^3 + 47820302*x^2 + 42668920*x + 6976948)/((5*x^2
+ 3*x + 2)^2*(2*x^2 - x + 3)) - 97/468512*log(5*x^2 + 3*x + 2) + 97/468512*
log(2*x^2 - x + 3)
```

## 3.51 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^3} \, dx$

**Optimal**. Leaf size=98

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424\left(2x^2-x+3\right)}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}-\frac{13915}{64}\log\left(2x^2-x+3\right)+\frac{2725x}{8}+\frac{63799791\tan^{-1}}{16928\sqrt{23}}$$

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) + (637997
91*ArcTan[(1 - 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2]
)/64
```

---

**Rubi [A]** time = 0.11253, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424\left(2x^2-x+3\right)}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}-\frac{13915}{64}\log\left(2x^2-x+3\right)+\frac{2725x}{8}+\frac{63799791\tan^{-1}}{16928\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) + (637997
91*ArcTan[(1 - 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2]
)/64
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^3}\,dx = -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{\frac{2173869}{128}-\frac{661181x}{32}-\frac{488267x^2}{16}+\frac{143635x^3}{8}+\frac{213325x^4}{4}+\frac{83375x^5}{2}+1}{\left(3-x+2x^2\right)^2}$$

$$= -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}+\frac{\int \frac{-\frac{5460539}{8}-\frac{626865x}{2}+\frac{5170975x^2}{8}+\frac{1124125x^3}{2}+\frac{330625x^4}{2}}{3-x+2x^2}\,d}{1058}$$

$$= -\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}+\frac{\int \left(\frac{1441525}{4}+\frac{2578875x}{8}+\frac{330625x^2}{4}-\frac{121(116609}{8(3-x-}\right.}{1058}$$

$$= \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}-\frac{121\int \frac{116609+60}{3-x+2x}}{8464}$$

$$= \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}-\frac{13915}{64}\int \frac{-1}{3-x}$$

$$= \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}-\frac{13915}{64}\log\left(3-\right.$$

$$= \frac{2725x}{8}+\frac{4875x^2}{32}+\frac{625x^3}{24}-\frac{14641(101+79x)}{5888\left(3-x+2x^2\right)^2}+\frac{1331(5229+76420x)}{135424\left(3-x+2x^2\right)}+\frac{63799791\tan^{-}}{16928\sqrt{2}}$$

**Mathematica [A]** time = 0.0370471, size = 98, normalized size = 1.

$$\frac{625x^3}{24}+\frac{4875x^2}{32}+\frac{1331(76420x+5229)}{135424\left(2x^2-x+3\right)}-\frac{14641(79x+101)}{5888\left(2x^2-x+3\right)^2}-\frac{13915}{64}\log\left(2x^2-x+3\right)+\frac{2725x}{8}-\frac{63799791\tan}{16928\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^3,x]
```

```
[Out] (2725*x)/8 + (4875*x^2)/32 + (625*x^3)/24 - (14641*(101 + 79*x))/(5888*(3 -
 x + 2*x^2)^2) + (1331*(5229 + 76420*x))/(135424*(3 - x + 2*x^2)) - (637997
```

```
91*ArcTan[(-1 + 4*x)/Sqrt[23]])/(16928*Sqrt[23]) - (13915*Log[3 - x + 2*x^2
])/64
```

---

**Maple [A]** time = 0.054, size = 73, normalized size = 0.7

$$\frac{625\,x^{3}}{24}+\frac{4875\,x^{2}}{32}+\frac{2725\,x}{8}-\frac{121}{4\,\left(2\,x^{2}-x+3\right)^{2}}\left(-\frac{210155\,x^{3}}{4232}+\frac{362791\,x^{2}}{16928}-\frac{561121\,x}{8464}+\frac{54263}{16928}\right)-\frac{13915\,\ln\left(2\,x^{2}-x+\right.}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x)
```

```
[Out] 625/24*x^3+4875/32*x^2+2725/8*x-121/4*(-210155/4232*x^3+362791/16928*x^2-56
1121/8464*x+54263/16928)/(2*x^2-x+3)^2-13915/64*ln(2*x^2-x+3)-63799791/3893
44*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.45374, size = 111, normalized size = 1.13

$$\frac{625}{24}\,x^{3}+\frac{4875}{32}\,x^{2}-\frac{63799791}{389344}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{2725}{8}\,x+\frac{1331\left(76420\,x^{3}-32981\,x^{2}+102022\,x-4933\right.}{67712\left(4\,x^{4}-4\,x^{3}+13\,x^{2}-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4
*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/
(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 13915/64*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 0.905483, size = 447, normalized size = 4.56

$$\frac{486680000\,x^{7}+2360398000\,x^{6}+5100406400\,x^{5}+2157209100\,x^{4}+24531516180\,x^{3}-765597492\,\sqrt{23}\left(4\,x^{4}-4\,x^{3}+13\right.}{467}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/4672128*(486680000*x^7 + 2360398000*x^6 + 5100406400*x^5 + 2157209100*x^4
 + 24531516180*x^3 - 765597492*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*
arctan(1/23*sqrt(23)*(4*x - 1)) - 6171678159*x^2 - 1015822830*(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 23692590858*x - 453041787)/(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Sympy [A]** time = 0.277565, size = 95, normalized size = 0.97

$$\frac{625x^{3}}{24}+\frac{4875x^{2}}{32}+\frac{2725x}{8}+\frac{101715020x^{3}-43897711x^{2}+135791282x-6565823}{270848x^{4}-270848x^{3}+880256x^{2}-406272x+609408}-\frac{13915\log\left(x^{2}-\frac{x}{2}+\frac{3}{2}\right)}{64}-\frac{63799}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**3,x)
```

```
[Out] 625*x**3/24 + 4875*x**2/32 + 2725*x/8 + (101715020*x**3 - 43897711*x**2 + 1
35791282*x - 6565823)/(270848*x**4 - 270848*x**3 + 880256*x**2 - 406272*x +
 609408) - 13915*log(x**2 - x/2 + 3/2)/64 - 63799791*sqrt(23)*atan(4*sqrt(2
3)*x/23 - sqrt(23)/23)/389344
```

---

**Giac [A]** time = 1.14796, size = 97, normalized size = 0.99

$$\frac{625}{24}\,x^3+\frac{4875}{32}\,x^2-\frac{63799791}{389344}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)+\frac{2725}{8}\,x+\frac{1331\left(76420\,x^3-32981\,x^2+102022\,x-49\right.}{67712\left(2\,x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 625/24*x^3 + 4875/32*x^2 - 63799791/389344*sqrt(23)*arctan(1/23*sqrt(23)*(4
*x - 1)) + 2725/8*x + 1331/67712*(76420*x^3 - 32981*x^2 + 102022*x - 4933)/
(2*x^2 - x + 3)^2 - 13915/64*log(2*x^2 - x + 3)
```

## 3.52 $\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^3} \, dx$

**Optimal**. Leaf size=84

$$\frac{121(21193-12828x)}{33856\left(2x^2-x+3\right)}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

```
[Out] (125*x)/8 - (1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + (121*(21193 - 128
28*x))/(33856*(3 - x + 2*x^2)) + (165099*ArcTan[(1 - 4*x)/Sqrt[23]])/(8464*
Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

---

**Rubi [A]** time = 0.0865576, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {1660, 1657, 634, 618, 204, 628}

$$\frac{121(21193-12828x)}{33856\left(2x^2-x+3\right)}-\frac{1331(17-45x)}{1472\left(2x^2-x+3\right)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 - (1331*(17 - 45*x))/(1472*(3 - x + 2*x^2)^2) + (121*(21193 - 128
28*x))/(33856*(3 - x + 2*x^2)) + (165099*ArcTan[(1 - 4*x)/Sqrt[23]])/(8464*
Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^3}\,dx = -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{-\frac{40885}{32}-\frac{19067x}{8}+\frac{22195x^2}{4}+\frac{13225x^3}{2}+2875x^4}{\left(3-x+2x^2\right)^2}\,dx$$

$$= -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{\int \frac{\frac{23997}{2}+92575x+\frac{66125x^2}{2}}{3-x+2x^2}\,dx}{1058}$$

$$= -\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{\int \left(\frac{66125}{4}-\frac{33(4557-13225x)}{4\left(3-x+2x^2\right)}\right)dx}{1058}$$

$$= \frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}-\frac{33\int \frac{4557-13225x}{3-x+2x^2}\,dx}{4232}$$

$$= \frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}-\frac{165099\int \frac{1}{3-x+2x^2}\,dx}{16928}+\frac{825}{32}\int \frac{-1+\cdots}{3-x\cdots}$$

$$= \frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{825}{32}\log\left(3-x+2x^2\right)+\frac{165099\,\mathrm{Sub}\cdots}{\cdots}$$

$$= \frac{125x}{8}-\frac{1331(17-45x)}{1472\left(3-x+2x^2\right)^2}+\frac{121(21193-12828x)}{33856\left(3-x+2x^2\right)}+\frac{165099\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{8464\sqrt{23}}+\frac{825}{32}\log\left(3-\cdots\right.$$

**Mathematica [A]** time = 0.0362954, size = 84, normalized size = 1.

$$-\frac{121(12828x-21193)}{33856\left(2x^2-x+3\right)}+\frac{1331(45x-17)}{1472\left(2x^2-x+3\right)^2}+\frac{825}{32}\log\left(2x^2-x+3\right)+\frac{125x}{8}-\frac{165099\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{8464\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^3,x]
```

```
[Out] (125*x)/8 + (1331*(-17 + 45*x))/(1472*(3 - x + 2*x^2)^2) - (121*(-21193 + 1
2828*x))/(33856*(3 - x + 2*x^2)) - (165099*ArcTan[(-1 + 4*x)/Sqrt[23]])/(84
64*Sqrt[23]) + (825*Log[3 - x + 2*x^2])/32
```

**Maple [A]** time = 0.05, size = 63, normalized size = 0.8

$$\frac{125x}{8}+\frac{11}{2\left(2x^2-x+3\right)^2}\left(-\frac{35277x^3}{2116}+\frac{303677x^2}{8464}-\frac{132803x}{4232}+\frac{326029}{8464}\right)+\frac{825\ln\left(2x^2-x+3\right)}{32}-\frac{165099\sqrt{23}}{194672}\operatorname{arcta}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x)
```

```
[Out] 125/8*x+11/2*(-35277/2116*x^3+303677/8464*x^2-132803/4232*x+326029/8464)/(2
*x^2-x+3)^2+825/32*ln(2*x^2-x+3)-165099/194672*23^(1/2)*arctan(1/23*(-1+4*x
)*23^(1/2))
```

**Maxima [A]** time = 1.45442, size = 97, normalized size = 1.15

$$-\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{125}{8}x-\frac{121\left(12828x^3-27607x^2+24146x-29639\right)}{16928\left(4x^4-4x^3+13x^2-6x+9\right)}+\frac{825}{32}\log\left(2x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/169
28*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x
+ 9) + 825/32*log(2*x^2 - x + 3)
```

**Fricas [A]** time = 0.985266, size = 377, normalized size = 4.49

$$\frac{24334000x^5-24334000x^4+43385176x^3-330198\sqrt{23}\left(4x^4-4x^3+13x^2-6x+9\right)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+4032}{389344\left(4x^4-4x^3+13x^2-6\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/389344*(24334000*x^5 - 24334000*x^4 + 43385176*x^3 - 330198*sqrt(23)*(4*x
^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) + 40329281*x
^2 + 10037775*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) - 12446
818*x + 82485337)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

**Sympy [A]** time = 0.300081, size = 82, normalized size = 0.98

$$\frac{125x}{8}-\frac{1552188x^3-3340447x^2+2921666x-3586319}{67712x^4-67712x^3+220064x^2-101568x+152352}+\frac{825\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{32}-\frac{165099\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{194672}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**3,x)
```

```
[Out] 125*x/8 - (1552188*x**3 - 3340447*x**2 + 2921666*x - 3586319)/(67712*x**4 -
 67712*x**3 + 220064*x**2 - 101568*x + 152352) + 825*log(x**2 - x/2 + 3/2)/
32 - 165099*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/194672
```

---

**Giac [A]** time = 1.13062, size = 84, normalized size = 1.

$$-\frac{165099}{194672}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{125}{8}x-\frac{121\left(12828x^3-27607x^2+24146x-29639\right)}{16928\left(2x^2-x+3\right)^2}+\frac{825}{32}\log\left(2x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] -165099/194672*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 125/8*x - 121/169
28*(12828*x^3 - 27607*x^2 + 24146*x - 29639)/(2*x^2 - x + 3)^2 + 825/32*log
(2*x^2 - x + 3)
```

## 3.53 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3} dx$

**Optimal**. Leaf size=64

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{55(332x+975)}{8464(2x^2-x+3)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

```
[Out] (121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (55*(975 + 332*x))/(8464*(3 - x
+ 2*x^2)) - (4330*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

---

**Rubi [A]** time = 0.0529804, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1660, 12, 618, 204}

$$\frac{121(19-7x)}{368(2x^2-x+3)^2}-\frac{55(332x+975)}{8464(2x^2-x+3)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (121*(19 - 7*x))/(368*(3 - x + 2*x^2)^2) - (55*(975 + 332*x))/(8464*(3 - x
+ 2*x^2)) - (4330*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^3}\,dx = \frac{121(19-7x)}{368(3-x+2x^2)^2}+\frac{1}{46}\int \frac{-\frac{195}{8}+\frac{1955x}{2}+575x^2}{(3-x+2x^2)^2}\,dx$$
$$= \frac{121(19-7x)}{368(3-x+2x^2)^2}-\frac{55(975+332x)}{8464(3-x+2x^2)}+\frac{\int \frac{4330}{3-x+2x^2}\,dx}{1058}$$
$$= \frac{121(19-7x)}{368(3-x+2x^2)^2}-\frac{55(975+332x)}{8464(3-x+2x^2)}+\frac{2165}{529}\int \frac{1}{3-x+2x^2}\,dx$$
$$= \frac{121(19-7x)}{368(3-x+2x^2)^2}-\frac{55(975+332x)}{8464(3-x+2x^2)}-\frac{4330}{529}\,\text{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right)$$
$$= \frac{121(19-7x)}{368(3-x+2x^2)^2}-\frac{55(975+332x)}{8464(3-x+2x^2)}-\frac{4330\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

**Mathematica [A]** time = 0.0290323, size = 51, normalized size = 0.8

$$\frac{4330\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{529\sqrt{23}}-\frac{11(1660x^3+4045x^2+938x+4909)}{4232(-2x^2+x-3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^3,x]
```

```
[Out] (-11*(4909 + 938*x + 4045*x^2 + 1660*x^3))/(4232*(-3 + x - 2*x^2)^2) + (433
0*ArcTan[(-1 + 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

---

**Maple [A]** time = 0.047, size = 47, normalized size = 0.7

$$4\,\frac{1}{(2x^2-x+3)^2}\left(-\frac{4565\,x^3}{4232}-\frac{44495\,x^2}{16928}-\frac{5159\,x}{8464}-\frac{53999}{16928}\right)+\frac{4330\,\sqrt{23}}{12167}\arctan\left(\frac{(-1+4\,x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x)
```

```
[Out] 4*(-4565/4232*x^3-44495/16928*x^2-5159/8464*x-53999/16928)/(2*x^2-x+3)^2+43
30/12167*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.43885, size = 76, normalized size = 1.19

$$\frac{4330}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)-\frac{11\,(1660\,x^3+4045\,x^2+938\,x+4909)}{4232\,(4\,x^4-4\,x^3+13\,x^2-6\,x+9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4
045*x^2 + 938*x + 4909)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Fricas [A]** time = 1.00681, size = 239, normalized size = 3.73

$$-\frac{419980\,x^3 - 34640\,\sqrt{23}\left(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9\right)\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x - 1)\right) + 1023385\,x^2 + 237314\,x + 1241977}{97336\left(4\,x^4 - 4\,x^3 + 13\,x^2 - 6\,x + 9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] -1/97336*(419980*x^3 - 34640*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*ar
ctan(1/23*sqrt(23)*(4*x - 1)) + 1023385*x^2 + 237314*x + 1241977)/(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Sympy [A]** time = 0.275541, size = 61, normalized size = 0.95

$$-\frac{18260x^3 + 44495x^2 + 10318x + 53999}{16928x^4 - 16928x^3 + 55016x^2 - 25392x + 38088} + \frac{4330\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23} - \frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**3,x)
```

```
[Out] -(18260*x**3 + 44495*x**2 + 10318*x + 53999)/(16928*x**4 - 16928*x**3 + 550
16*x**2 - 25392*x + 38088) + 4330*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/
23)/12167
```

---

**Giac [A]** time = 1.29135, size = 62, normalized size = 0.97

$$\frac{4330}{12167}\,\sqrt{23}\arctan\left(\frac{1}{23}\,\sqrt{23}(4\,x - 1)\right) - \frac{11\left(1660\,x^3 + 4045\,x^2 + 938\,x + 4909\right)}{4232\left(2\,x^2 - x + 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 4330/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 11/4232*(1660*x^3 + 4
045*x^2 + 938*x + 4909)/(2*x^2 - x + 3)^2
```

## 3.54 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^3} \, dx$

**Optimal**. Leaf size=64

$$-\frac{131(1-4x)}{2116(2x^2-x+3)}-\frac{11(3x+5)}{92(2x^2-x+3)^2}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

```
[Out] (-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) - (131*(1 - 4*x))/(2116*(3 - x + 2*x
^2)) - (262*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

---

**Rubi [A]** time = 0.0331539, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1660, 12, 614, 618, 204}

$$-\frac{131(1-4x)}{2116(2x^2-x+3)}-\frac{11(3x+5)}{92(2x^2-x+3)^2}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] (-11*(5 + 3*x))/(92*(3 - x + 2*x^2)^2) - (131*(1 - 4*x))/(2116*(3 - x + 2*x
^2)) - (262*ArcTan[(1 - 4*x)/Sqrt[23]])/(529*Sqrt[23])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^3}\,dx &= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}+\frac{1}{46}\int \frac{131}{2\left(3-x+2x^2\right)^2}\,dx \\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}+\frac{131}{92}\int \frac{1}{\left(3-x+2x^2\right)^2}\,dx \\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}+\frac{131}{529}\int \frac{1}{3-x+2x^2}\,dx \\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}-\frac{262}{529}\operatorname{Subst}\left(\int \frac{1}{-23-x^2}\,dx,x,-1+4x\right) \\
&= -\frac{11(5+3x)}{92\left(3-x+2x^2\right)^2}-\frac{131(1-4x)}{2116\left(3-x+2x^2\right)}-\frac{262\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{529\sqrt{23}}
\end{aligned}$$

**Mathematica [A]** time = 0.0280116, size = 51, normalized size = 0.8

$$\frac{\frac{46\left(524x^3-393x^2+472x-829\right)}{\left(-2x^2+x-3\right)^2}+1048\sqrt{23}\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{48668}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^3,x]
```

```
[Out] ((46*(-829 + 472*x - 393*x^2 + 524*x^3))/(-3 + x - 2*x^2)^2 + 1048*Sqrt[23]
*ArcTan[(-1 + 4*x)/Sqrt[23]])/48668
```

---

**Maple [A]** time = 0.046, size = 47, normalized size = 0.7

$$4\frac{1}{\left(2x^2-x+3\right)^2}\left(\frac{131\,x^3}{1058}-\frac{393\,x^2}{4232}+\frac{59\,x}{529}-\frac{829}{4232}\right)+\frac{262\sqrt{23}}{12167}\arctan\left(\frac{(-1+4\,x)\sqrt{23}}{23}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^3,x)
```

```
[Out] 4*(131/1058*x^3-393/4232*x^2+59/529*x-829/4232)/(2*x^2-x+3)^2+262/12167*23^
(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.45189, size = 76, normalized size = 1.19

$$\frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{524x^3-393x^2+472x-829}{1058\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="maxima")
```

```
[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*
x^2 + 472*x - 829)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Fricas [A]** time = 1.05006, size = 225, normalized size = 3.52

$$\frac{12052x^3+524\sqrt{23}\left(4x^4-4x^3+13x^2-6x+9\right)\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-9039x^2+10856x-19067}{24334\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="fricas")
```

```
[Out] 1/24334*(12052*x^3 + 524*sqrt(23)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*arctan
(1/23*sqrt(23)*(4*x - 1)) - 9039*x^2 + 10856*x - 19067)/(4*x^4 - 4*x^3 + 13
*x^2 - 6*x + 9)
```

---

**Sympy [A]** time = 0.19223, size = 61, normalized size = 0.95

$$\frac{524x^3-393x^2+472x-829}{4232x^4-4232x^3+13754x^2-6348x+9522}+\frac{262\sqrt{23}\operatorname{atan}\left(\frac{4\sqrt{23}x}{23}-\frac{\sqrt{23}}{23}\right)}{12167}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**3,x)
```

```
[Out] (524*x**3 - 393*x**2 + 472*x - 829)/(4232*x**4 - 4232*x**3 + 13754*x**2 - 6
348*x + 9522) + 262*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/12167
```

---

**Giac [A]** time = 1.20159, size = 62, normalized size = 0.97

$$\frac{262}{12167}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{524x^3-393x^2+472x-829}{1058\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^3,x, algorithm="giac")
```

```
[Out] 262/12167*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/1058*(524*x^3 - 393*
x^2 + 472*x - 829)/(2*x^2 - x + 3)^2
```

## 3.55 $\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)}\,dx$

**Optimal**. Leaf size=115

$$\frac{3625-746x}{256036\left(2x^2-x+3\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}-\frac{119\log\left(2x^2-x+3\right)}{21296}+\frac{119\log\left(5x^2+3x+2\right)}{21296}-\frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}+$$

```
[Out] (13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + (3625 - 746*x)/(256036*(3 - x + 2*x^2
)) - (53403*ArcTan[(1 - 4*x)/Sqrt[23]])/(5632792*Sqrt[23]) + (247*ArcTan[(3
 + 10*x)/Sqrt[31]])/(10648*Sqrt[31]) - (119*Log[3 - x + 2*x^2])/21296 + (11
9*Log[2 + 3*x + 5*x^2])/21296
```

---

**Rubi [A]** time = 0.1235, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{3625-746x}{256036\left(2x^2-x+3\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2}-\frac{119\log\left(2x^2-x+3\right)}{21296}+\frac{119\log\left(5x^2+3x+2\right)}{21296}-\frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(1012*(3 - x + 2*x^2)^2) + (3625 - 746*x)/(256036*(3 - x + 2*x^2
)) - (53403*ArcTan[(1 - 4*x)/Sqrt[23]])/(5632792*Sqrt[23]) + (247*ArcTan[(3
 + 10*x)/Sqrt[31]])/(10648*Sqrt[31]) - (119*Log[3 - x + 2*x^2])/21296 + (11
9*Log[2 + 3*x + 5*x^2])/21296
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
```

```
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)}dx = \frac{13-6x}{1012\left(3-x+2x^2\right)^2} - \frac{\int \frac{-3652-1936x+990x^2}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)}dx}{11132}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{\int \frac{-6551908-7779574x+902660x^2}{\left(3-x+2x^2\right)\left(2+3x+5x^2\right)}dx}{61960712}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{\int \frac{-154867174+335151124x}{3-x+2x^2}dx}{14994492304} - \frac{\int \frac{-425}{}}{}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} + \frac{53403\int \frac{1}{3-x+2x^2}dx}{11265584} - \frac{119\int \frac{-1+4x}{3-x+2x}}{21296}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{119\log\left(3-x+2x^2\right)}{21296} + \frac{119\log\left(2\right.}{21}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2} + \frac{3625-746x}{256036\left(3-x+2x^2\right)} - \frac{53403\tan^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{5632792\sqrt{23}} + \frac{247\tan^{-1}\left(\right.}{10648\sqrt{}}$$

**Mathematica [A]** time = 0.154692, size = 99, normalized size = 0.86

$$\frac{713\left(-\frac{44\left(1492x^3-7996x^2+7381x-14164\right)}{\left(-2x^2+x-3\right)^2} - 62951\log\left(2x^2-x+3\right) + 62951\log\left(5x^2+3x+2\right)\right) + 3310986\sqrt{23}\tan^{-1}\left(\frac{4x-1}{\sqrt{23}}\right) +}{8032361392}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (3310986*Sqrt[23]*ArcTan[(-1 + 4*x)/Sqrt[23]] + 6010498*Sqrt[31]*ArcTan[(3
+ 10*x)/Sqrt[31]] + 713*((-44*(-14164 + 7381*x - 7996*x^2 + 1492*x^3))/(-3
+ x - 2*x^2)^2 - 62951*Log[3 - x + 2*x^2] + 62951*Log[2 + 3*x + 5*x^2]))/80
32361392
```

---

**Maple [A]** time = 0.053, size = 89, normalized size = 0.8

$$\frac{119\ln\left(5x^2+3x+2\right)}{21296} + \frac{247\sqrt{31}}{330088}\arctan\left(\frac{(3+10x)\sqrt{31}}{31}\right) - \frac{1}{2662\left(2x^2-x+3\right)^2}\left(\frac{8206x^3}{529} - \frac{43978x^2}{529} + \frac{81191x}{1058} - \frac{77}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x)
```

```
[Out] 119/21296*ln(5*x^2+3*x+2)+247/330088*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2
)-1/2662*(8206/529*x^3-43978/529*x^2+81191/1058*x-77902/529)/(2*x^2-x+3)^2-
119/21296*ln(2*x^2-x+3)+53403/129554216*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1
/2))
```

---

**Maxima [A]** time = 1.43151, size = 132, normalized size = 1.15

$$\frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1492x^3-7996x^2+7381x}{256036\left(4x^4-4x^3+13x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381
*x - 14164)/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) + 119/21296*log(5*x^2 + 3*x
+ 2) - 119/21296*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.02491, size = 556, normalized size = 4.83

$$-\frac{46807024x^3-6010498\sqrt{31}\left(4x^4-4x^3+13x^2-6x+9\right)\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-3310986\sqrt{23}\left(4x^4-4x^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/8032361392*(46807024*x^3 - 6010498*sqrt(31)*(4*x^4 - 4*x^3 + 13*x^2 - 6*
x + 9)*arctan(1/31*sqrt(31)*(10*x + 3)) - 3310986*sqrt(23)*(4*x^4 - 4*x^3 +
 13*x^2 - 6*x + 9)*arctan(1/23*sqrt(23)*(4*x - 1)) - 250850512*x^2 - 448840
63*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(5*x^2 + 3*x + 2) + 44884063*(4*x^
4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(2*x^2 - x + 3) + 231556732*x - 444353008)
/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Sympy [A]** time = 0.396684, size = 122, normalized size = 1.06

$$-\frac{1492x^3-7996x^2+7381x-14164}{1024144x^4-1024144x^3+3328468x^2-1536216x+2304324}-\frac{119\log\left(x^2-\frac{x}{2}+\frac{3}{2}\right)}{21296}+\frac{119\log\left(x^2+\frac{3x}{5}+\frac{2}{5}\right)}{21296}+\frac{534}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2),x)
```

```
[Out] -(1492*x**3 - 7996*x**2 + 7381*x - 14164)/(1024144*x**4 - 1024144*x**3 + 33
28468*x**2 - 1536216*x + 2304324) - 119*log(x**2 - x/2 + 3/2)/21296 + 119*l
og(x**2 + 3*x/5 + 2/5)/21296 + 53403*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(2
3)/23)/129554216 + 247*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/3300
88
```

---

**Giac [A]** time = 1.15984, size = 119, normalized size = 1.03

$$\frac{247}{330088}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{53403}{129554216}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1492x^3-7996x^2+7381x-}{256036\left(2x^2-x+3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 247/330088*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 53403/129554216*sqrt
(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/256036*(1492*x^3 - 7996*x^2 + 7381
*x - 14164)/(2*x^2 - x + 3)^2 + 119/21296*log(5*x^2 + 3*x + 2) - 119/21296*
log(2*x^2 - x + 3)
```

## 3.56 $\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^2}\,dx$

**Optimal**. Leaf size=160

$$\frac{9665-1446x}{512072\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}-\frac{252815x+2328909}{174616552\left(5x^2+3x+2\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}+\frac{181\log\left(2\ldots\right.}{468\ldots}$$

```
[Out] -(2328909 + 252815*x)/(174616552*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(1012*(3 -
 x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + (9665 - 1446*x)/(512072*(3 - x + 2*x^2)*
(2 + 3*x + 5*x^2)) + (2038497*ArcTan[(1 - 4*x)/Sqrt[23]])/(123921424*Sqrt[2
3]) + (246757*ArcTan[(3 + 10*x)/Sqrt[31]])/(7261936*Sqrt[31]) + (181*Log[3
- x + 2*x^2])/468512 - (181*Log[2 + 3*x + 5*x^2])/468512
```

---

**Rubi [A]** time = 0.160139, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{9665-1446x}{512072\left(2x^2-x+3\right)\left(5x^2+3x+2\right)}-\frac{252815x+2328909}{174616552\left(5x^2+3x+2\right)}+\frac{13-6x}{1012\left(2x^2-x+3\right)^2\left(5x^2+3x+2\right)}+\frac{181\log\left(2\ldots\right.}{468\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(2328909 + 252815*x)/(174616552*(2 + 3*x + 5*x^2)) + (13 - 6*x)/(1012*(3 -
 x + 2*x^2)^2*(2 + 3*x + 5*x^2)) + (9665 - 1446*x)/(512072*(3 - x + 2*x^2)*
(2 + 3*x + 5*x^2)) + (2038497*ArcTan[(1 - 4*x)/Sqrt[23]])/(123921424*Sqrt[2
3]) + (246757*ArcTan[(3 + 10*x)/Sqrt[31]])/(7261936*Sqrt[31]) + (181*Log[3
- x + 2*x^2])/468512 - (181*Log[2 + 3*x + 5*x^2])/468512
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
```

```
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^3\left(2+3x+5x^2\right)^2}\,dx = \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} - \frac{\int \frac{-4081-3168x+1650x^2}{\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)^2}\,dx}{11132}$$

$$= \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{9665-1446x}{512072\left(3-x+2x^2\right)\left(2+3x+5x^2\right)} -$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{512072\,(}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{512072\,(}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{512072\,(}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{512072\,(}$$

$$= -\frac{2328909+252815x}{174616552\left(2+3x+5x^2\right)} + \frac{13-6x}{1012\left(3-x+2x^2\right)^2\left(2+3x+5x^2\right)} + \frac{}{512072\,(}$$

**Mathematica [A]** time = 0.101724, size = 136, normalized size = 0.85

$$\frac{-2923x-1782}{1408198\left(2x^2-x+3\right)} + \frac{1235x-1474}{330088\left(5x^2+3x+2\right)} + \frac{-14x-31}{22264\left(2x^2-x+3\right)^2} + \frac{181\log\left(2x^2-x+3\right)}{468512} - \frac{181\log\left(5x^2+3x\right.}{468512}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (-31 - 14*x)/(22264*(3 - x + 2*x^2)^2) + (-1782 - 2923*x)/(1408198*(3 - x +
 2*x^2)) + (-1474 + 1235*x)/(330088*(2 + 3*x + 5*x^2)) - (2038497*ArcTan[(-
1 + 4*x)/Sqrt[23]])/(123921424*Sqrt[23]) + (246757*ArcTan[(3 + 10*x)/Sqrt[3
1]])/(7261936*Sqrt[31]) + (181*Log[3 - x + 2*x^2])/468512 - (181*Log[2 + 3*
x + 5*x^2])/468512
```

---

**Maple [A]** time = 0.056, size = 106, normalized size = 0.7

$$-\frac{1}{234256}\left(-\frac{5434\,x}{31}+\frac{32428}{155}\right)\left(x^2+\frac{3\,x}{5}+\frac{2}{5}\right)^{-1} - \frac{181\,\ln\left(5\,x^2+3\,x+2\right)}{468512} + \frac{246757\,\sqrt{31}}{225120016}\arctan\left(\frac{(3+10\,x)\sqrt{31}}{31}\right) + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x)
```

```
[Out] -1/234256*(-5434/31*x+32428/155)/(x^2+3/5*x+2/5)-181/468512*ln(5*x^2+3*x+2)
+246757/225120016*arctan(1/31*(3+10*x)*31^(1/2))*31^(1/2)+1/58564*(-128612/
529*x^3-14102/529*x^2-173195/529*x-321497/1058)/(2*x^2-x+3)^2+181/468512*ln
```

```
(2*x^2-x+3)-2038497/2850192752*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.46593, size = 157, normalized size = 0.98

$$\frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1011260x^5+8304376x^4-}{174616552\,(20x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/285019
2752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 +
8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/(20*x^6 - 8
*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) - 181/468512*log(5*x^2 + 3*x + 2)
 + 181/468512*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.03044, size = 763, normalized size = 4.77

$$-\frac{31725248720x^5+260524883872x^4-158204886268x^3-6004584838\sqrt{31}\left(20x^6-8x^5+61x^4+x^3+53x^2+15x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/5478070469344*(31725248720*x^5 + 260524883872*x^4 - 158204886268*x^3 - 6
004584838*sqrt(31)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*arc
tan(1/31*sqrt(31)*(10*x + 3)) + 3917991234*sqrt(23)*(20*x^6 - 8*x^5 + 61*x^
4 + x^3 + 53*x^2 + 15*x + 18)*arctan(1/23*sqrt(23)*(4*x - 1)) + 67996648469
2*x^2 + 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*log
(5*x^2 + 3*x + 2) - 2116340147*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15
*x + 18)*log(2*x^2 - x + 3) - 184712689040*x + 277008109136)/(20*x^6 - 8*x^
5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

---

**Sympy [A]** time = 0.534726, size = 143, normalized size = 0.89

$$-\frac{1011260x^5+8304376x^4-5042869x^3+21674311x^2-5887820x+8829788}{3492331040x^6-1396932416x^5+10651609672x^4+174616552x^3+9254677256x^2+2619248280x+3143097936}+\frac{1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**2,x)
```

```
[Out] -(1011260*x**5 + 8304376*x**4 - 5042869*x**3 + 21674311*x**2 - 5887820*x +
8829788)/(3492331040*x**6 - 1396932416*x**5 + 10651609672*x**4 + 174616552*
x**3 + 9254677256*x**2 + 2619248280*x + 3143097936) + 181*log(x**2 - x/2 +
3/2)/468512 - 181*log(x**2 + 3*x/5 + 2/5)/468512 - 2038497*sqrt(23)*atan(4*
sqrt(23)*x/23 - sqrt(23)/23)/2850192752 + 246757*sqrt(31)*atan(10*sqrt(31)*
x/31 + 3*sqrt(31)/31)/225120016
```

---

**Giac [A]** time = 1.1564, size = 149, normalized size = 0.93

$$\frac{246757}{225120016}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)-\frac{2038497}{2850192752}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{1011260x^5+8304376x^{\cdot}}{17461}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 246757/225120016*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) - 2038497/285019
2752*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) - 1/174616552*(1011260*x^5 +
8304376*x^4 - 5042869*x^3 + 21674311*x^2 - 5887820*x + 8829788)/((5*x^2 + 3
*x + 2)*(2*x^2 - x + 3)^2) - 181/468512*log(5*x^2 + 3*x + 2) + 181/468512*l
og(2*x^2 - x + 3)
```

## 3.57 $\int \frac{1}{(3-x+2x^2)^3 (2+3x+5x^2)^3} \, dx$

**Optimal**. Leaf size=181

$$\frac{5(302-35x)}{64009\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{15(7140435x+2618306)}{14886061058\left(5x^2+3x+2\right)}-\frac{5(77020x+223707)}{87308276\left(5x^2+3x+2\right)^2}+\frac{13-6}{1012\left(2x^2-x+3\right)^2}$$

```
[Out] (-5*(223707 + 77020*x))/(87308276*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(1012*(
3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*(302 - 35*x))/(64009*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (15*(2618306 + 7140435*x))/(14886061058*(2 + 3*
x + 5*x^2)) - (880575*ArcTan[(1 - 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2
768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x +
 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408
```

---

**Rubi [A]** time = 0.204182, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {974, 1060, 1072, 634, 618, 204, 628}

$$\frac{5(302-35x)}{64009\left(2x^2-x+3\right)\left(5x^2+3x+2\right)^2}+\frac{15(7140435x+2618306)}{14886061058\left(5x^2+3x+2\right)}-\frac{5(77020x+223707)}{87308276\left(5x^2+3x+2\right)^2}+\frac{13-6}{1012\left(2x^2-x+3\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```


```
[Out] (-5*(223707 + 77020*x))/(87308276*(2 + 3*x + 5*x^2)^2) + (13 - 6*x)/(1012*(
3 - x + 2*x^2)^2*(2 + 3*x + 5*x^2)^2) + (5*(302 - 35*x))/(64009*(3 - x + 2*
x^2)*(2 + 3*x + 5*x^2)^2) + (15*(2618306 + 7140435*x))/(14886061058*(2 + 3*
x + 5*x^2)) - (880575*ArcTan[(1 - 4*x)/Sqrt[23]])/(340783916*Sqrt[23]) + (2
768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(619080044*Sqrt[31]) + (405*Log[3 - x +
 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2])/1288408
```


Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
```

```
c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1072

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = c^2*d^2 - b*c*
d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2}, Dist[1/q, Int[(A*
c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f
 + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(a + b*x + c*x^2),
x], x] + Dist[1/q, Int[(c*C*d^2 - B*c*d*e + A*c*e^2 + b*B*d*f - A*c*d*f - a
*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d - b*C*d - A*c*e + a*C*e + A*b*f - a*B
*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, e, f,
A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^3(2+3x+5x^2)^3}\,dx = \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} - \frac{\int \frac{-4510-4400x+2310x^2}{(3-x+2x^2)^2(2+3x+5x^2)^3}\,dx}{11132}$$

$$= \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{5(302-35x)}{64009(3-x+2x^2)(2+3x+5x^2)^2} - - \int$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64009(3-}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64009(3-}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64009(3-}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64009(3-}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64009(3-}$$

$$= -\frac{5(223707+77020x)}{87308276(2+3x+5x^2)^2} + \frac{13-6x}{1012(3-x+2x^2)^2(2+3x+5x^2)^2} + \frac{}{64009(3-}$$

**Mathematica [A]** time = 0.0852828, size = 151, normalized size = 0.83

$$\frac{6850x^3-9275x^2+11154x-4342}{345092(10x^4+x^3+16x^2+7x+6)^2} + \frac{5(42842610x^3-5711469x^2+51156233x+14085977)}{14886061058(10x^4+x^3+16x^2+7x+6)} + \frac{405\log(2x^2-x+3)}{1288408}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^3*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (-4342 + 11154*x - 9275*x^2 + 6850*x^3)/(345092*(6 + 7*x + 16*x^2 + x^3 + 1
0*x^4)^2) + (5*(14085977 + 51156233*x - 5711469*x^2 + 42842610*x^3))/(14886
061058*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)) + (880575*ArcTan[(-1 + 4*x)/Sqrt[
23]])/(340783916*Sqrt[23]) + (2768835*ArcTan[(3 + 10*x)/Sqrt[31]])/(6190800
44*Sqrt[31]) + (405*Log[3 - x + 2*x^2])/1288408 - (405*Log[2 + 3*x + 5*x^2]
)/1288408
```

---

**Maple [A]** time = 0.053, size = 118, normalized size = 0.7

$$-\frac{25}{2576816(5x^2+3x+2)^2}\left(-\frac{3013197x^3}{961} - \frac{14516062x^2}{4805} - \frac{51193868x}{24025} - \frac{5423968}{24025}\right) - \frac{405\ln(5x^2+3x+2)}{1288408} + \frac{27688}{19191}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x)
```

```
[Out] -25/2576816*(-3013197/961*x^3-14516062/4805*x^2-51193868/24025*x-5423968/24
025)/(5*x^2+3*x+2)^2-405/1288408*ln(5*x^2+3*x+2)+2768835/19191481364*arctan
(1/31*(3+10*x)*31^(1/2))*31^(1/2)+1/644204*(302907/529*x^3-368291/529*x^2+2
501587/2116*x-665819/1058)/(2*x^2-x+3)^2+405/1288408*ln(2*x^2-x+3)+880575/7
838030068*23^(1/2)*arctan(1/23*(-1+4*x)*23^(1/2))
```

---

**Maxima [A]** time = 1.44651, size = 186, normalized size = 1.03

$$\frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{4284261000x^7-142\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838
030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000
*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 +
 4691822415*x^2 + 5017681412*x + 470561254)/(100*x^8 + 20*x^7 + 321*x^6 + 1
72*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) - 405/1288408*log(5*x^2 +
 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.17243, size = 1035, normalized size = 5.72

$$\frac{67202918046000x^7-2238718468800x^6+186872434930060x^5+62827256425340x^4+173919793526820x^3+67\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/467005507511576*(67202918046000*x^7 - 2238718468800*x^6 + 186872434930060
*x^5 + 62827256425340*x^4 + 173919793526820*x^3 + 67376830890*sqrt(31)*(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)*arctan(1/31*sqrt(31)*(10*x + 3)) + 52466419650*sqrt(23)*(100*x^8 + 20*x^7
 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*arctan(1/23
*sqrt(23)*(4*x - 1)) + 73595926401690*x^2 - 146799174285*(100*x^8 + 20*x^7
+ 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*log(5*x^2 +
3*x + 2) + 146799174285*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 2
36*x^3 + 241*x^2 + 84*x + 36)*log(2*x^2 - x + 3) + 78707350628632*x + 73812
23830244)/(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x
^2 + 84*x + 36)
```

---

**Sympy [A]** time = 0.539186, size = 163, normalized size = 0.9

$$\frac{4284261000x^7-142720800x^6+11913326210x^5+4005307690x^4+11087580870x^3+4691\ldots}{2977212211600x^8+595442442320x^7+9556851199236x^6+5120805003952x^5+11611127625240x^4+7026220819\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**3/(5*x**2+3*x+2)**3,x)
```

```
[Out] (4284261000*x**7 - 142720800*x**6 + 11913326210*x**5 + 4005307690*x**4 + 11
087580870*x**3 + 4691822415*x**2 + 5017681412*x + 470561254)/(2977212211600
*x**8 + 595442442320*x**7 + 9556851199236*x**6 + 5120805003952*x**5 + 11611
127625240*x**4 + 7026220819376*x**3 + 7175081429956*x**2 + 2500858257744*x
+ 1071796396176) + 405*log(x**2 - x/2 + 3/2)/1288408 - 405*log(x**2 + 3*x/5
 + 2/5)/1288408 + 880575*sqrt(23)*atan(4*sqrt(23)*x/23 - sqrt(23)/23)/78380
30068 + 2768835*sqrt(31)*atan(10*sqrt(31)*x/31 + 3*sqrt(31)/31)/19191481364
```

---

**Giac [A]** time = 1.16417, size = 157, normalized size = 0.87

$$\frac{2768835}{19191481364}\sqrt{31}\arctan\left(\frac{1}{31}\sqrt{31}(10x+3)\right)+\frac{880575}{7838030068}\sqrt{23}\arctan\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{4284261000x^7-142720$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^3/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 2768835/19191481364*sqrt(31)*arctan(1/31*sqrt(31)*(10*x + 3)) + 880575/7838
030068*sqrt(23)*arctan(1/23*sqrt(23)*(4*x - 1)) + 1/29772122116*(4284261000
*x^7 - 142720800*x^6 + 11913326210*x^5 + 4005307690*x^4 + 11087580870*x^3 +
 4691822415*x^2 + 5017681412*x + 470561254)/(10*x^4 + x^3 + 16*x^2 + 7*x +
6)^2 - 405/1288408*log(5*x^2 + 3*x + 2) + 405/1288408*log(2*x^2 - x + 3)
```

## 3.58 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx$

**Optimal**. Leaf size=208

$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7+\frac{14125}{144}\left(2x^2-x+3\right)^{3/2}x^6+\frac{233225\left(2x^2-x+3\right)^{3/2}x^5}{1536}+\frac{4796405\left(2x^2-x+3\right)^{3/2}x^4}{43008}+$$

```
[Out] (-359471503*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/67108864 + (27185733541*(3 - x +
 2*x^2)^(3/2))/440401920 + (804243809*x*(3 - x + 2*x^2)^(3/2))/36700160 - (
83948353*x^2*(3 - x + 2*x^2)^(3/2))/2293760 + (8325631*x^3*(3 - x + 2*x^2)^
(3/2))/1032192 + (4796405*x^4*(3 - x + 2*x^2)^(3/2))/43008 + (233225*x^5*(3
 - x + 2*x^2)^(3/2))/1536 + (14125*x^6*(3 - x + 2*x^2)^(3/2))/144 + (125*x^
7*(3 - x + 2*x^2)^(3/2))/4 - (8267844569*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1342
17728*Sqrt[2])
```

---

**Rubi [A]** time = 0.309322, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{4}\left(2x^2-x+3\right)^{3/2}x^7+\frac{14125}{144}\left(2x^2-x+3\right)^{3/2}x^6+\frac{233225\left(2x^2-x+3\right)^{3/2}x^5}{1536}+\frac{4796405\left(2x^2-x+3\right)^{3/2}x^4}{43008}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (-359471503*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/67108864 + (27185733541*(3 - x +
 2*x^2)^(3/2))/440401920 + (804243809*x*(3 - x + 2*x^2)^(3/2))/36700160 - (
83948353*x^2*(3 - x + 2*x^2)^(3/2))/2293760 + (8325631*x^3*(3 - x + 2*x^2)^
(3/2))/1032192 + (4796405*x^4*(3 - x + 2*x^2)^(3/2))/43008 + (233225*x^5*(3
 - x + 2*x^2)^(3/2))/1536 + (14125*x^6*(3 - x + 2*x^2)^(3/2))/144 + (125*x^
7*(3 - x + 2*x^2)^(3/2))/4 - (8267844569*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1342
17728*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^4\,dx = \frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+\frac{1}{20}\int \sqrt{3-x+2x^2}\left(320+1920x+7520x^2+18720x^3+\right.$$

$$=\frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2}+\frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+\frac{1}{360}\int \sqrt{3-x+2x^2}\left(5760+\right.$$

$$=\frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{14125}{144}x^6\left(3-x+2x^2\right)^{3/2}+\frac{125}{4}x^7\left(3-x+2x^2\right)^{3/2}+$$

$$=\frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008}+\frac{233225x^5\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{14125}{144}x^6\left(3-x+2x^2\right)$$

$$=\frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192}+\frac{4796405x^4\left(3-x+2x^2\right)^{3/2}}{43008}+\frac{233225x^5\left(3-x+2x\right.}{1536}$$

$$=-\frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760}+\frac{8325631x^3\left(3-x+2x^2\right)^{3/2}}{1032192}+\frac{4796405x^4\left(3-x\right.}{43008}$$

$$=\frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160}-\frac{83948353x^2\left(3-x+2x^2\right)^{3/2}}{2293760}+\frac{8325631x^3\left(3-x\right.}{1032192}$$

$$=\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804243809x\left(3-x+2x^2\right)^{3/2}}{36700160}-\frac{83948353x^2\left(3-\right.}{229376}$$

$$=-\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804243809x}{36}$$

$$=-\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804243809x}{36}$$

$$=-\frac{359471503(1-4x)\sqrt{3-x+2x^2}}{67108864}+\frac{27185733541\left(3-x+2x^2\right)^{3/2}}{440401920}+\frac{804243809x}{36}$$

**Mathematica [A]** time = 0.291108, size = 85, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(1321205760000x^9+3486515200000x^8+6327795712000x^7+7725962035200x^6+7612808028160x^5+\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3801512106459 + 537752185764*x - 174418077792*x^2 +
 2211683657856*x^3 + 5354741991424*x^4 + 7612808028160*x^5 + 7725962035200*
x^6 + 6327795712000*x^7 + 3486515200000*x^8 + 1321205760000*x^9) - 26043710
39235*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/84557168640
```

---

**Maple [A]** time = 0.069, size = 166, normalized size = 0.8

$$\frac{125\,x^7}{4}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{14125\,x^6}{144}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{233225\,x^5}{1536}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{4796405\,x^4}{43008}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{83256}{103}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 125/4*x^7*(2*x^2-x+3)^(3/2)+14125/144*x^6*(2*x^2-x+3)^(3/2)+233225/1536*x^5
*(2*x^2-x+3)^(3/2)+4796405/43008*x^4*(2*x^2-x+3)^(3/2)+8325631/1032192*x^3*
(2*x^2-x+3)^(3/2)-83948353/2293760*x^2*(2*x^2-x+3)^(3/2)+804243809/36700160
*x*(2*x^2-x+3)^(3/2)+359471503/67108864*(-1+4*x)*(2*x^2-x+3)^(1/2)+82678445
69/268435456*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+27185733541/440401920*(
2*x^2-x+3)^(3/2)
```

---

**Maxima [A]** time = 1.48403, size = 239, normalized size = 1.15

$$\frac{125}{4}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^7+\frac{14125}{144}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^6+\frac{233225}{1536}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^5+\frac{4796405}{43008}\left(2\,x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{832563}{103219}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/4*(2*x^2 - x + 3)^(3/2)*x^7 + 14125/144*(2*x^2 - x + 3)^(3/2)*x^6 + 233
225/1536*(2*x^2 - x + 3)^(3/2)*x^5 + 4796405/43008*(2*x^2 - x + 3)^(3/2)*x^
4 + 8325631/1032192*(2*x^2 - x + 3)^(3/2)*x^3 - 83948353/2293760*(2*x^2 - x
 + 3)^(3/2)*x^2 + 804243809/36700160*(2*x^2 - x + 3)^(3/2)*x + 27185733541/
440401920*(2*x^2 - x + 3)^(3/2) + 359471503/16777216*sqrt(2*x^2 - x + 3)*x
+ 8267844569/268435456*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 359471503
/67108864*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.40846, size = 439, normalized size = 2.11

$$\frac{1}{21139292160}\left(1321205760000\,x^9+3486515200000\,x^8+6327795712000\,x^7+7725962035200\,x^6+7612808028160\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/21139292160*(1321205760000*x^9 + 3486515200000*x^8 + 6327795712000*x^7 +
7725962035200*x^6 + 7612808028160*x^5 + 5354741991424*x^4 + 2211683657856*x
^3 - 174418077792*x^2 + 537752185764*x + 3801512106459)*sqrt(2*x^2 - x + 3)
 + 8267844569/536870912*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1
```

```
) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x^2 - x + 3}\left(5x^2 + 3x + 2\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**4, x)
```

---

**Giac [A]** time = 1.13665, size = 126, normalized size = 0.61

$$\frac{1}{21139292160}(4(8(4(16(20(40(140(160(36x + 95)x + 27587)x + 4715553)x + 185859571)x + 2614620113)x + 17278$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/21139292160*(4*(8*(4*(16*(20*(40*(140*(160*(36*x + 95)*x + 27587)*x + 471
5553)*x + 185859571)*x + 2614620113)*x + 17278778577)*x - 5450564931)*x + 1
34438046441)*x + 3801512106459)*sqrt(2*x^2 - x + 3) - 8267844569/268435456*
sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.59 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3\,dx$

**Optimal**. Leaf size=166

$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5+\frac{8825}{448}\left(2x^2-x+3\right)^{3/2}x^4+\frac{247435\left(2x^2-x+3\right)^{3/2}x^3}{10752}+\frac{531681\left(2x^2-x+3\right)^{3/2}x^2}{71680}-\frac{96273}{}$$

```
[Out] (-6766097*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (22548119*(3 - x + 2*x^2
)^(3/2))/4587520 - (9627393*x*(3 - x + 2*x^2)^(3/2))/1146880 + (531681*x^2*
(3 - x + 2*x^2)^(3/2))/71680 + (247435*x^3*(3 - x + 2*x^2)^(3/2))/10752 + (
8825*x^4*(3 - x + 2*x^2)^(3/2))/448 + (125*x^5*(3 - x + 2*x^2)^(3/2))/16 -
(155620231*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```

---

**Rubi [A]** time = 0.181374, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{16}\left(2x^2-x+3\right)^{3/2}x^5+\frac{8825}{448}\left(2x^2-x+3\right)^{3/2}x^4+\frac{247435\left(2x^2-x+3\right)^{3/2}x^3}{10752}+\frac{531681\left(2x^2-x+3\right)^{3/2}x^2}{71680}-\frac{96273}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```


```
[Out] (-6766097*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2097152 - (22548119*(3 - x + 2*x^2
)^(3/2))/4587520 - (9627393*x*(3 - x + 2*x^2)^(3/2))/1146880 + (531681*x^2*
(3 - x + 2*x^2)^(3/2))/71680 + (247435*x^3*(3 - x + 2*x^2)^(3/2))/10752 + (
8825*x^4*(3 - x + 2*x^2)^(3/2))/448 + (125*x^5*(3 - x + 2*x^2)^(3/2))/16 -
(155620231*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4194304*Sqrt[2])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3 dx &= \frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2}+\frac{1}{16}\int \sqrt{3-x+2x^2}\left(128+576x+1824x^2+3312x^3+26\right. \\
&= \frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2}+\frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2}+\frac{1}{224}\int \sqrt{3-x+2x^2}\left(1792+\right. \\
&= \frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752}+\frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2}+\frac{125}{16}x^5\left(3-x+2x^2\right)^{3/2}+ \\
&= \frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680}+\frac{247435x^3\left(3-x+2x^2\right)^{3/2}}{10752}+\frac{8825}{448}x^4\left(3-x+2x^2\right)^{3/2} \\
&= -\frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880}+\frac{531681x^2\left(3-x+2x^2\right)^{3/2}}{71680}+\frac{247435x^3\left(3-x+2x^2\right.}{10752} \\
&= -\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{9627393x\left(3-x+2x^2\right)^{3/2}}{1146880}+\frac{531681x^2\left(3-x+2x^2\right.}{71680} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{9627393x\left(3-x+\right.}{1146880} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{9627393x\left(3-x+\right.}{1146880} \\
&= -\frac{6766097(1-4x)\sqrt{3-x+2x^2}}{2097152}-\frac{22548119\left(3-x+2x^2\right)^{3/2}}{4587520}-\frac{9627393x\left(3-x+\right.}{1146880}
\end{aligned}$$

**Mathematica [A]** time = 0.172468, size = 75, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(3440640000x^7+6955008000x^6+10958233600x^5+11212171264x^4+9872163456x^3+4583812128x^2-\right.}{880803840}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-3957369321 - 1621307916*x + 4583812128*x^2 + 98721
63456*x^3 + 11212171264*x^4 + 10958233600*x^5 + 6955008000*x^6 + 3440640000
*x^7) - 16340124255*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/880803840
```

---

**Maple [A]** time = 0.059, size = 132, normalized size = 0.8

$$\frac{125\,x^5}{16}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{8825\,x^4}{448}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{247435\,x^3}{10752}\left(2\,x^2-x+3\right)^{\frac{3}{2}}+\frac{531681\,x^2}{71680}\left(2\,x^2-x+3\right)^{\frac{3}{2}}-\frac{9627393\,x}{1146880}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 125/16*x^5*(2*x^2-x+3)^(3/2)+8825/448*x^4*(2*x^2-x+3)^(3/2)+247435/10752*x^
3*(2*x^2-x+3)^(3/2)+531681/71680*x^2*(2*x^2-x+3)^(3/2)-9627393/1146880*x*(2
*x^2-x+3)^(3/2)+6766097/2097152*(-1+4*x)*(2*x^2-x+3)^(1/2)+155620231/838860
8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-22548119/4587520*(2*x^2-x+3)^(3/2)
```

---

**Maxima [A]** time = 1.47232, size = 193, normalized size = 1.16

$$\frac{125}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^5+\frac{8825}{448}\left(2x^2-x+3\right)^{\frac{3}{2}}x^4+\frac{247435}{10752}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{531681}{71680}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2-\frac{9627393}{1146880}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/16*(2*x^2 - x + 3)^(3/2)*x^5 + 8825/448*(2*x^2 - x + 3)^(3/2)*x^4 + 247
435/10752*(2*x^2 - x + 3)^(3/2)*x^3 + 531681/71680*(2*x^2 - x + 3)^(3/2)*x^
2 - 9627393/1146880*(2*x^2 - x + 3)^(3/2)*x - 22548119/4587520*(2*x^2 - x +
 3)^(3/2) + 6766097/524288*sqrt(2*x^2 - x + 3)*x + 155620231/8388608*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 6766097/2097152*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.52335, size = 352, normalized size = 2.12

$$\frac{1}{220200960}\left(3440640000\,x^7+6955008000\,x^6+10958233600\,x^5+11212171264\,x^4+9872163456\,x^3+4583812128\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/220200960*(3440640000*x^7 + 6955008000*x^6 + 10958233600*x^5 + 1121217126
4*x^4 + 9872163456*x^3 + 4583812128*x^2 - 1621307916*x - 3957369321)*sqrt(2
*x^2 - x + 3) + 155620231/16777216*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3, x)
```

---

**Giac [A]** time = 1.32715, size = 112, normalized size = 0.67

$$\frac{1}{220200960}\left(4\left(8\left(4\left(16\left(100\left(120\left(140\,x+283\right)x+53507\right)x+5474693\right)x+77126277\right)x+143244129\right)x-405326979\right)x-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/220200960*(4*(8*(4*(16*(100*(120*(140*x + 283)*x + 53507)*x + 5474693)*x
+ 77126277)*x + 143244129)*x - 405326979)*x - 3957369321)*sqrt(2*x^2 - x +
3) - 155620231/8388608*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x +
 3)) + 1)
```

## 3.60 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2\,dx$

**Optimal**. Leaf size=124

$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{3/2}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{3/2}x-\frac{2107\left(2x^2-x+3\right)^{3/2}}{3072}+\frac{12371(1-4x)\sqrt{2x^2}}{16384}$$

```
[Out] (12371*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 - (2107*(3 - x + 2*x^2)^(3/2))/
3072 + (769*x*(3 - x + 2*x^2)^(3/2))/256 + (63*x^2*(3 - x + 2*x^2)^(3/2))/1
6 + (25*x^3*(3 - x + 2*x^2)^(3/2))/12 + (284533*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/(32768*Sqrt[2])
```

---

**Rubi [A]** time = 0.0990913, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{12}\left(2x^2-x+3\right)^{3/2}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{3/2}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{3/2}x-\frac{2107\left(2x^2-x+3\right)^{3/2}}{3072}+\frac{12371(1-4x)\sqrt{2x^2}}{16384}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (12371*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16384 - (2107*(3 - x + 2*x^2)^(3/2))/
3072 + (769*x*(3 - x + 2*x^2)^(3/2))/256 + (63*x^2*(3 - x + 2*x^2)^(3/2))/1
6 + (25*x^3*(3 - x + 2*x^2)^(3/2))/12 + (284533*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/(32768*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2 dx = \frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{1}{12}\int \sqrt{3-x+2x^2}\left(48+144x+123x^2+\frac{945x^3}{2}\right) dx$$

$$= \frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2} + \frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{1}{120}\int \sqrt{3-x+2x^2}\left(480-1395\right.$$

$$= \frac{769}{256}x\left(3-x+2x^2\right)^{3/2} + \frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2} + \frac{25}{12}x^3\left(3-x+2x^2\right)^{3/2} + \frac{1}{960}\int\left(\right.$$

$$= -\frac{2107\left(3-x+2x^2\right)^{3/2}}{3072} + \frac{769}{256}x\left(3-x+2x^2\right)^{3/2} + \frac{63}{16}x^2\left(3-x+2x^2\right)^{3/2} + \frac{25}{12}x^3\left(3\right.$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107\left(3-x+2x^2\right)^{3/2}}{3072} + \frac{769}{256}x\left(3-x+2x^2\right)^{3/2} + \frac{63}{16}$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107\left(3-x+2x^2\right)^{3/2}}{3072} + \frac{769}{256}x\left(3-x+2x^2\right)^{3/2} + \frac{63}{16}$$

$$= \frac{12371(1-4x)\sqrt{3-x+2x^2}}{16384} - \frac{2107\left(3-x+2x^2\right)^{3/2}}{3072} + \frac{769}{256}x\left(3-x+2x^2\right)^{3/2} + \frac{63}{16}$$

**Mathematica [A]** time = 0.10554, size = 65, normalized size = 0.52

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023\right)+853599\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{196608}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-64023 + 328204*x + 365536*x^2 + 408960*x^3 + 28467
2*x^4 + 204800*x^5) + 853599*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/196608
```

---

**Maple [A]** time = 0.053, size = 98, normalized size = 0.8

$$\frac{25\,x^3}{12}\left(2\,x^2-x+3\right)^{\frac{3}{2}} + \frac{63\,x^2}{16}\left(2\,x^2-x+3\right)^{\frac{3}{2}} + \frac{769\,x}{256}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{2107}{3072}\left(2\,x^2-x+3\right)^{\frac{3}{2}} - \frac{-12371+49484\,x}{16384}\sqrt{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 25/12*x^3*(2*x^2-x+3)^(3/2)+63/16*x^2*(2*x^2-x+3)^(3/2)+769/256*x*(2*x^2-x+
3)^(3/2)-2107/3072*(2*x^2-x+3)^(3/2)-12371/16384*(-1+4*x)*(2*x^2-x+3)^(1/2)
```

```
-284533/65536*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

---

**Maxima [A]** time = 1.53431, size = 147, normalized size = 1.19

$$\frac{25}{12}\left(2x^2-x+3\right)^{\frac{3}{2}}x^3+\frac{63}{16}\left(2x^2-x+3\right)^{\frac{3}{2}}x^2+\frac{769}{256}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{2107}{3072}\left(2x^2-x+3\right)^{\frac{3}{2}}-\frac{12371}{4096}\sqrt{2x^2-x+3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 25/12*(2*x^2 - x + 3)^(3/2)*x^3 + 63/16*(2*x^2 - x + 3)^(3/2)*x^2 + 769/256
*(2*x^2 - x + 3)^(3/2)*x - 2107/3072*(2*x^2 - x + 3)^(3/2) - 12371/4096*sqr
t(2*x^2 - x + 3)*x - 284533/65536*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
+ 12371/16384*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.52922, size = 257, normalized size = 2.07

$$\frac{1}{49152}\left(204800x^5+284672x^4+408960x^3+365536x^2+328204x-64023\right)\sqrt{2x^2-x+3}+\frac{284533}{131072}\sqrt{2}\log\left(4\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/49152*(204800*x^5 + 284672*x^4 + 408960*x^3 + 365536*x^2 + 328204*x - 640
23)*sqrt(2*x^2 - x + 3) + 284533/131072*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2, x)
```

---

**Giac [A]** time = 1.19608, size = 99, normalized size = 0.8

$$\frac{1}{49152}(4(8(4(16(100x+139)x+3195)x+11423)x+82051)x-64023)\sqrt{2x^2-x+3}+\frac{284533}{65536}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/49152*(4*(8*(4*(16*(100*x + 139)*x + 3195)*x + 11423)*x + 82051)*x - 6402
3)*sqrt(2*x^2 - x + 3) + 284533/65536*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - s
qrt(2*x^2 - x + 3)) + 1)
```

## 3.61 $\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)\,dx$

**Optimal**. Leaf size=82

$$\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}+\frac{73}{96}\left(2x^2-x+3\right)^{3/2}-\frac{81}{512}(1-4x)\sqrt{2x^2-x+3}-\frac{1863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

```
[Out] (-81*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/512 + (73*(3 - x + 2*x^2)^(3/2))/96 + (
5*x*(3 - x + 2*x^2)^(3/2))/8 - (1863*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqr
t[2])
```

---

**Rubi [A]** time = 0.039791, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{8}x\left(2x^2-x+3\right)^{3/2}+\frac{73}{96}\left(2x^2-x+3\right)^{3/2}-\frac{81}{512}(1-4x)\sqrt{2x^2-x+3}-\frac{1863\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1024\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-81*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/512 + (73*(3 - x + 2*x^2)^(3/2))/96 + (
5*x*(3 - x + 2*x^2)^(3/2))/8 - (1863*ArcSinh[(1 - 4*x)/Sqrt[23]])/(1024*Sqr
t[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \sqrt{3-x+2x^2}\left(2+3x+5x^2\right)dx = \frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+\frac{1}{8}\int\left(1+\frac{73x}{2}\right)\sqrt{3-x+2x^2}\,dx$$

$$=\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+\frac{81}{64}\int\sqrt{3-x+2x^2}\,dx$$

$$=-\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+\frac{1863}{\ldots}$$

$$=-\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}+\frac{\left(81\sqrt{\ldots}\right.}{\ldots}$$

$$=-\frac{81}{512}(1-4x)\sqrt{3-x+2x^2}+\frac{73}{96}\left(3-x+2x^2\right)^{3/2}+\frac{5}{8}x\left(3-x+2x^2\right)^{3/2}-\frac{1863\,\mathrm{s}\ldots}{1\ldots}$$

**Mathematica [A]** time = 0.0534298, size = 55, normalized size = 0.67

$$\frac{4\sqrt{2x^2-x+3}\left(1920x^3+1376x^2+2684x+3261\right)-5589\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{6144}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(3261 + 2684*x + 1376*x^2 + 1920*x^3) - 5589*Sqrt[2]
*ArcSinh[(1 - 4*x)/Sqrt[23]])/6144
```

---

**Maple [A]** time = 0.048, size = 64, normalized size = 0.8

$$\frac{5x}{8}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{73}{96}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{-81+324x}{512}\sqrt{2x^2-x+3}+\frac{1863\sqrt{2}}{2048}\mathrm{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/8*x*(2*x^2-x+3)^(3/2)+73/96*(2*x^2-x+3)^(3/2)+81/512*(-1+4*x)*(2*x^2-x+3)
^(1/2)+1863/2048*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

---

**Maxima [A]** time = 1.50358, size = 101, normalized size = 1.23

$$\frac{5}{8}\left(2x^2-x+3\right)^{\frac{3}{2}}x+\frac{73}{96}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{81}{128}\sqrt{2x^2-x+3}x+\frac{1863}{2048}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{81}{512}\sqrt{2x^2-x\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/8*(2*x^2 - x + 3)^(3/2)*x + 73/96*(2*x^2 - x + 3)^(3/2) + 81/128*sqrt(2*x
^2 - x + 3)*x + 1863/2048*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 81/512
*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.64869, size = 207, normalized size = 2.52

$$\frac{1}{1536}\left(1920\,x^3+1376\,x^2+2684\,x+3261\right)\sqrt{2\,x^2-x+3}+\frac{1863}{4096}\,\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/1536*(1920*x^3 + 1376*x^2 + 2684*x + 3261)*sqrt(2*x^2 - x + 3) + 1863/409
6*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)*(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2), x)
```

---

**Giac [A]** time = 1.16374, size = 85, normalized size = 1.04

$$\frac{1}{1536}\,(4\,(8\,(60\,x+43)x+671)x+3261)\sqrt{2\,x^2-x+3}-\frac{1863}{2048}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)*(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1536*(4*(8*(60*x + 43)*x + 671)*x + 3261)*sqrt(2*x^2 - x + 3) - 1863/2048
*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.62 $\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$

**Optimal**. Leaf size=174

$$\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{\frac{11}{31}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(10\sqrt{2}-13\right)}}\left(\cdots\right.}{\cdots}\right.$$

```
[Out] -(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/5 + (Sqrt[(11*(13 + 10*Sqrt[2]))/31]
*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2])
*x))/Sqrt[3 - x + 2*x^2]])/5 - (Sqrt[(11*(-13 + 10*Sqrt[2]))/31]*ArcTanh[(S
qrt[11/(62*(-13 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sqrt
[3 - x + 2*x^2]])/5
```

---

**Rubi [A]** time = 0.441296, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {989, 619, 215, 1035, 1029, 206, 204}

$$\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(\left(20+13\sqrt{2}\right)x+7\sqrt{2}+6\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{5}\sqrt{\frac{11}{31}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{62\left(10\sqrt{2}-13\right)}}\left(\cdots\right.}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/5 + (Sqrt[(11*(13 + 10*Sqrt[2]))/31]
*ArcTan[(Sqrt[11/(62*(13 + 10*Sqrt[2]))]*(6 + 7*Sqrt[2] + (20 + 13*Sqrt[2])
*x))/Sqrt[3 - x + 2*x^2]])/5 - (Sqrt[(11*(-13 + 10*Sqrt[2]))/31]*ArcTanh[(S
qrt[11/(62*(-13 + 10*Sqrt[2]))]*(6 - 7*Sqrt[2] + (20 - 13*Sqrt[2])*x))/Sqrt
[3 - x + 2*x^2]])/5
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx &= -\left(\frac{1}{5}\int \frac{-11+11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx\right)+\frac{2}{5}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx\\ &= \frac{1}{5}\sqrt{\frac{2}{23}}\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)+\frac{\int \frac{121\left(2+\sqrt{2}\right)-121\sqrt{2}x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{110\sqrt{2}}-\frac{\int \frac{121\left(2-\sqrt{2}\right)+121\sqrt{2}x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{110\sqrt{2}}\\ &= -\frac{1}{5}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)-\frac{1}{5}\left(1331\left(20-13\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-907742\left(13-10\sqrt{2}\right)-11x^2}\,dx, x, \frac{121}{}\right.\\ &= -\frac{1}{5}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)+\frac{1}{5}\sqrt{\frac{11}{31}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(13+10\sqrt{2}\right)}}\left(6+7\sqrt{2}+\left(20+13\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right)-\end{aligned}$$

**Mathematica [C]** time = 0.463022, size = 185, normalized size = 1.06

$$-\frac{1}{5}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)-\frac{1}{5}i\sqrt{\frac{11}{62}}\left(\sqrt{13+i\sqrt{31}}\tanh^{-1}\left(\frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)-\sqrt{13-i\sqrt{31}}\tanh^{-1}\left(\frac{4i\sqrt{31}}{2\sqrt{286}}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2),x]
```

```
[Out] -(Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/5 - (I/5)*Sqrt[11/62]*(Sqrt[13 + I*S
qrt[31]]*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 +
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - Sqrt[13 - I*Sqrt[31]]*ArcTanh[(63
- I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt
[3 - x + 2*x^2])])
```

---

**Maple [B]** time = 0.276, size = 2065, normalized size = 11.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-1/52855*(8*(2^(1/2)-1+x)^2/(2^(1
/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(
1/2)*(285*(-775687+549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)
*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)
+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x
)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+386*(-775687+549362*2^(1
/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(
1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1
/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)
-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1
-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2
^(1/2)+1-x))-274846*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^
(1/2))*2^(1/2)-1543366*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2
))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1
-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(
1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+1/21142*(
8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)
+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*(-775687+549362*2^(1/2))^(1/2)*2^(1/2)*(-8
866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-
23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)
*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+2
18*(-775687+549362*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692
487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2
*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^
(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^
(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+401698*arctanh(31/2*(8*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2
)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-63426*arctanh(31/2*(8*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/
2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1
-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(
1/2))^(1/2)+3/21142*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*(-775687+549362*2^(1/
```

```
2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549
362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^
(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-
1+x)/(2^(1/2)+1-x))+520*(-775687+549362*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))
^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*
(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1
+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2
^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+465124*arctanh(31/
2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1
/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arctanh(3
1/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^
(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2
^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^
(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2\,x^2 - x + 3}}{5\,x^2 + 3\,x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2), x)
```

---

**Fricas [B]** time = 4.39329, size = 7056, normalized size = 40.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/1550*6050^(1/4)*sqrt(31)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/8
9125*(460*sqrt(5)*(4*6050^(3/4)*sqrt(31)*(4702*x^7 - 19541*x^6 + 40352*x^5
- 68777*x^4 + 35480*x^3 - 19080*x^2 - sqrt(2)*(4028*x^7 - 14488*x^6 + 30919
*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 17280) - 34560*x + 2764
8) + 5*6050^(1/4)*sqrt(31)*(22836*x^7 - 355266*x^6 + 1914360*x^5 - 4475096*
x^4 + 5840640*x^3 - 4011840*x^2 - sqrt(2)*(18463*x^7 - 280047*x^6 + 1453472
*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 3068928*x + 1990656) - 398
1312*x + 3068928))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) + 253000*sqrt(
31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^
4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 -
783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1
154304*x - 456192) - 2*sqrt(10)*(sqrt(5)*(4*6050^(3/4)*sqrt(31)*(15454*x^7
- 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - sqrt(2)*(1543
8*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 + 44928*x^2 - 44928*x
) - 13824*x) + 5*6050^(1/4)*sqrt(31)*(77254*x^7 - 1000024*x^6 + 3868360*x^5
 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(69479*x^7 - 898236*x^
6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 - 4478976*x) - 24
```

```
05376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) + 550*sqrt(31)*sqrt(2)*
(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*
x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x
^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 25
*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087819
20*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517
*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*
sqrt(-(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)
*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*sqrt(2)*(2*x^2 - x + 3) + 755*x - 10
00)/x^2) + 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835
344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348
*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 +
4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 1419
1920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 247726
08*x + 18579456)) + 1/1550*6050^(1/4)*sqrt(31)*sqrt(5)*sqrt(2)*sqrt(13*sqrt
(2) + 20)*arctan(1/89125*(460*sqrt(5)*(4*6050^(3/4)*sqrt(31)*(4702*x^7 - 19
541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - sqrt(2)*(4028*x^7
 - 14488*x^6 + 30919*x^5 - 46671*x^4 + 22688*x^3 - 9144*x^2 - 27648*x + 172
80) - 34560*x + 27648) + 5*6050^(1/4)*sqrt(31)*(22836*x^7 - 355266*x^6 + 19
14360*x^5 - 4475096*x^4 + 5840640*x^3 - 4011840*x^2 - sqrt(2)*(18463*x^7 -
280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - 306892
8*x + 1990656) - 3981312*x + 3068928))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2)
+ 20) - 253000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385
256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335
*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 54
6048*x - 539136) + 1154304*x - 456192) - 2*sqrt(10)*(sqrt(5)*(4*6050^(3/4)*
sqrt(31)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824
*x^2 - sqrt(2)*(15438*x^7 - 22007*x^6 + 69837*x^5 - 21232*x^4 + 19368*x^3 +
 44928*x^2 - 44928*x) - 13824*x) + 5*6050^(1/4)*sqrt(31)*(77254*x^7 - 10000
24*x^6 + 3868360*x^5 - 5120640*x^4 + 7012800*x^3 + 2405376*x^2 - sqrt(2)*(6
9479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x
^2 - 4478976*x) - 2405376*x))*sqrt(2*x^2 - x + 3)*sqrt(13*sqrt(2) + 20) - 5
50*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt((6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(2)*(2*x^2 -
x + 3) - 755*x + 1000)/x^2) - 2875*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53
385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 -
 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 556
8*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 -
 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34
615296*x^2 - 24772608*x + 18579456)) - 1/6200*6050^(1/4)*sqrt(5)*sqrt(13*sq
rt(2) + 20)*(13*sqrt(2) - 20)*log(40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) + 245*x^2 + 220*sqrt(
2)*(2*x^2 - x + 3) - 755*x + 1000)/x^2) + 1/6200*6050^(1/4)*sqrt(5)*sqrt(13
*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-40*(6050^(1/4)*sqrt(5)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(3*x + 5) - 8*x + 2)*sqrt(13*sqrt(2) + 20) - 245*x^2 - 220*s
qrt(2)*(2*x^2 - x + 3) + 755*x - 1000)/x^2) + 1/10*sqrt(2)*log(-4*sqrt(2)*s
qrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{5x^2 + 3x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.63 $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$

**Optimal**. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\sqrt{2}+419\right.}{\sqrt{2x^2-x+3}}\right.$$

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 49
942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))]*(419 + 277*
Sqrt[2] + (973 + 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62 - (Sqrt[(-70517
+ 49942*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))]*(419
- 277*Sqrt[2] + (973 - 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62
```

---

**Rubi [A]** time = 0.393492, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {971, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{31\left(5x^2+3x+2\right)}+\frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(\left(973+696\sqrt{2}\right)x+277\sqrt{2}+419\right.}{\sqrt{2x^2-x+3}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```


```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(31*(2 + 3*x + 5*x^2)) + (Sqrt[(70517 + 49
942*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(70517 + 49942*Sqrt[2]))]*(419 + 277*
Sqrt[2] + (973 + 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62 - (Sqrt[(-70517
+ 49942*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-70517 + 49942*Sqrt[2]))]*(419
- 277*Sqrt[2] + (973 - 696*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/62
```


Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
```

```
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{-\frac{63}{2}+11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{11}{2}\left(85-63\sqrt{2}\right)-\frac{11}{2}\left(41-22\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{682\sqrt{2}} + \frac{\int \frac{\frac{11}{2}\left(85+63\sqrt{2}\right)-\frac{11}{2}\left(41+22\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{682\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} - \frac{1}{248}\left(11\left(99884-70517\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{3751}{4}\left(70517-49942\sqrt{2}\right)-$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{31\left(2+3x+5x^2\right)} + \frac{1}{62}\sqrt{\frac{1}{682}\left(70517+49942\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(70517+49942\sqrt{2}\right)}}\left(419+277\right.}{\sqrt{3-x+2}$$

**Mathematica [C]** time = 1.08304, size = 214, normalized size = 1.14

$$\frac{\frac{27280\sqrt{2x^2-x+3}(10x+3)}{5x^2+3x+2} + i\sqrt{286-22i\sqrt{31}}\left(973\sqrt{31}+1271i\right)\tanh^{-1}\left(\frac{4i\sqrt{31}x-22x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - i\sqrt{286+22i\sqrt{31}}\left(973\sqrt{31}-1\right.}{845680}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] ((27280*(3 + 10*x)*Sqrt[3 - x + 2*x^2])/(2 + 3*x + 5*x^2) + I*Sqrt[286 - (2
2*I)*Sqrt[31]]*(1271*I + 973*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4
*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - I*Sq
rt[286 + (22*I)*Sqrt[31]]*(-1271*I + 973*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31]
 + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
```

```
x^2])])/845680
```

---

**Maple [B]** time = 0.292, size = 16357, normalized size = 87.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^2, x)
```

---

**Fricas [B]** time = 5.82357, size = 8142, normalized size = 43.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] -1/186703822445536*(88412*4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(2)*(5
*x^2 + 3*x + 2)*sqrt(70517*sqrt(2) + 99884)*arctan(1/10668926457462302923*(
3096404*sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(537184*x^7 - 2047820*x^
6 + 4310846*x^5 - 6853210*x^4 + 3421536*x^3 - 1589328*x^2 - sqrt(2)*(370014
*x^7 - 1438653*x^6 + 3014868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2
- 2633472*x + 1893888) - 3787776*x + 2633472) + 774101*4988406728^(1/4)*sqr
t(341)*(40625*x^7 - 622509*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 -
5992992*x^2 - sqrt(2)*(28204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 +
 6800832*x^3 - 4341024*x^2 - 4810752*x + 3442176) - 6884352*x + 4810752))*s
qrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) + 30285984782473634104*sqrt(
31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^
4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 -
783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1
154304*x - 456192) - 2*sqrt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(3
41)*(84604*x^7 - 121310*x^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624
*x^2 - sqrt(2)*(57082*x^7 - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^
3 + 110592*x^2 - 110592*x) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*
(6379*x^7 - 82508*x^6 + 318020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 -
 sqrt(2)*(4365*x^7 - 56468*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207
360*x^2 - 207360*x) - 331776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 9
```

```
9884) + 425261673562*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^
6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^
8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209
600*x^2 - 1036800*x) + 3276288*x) + 19330076071*sqrt(31)*(254591*x^8 - 4815
126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689
56928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 -
 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(4988406728^(1/4)*sqrt
(24971)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x -
 7)*sqrt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^
2 - x + 3) + 3623566781*x - 4799426200)/x^2) + 344158917982654933*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 88412
*4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(7051
7*sqrt(2) + 99884)*arctan(1/10668926457462302923*(3096404*sqrt(24971)*(11*4
988406728^(3/4)*sqrt(341)*(537184*x^7 - 2047820*x^6 + 4310846*x^5 - 6853210
*x^4 + 3421536*x^3 - 1589328*x^2 - sqrt(2)*(370014*x^7 - 1438653*x^6 + 3014
868*x^5 - 4873381*x^4 + 2452952*x^3 - 1184616*x^2 - 2633472*x + 1893888) -
3787776*x + 2633472) + 774101*4988406728^(1/4)*sqrt(341)*(40625*x^7 - 62250
9*x^6 + 3280912*x^5 - 7459052*x^4 + 9621216*x^3 - 5992992*x^2 - sqrt(2)*(28
204*x^7 - 433677*x^6 + 2297444*x^5 - 5257628*x^4 + 6800832*x^3 - 4341024*x^
2 - 4810752*x + 3442176) - 6884352*x + 4810752))*sqrt(2*x^2 - x + 3)*sqrt(7
0517*sqrt(2) + 99884) - 30285984782473634104*sqrt(31)*sqrt(2)*(28180*x^8 -
254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^
2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4
- 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sq
rt(49942)*(sqrt(24971)*(11*4988406728^(3/4)*sqrt(341)*(84604*x^7 - 121310*x
^6 + 389610*x^5 - 147168*x^4 + 168912*x^3 + 186624*x^2 - sqrt(2)*(57082*x^7
 - 82029*x^6 + 264639*x^5 - 107216*x^4 + 130104*x^3 + 110592*x^2 - 110592*x
) - 186624*x) + 774101*4988406728^(1/4)*sqrt(341)*(6379*x^7 - 82508*x^6 + 3
18020*x^5 - 410688*x^4 + 523872*x^3 + 331776*x^2 - sqrt(2)*(4365*x^7 - 5646
8*x^6 + 217820*x^5 - 282816*x^4 + 366624*x^3 + 207360*x^2 - 207360*x) - 331
776*x))*sqrt(2*x^2 - x + 3)*sqrt(70517*sqrt(2) + 99884) - 425261673562*sqrt
(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*
x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x
^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32
76288*x) - 19330076071*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 -
90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)
*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 19
44*x) + 144820224*x))*sqrt((4988406728^(1/4)*sqrt(24971)*sqrt(341)*sqrt(31)
*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sqrt(70517*sqrt(2) + 9
9884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x + 3) - 3623566781*x
+ 4799426200)/x^2) - 344158917982654933*sqrt(31)*(2828123*x^8 - 9696916*x^7
 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*
x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4
- 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*
x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3
 - 34615296*x^2 - 24772608*x + 18579456)) - 4988406728^(1/4)*sqrt(24971)*sq
rt(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x +
 199768)*sqrt(70517*sqrt(2) + 99884)*log(199768*(4988406728^(1/4)*sqrt(2497
1)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*s
qrt(70517*sqrt(2) + 99884) + 1175859419*x^2 + 1055873764*sqrt(2)*(2*x^2 - x
 + 3) - 3623566781*x + 4799426200)/x^2) + 4988406728^(1/4)*sqrt(24971)*sqrt
(341)*sqrt(31)*(499420*x^2 - 70517*sqrt(2)*(5*x^2 + 3*x + 2) + 299652*x + 1
99768)*sqrt(70517*sqrt(2) + 99884)*log(-199768*(4988406728^(1/4)*sqrt(24971
)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(10*x + 3) - 13*x - 7)*sq
rt(70517*sqrt(2) + 99884) - 1175859419*x^2 - 1055873764*sqrt(2)*(2*x^2 - x
```

```
+ 3) + 3623566781*x - 4799426200)/x^2) - 6022703949856*sqrt(2*x^2 - x + 3)*
(10*x + 3))/(5*x^2 + 3*x + 2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{(5x^2 + 3x + 2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**2, x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.64 $\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx$

**Optimal**. Leaf size=223

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}+\frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{}{31(11228586946}}\right.}{169136}$$

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + ((3464 + 13665*
x)*Sqrt[3 - x + 2*x^2])/(84568*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 + 7
9399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*Sq
rt[2]))]*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/169136 - (Sqrt[(-112285869463 + 79399380740*Sqrt[2])/682]*Arc
Tanh[(Sqrt[11/(31*(-112285869463 + 79399380740*Sqrt[2]))]*(509587 - 362788*
Sqrt[2] + (1235163 - 872375*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/169136
```

---

**Rubi [A]** time = 0.45864, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {971, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(10x+3)}{62\left(5x^2+3x+2\right)^2}+\frac{(13665x+3464)\sqrt{2x^2-x+3}}{84568\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(112285869463+79399380740\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{}{31(11228586946}}\right.}{169136}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((3 + 10*x)*Sqrt[3 - x + 2*x^2])/(62*(2 + 3*x + 5*x^2)^2) + ((3464 + 13665*
x)*Sqrt[3 - x + 2*x^2])/(84568*(2 + 3*x + 5*x^2)) + (Sqrt[(112285869463 + 7
9399380740*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(112285869463 + 79399380740*Sq
rt[2]))]*(509587 + 362788*Sqrt[2] + (1235163 + 872375*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/169136 - (Sqrt[(-112285869463 + 79399380740*Sqrt[2])/682]*Arc
Tanh[(Sqrt[11/(31*(-112285869463 + 79399380740*Sqrt[2]))]*(509587 - 362788*
Sqrt[2] + (1235163 - 872375*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/169136
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
```

```
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{-\frac{183}{2}+31x-40x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\int \frac{-213004+\frac{358655x}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}dx}{465124}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{121}{4}\left(110061-77456\sqrt{2}\right)-\frac{121}{4}\left(44851-32605\sqrt{\right.}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}}{10232728\sqrt{2}}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} - \frac{\left(11\left(158798761480-112285869463\right.\right.}{}$$

$$= \frac{(3+10x)\sqrt{3-x+2x^2}}{62\left(2+3x+5x^2\right)^2} + \frac{(3464+13665x)\sqrt{3-x+2x^2}}{84568\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{682}\left(112285869463+793993807\right.}}{}$$

**Mathematica [C]** time = 2.06866, size = 299, normalized size = 1.34

$$\frac{5\left(\frac{i\sqrt{286+22i\sqrt{31}}\left(258253\sqrt{31}+1004586i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\left(\sqrt{31}-13i\right)^2} + \frac{2000\left(1364\left(\sqrt{31}+13i\right)\sqrt{2x^2-x+3}\left(68325x^3+58315x^2+51362x+11020\right)-5\sqrt{286-\right.}{\left(\sqrt{31}-13i\right)\left(\sqrt{31}+13i\right)^2\left(-10ix\right.}\right.}{14418844}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[3 - x + 2*x^2]/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (5*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1004586*I + 258253*Sqrt[31])*ArcTanh[(6
3 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*S
qrt[3 - x + 2*x^2])])/(-13*I + Sqrt[31])^2 + (2000*(1364*(13*I + Sqrt[31])*
Sqrt[3 - x + 2*x^2]*(11020 + 51362*x + 58315*x^2 + 68325*x^3) - 5*Sqrt[286
- (22*I)*Sqrt[31]]*(-202151*I + 174475*Sqrt[31])*(2 + 3*x + 5*x^2)^2*ArcTan
h[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31
]]*Sqrt[3 - x + 2*x^2])]))/((-13*I + Sqrt[31])*(13*I + Sqrt[31])^2*(-3*I +
Sqrt[31] - (10*I)*x)^2*(3*I + Sqrt[31] + (10*I)*x)^2)))/14418844
```

---

**Maple [B]** time = 0.428, size = 44343, normalized size = 198.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{\left(5x^2 + 3x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^2 - x + 3)/(5*x^2 + 3*x + 2)^3, x)
```

---

**Fricas [B]** time = 6.15615, size = 10298, normalized size = 46.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/6505215189695292642599671424 0*(14205421276*788032707736935368450^(1/4)*sq
rt(39699690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqr
t(112285869463*sqrt(2) + 158798761480)*arctan(1/8610476622139712875910576 59
551879544939625*(2461380802940*sqrt(39699690370)*(22*788032707736935368450^
(3/4)*sqrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 86714302
12*x^4 + 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195
871*x^6 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2
 - 3352976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*7
88032707736935368450^(1/4)*sqrt(341)*(50730703*x^7 - 778833417*x^6 + 411636
7112*x^5 - 9392273180*x^4 + 12133646496*x^3 - 7660912032*x^2 - sqrt(2)*(359
38543*x^7 - 551546778*x^6 + 2913578540*x^5 - 6643469608*x^4 + 8580088800*x^
3 - 5403919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216
))*sqrt(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) + 24442643
314461120421939701303408193533770 00*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^
7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(
2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*
x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(396996
90370/160673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*
(104024992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*
x^3 + 214417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5
 - 133050960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152
*x) + 615345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102
233612*x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2
- 4*sqrt(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116
274240*x^3 + 70118784*x^2 - 70118784*x) - 391744512*x))*sqrt(2*x^2 - x + 3)
*sqrt(112285869463*sqrt(2) + 158798761480) + 431759125243238662111 43695850*
sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396
480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2440
47*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
+ 3276288*x) + 19625414783783575550519 86175*sqrt(31)*(254591*x^8 - 4815126*
x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 16895692
8*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 361
8*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(788032707736935368450^(1/
4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(12053
*x + 5138) - 17191*x - 6915)*sqrt(112285869463*sqrt(2) + 158798761480) - 15
018255698585818094 5*x^2 - 13485780627301550942 0*sqrt(2)*(2*x^2 - x + 3) + 4
62807471527848680055*x - 612990028513706861000)/x^2) + 2777573103916036411 5
```

```
8405696629638562 88375*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 -
142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)
*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*
x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7
+ 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 -
24772608*x + 18579456)) + 14205421276*788032707736935368450^(1/4)*sqrt(3969
9690370)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(11228
5869463*sqrt(2) + 158798761480)*arctan(1/861047662213971287591057659551879 5
44939625*(2461380802940*sqrt(39699690370)*(22*788032707736935368450^(3/4)*s
qrt(341)*(667937076*x^7 - 2573871186*x^6 + 5404850058*x^5 - 8671430212*x^4
+ 4348809776*x^3 - 2064441888*x^2 - sqrt(2)*(473555282*x^7 - 1821195871*x^6
 + 3826055542*x^5 - 6128133137*x^4 + 3070797960*x^3 - 1452037320*x^2 - 3352
976640*x + 2366869248) - 4733738496*x + 3352976640) + 615345200735*78803270
7736935368450^(1/4)*sqrt(341)*(50730703*x^7 - 778833417*x^6 + 4116367112*x^
5 - 9392273180*x^4 + 12133646496*x^3 - 7660912032*x^2 - sqrt(2)*(35938543*x
^7 - 551546778*x^6 + 2913578540*x^5 - 6643469608*x^4 + 8580088800*x^3 - 540
3919680*x^2 - 6107913216*x + 4313793024) - 8627586048*x + 6107913216))*sqrt
(2*x^2 - x + 3)*sqrt(112285869463*sqrt(2) + 158798761480) - 244426433144611
20421939701303408193533770 00*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3
96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(39699690370/1
60673)*(sqrt(39699690370)*(22*788032707736935368450^(3/4)*sqrt(341)*(104024
992*x^7 - 149335248*x^6 + 480784368*x^5 - 188730368*x^4 + 223535232*x^3 + 2
14417152*x^2 - sqrt(2)*(73906058*x^7 - 106073653*x^6 + 341348823*x^5 - 1330
50960*x^4 + 156704760*x^3 + 154338048*x^2 - 154338048*x) - 214417152*x) + 6
15345200735*788032707736935368450^(1/4)*sqrt(341)*(7903323*x^7 - 102233612*
x^6 + 394216580*x^5 - 510585408*x^4 + 657060192*x^3 + 391744512*x^2 - 4*sqr
t(2)*(1401761*x^7 - 18132196*x^6 + 69912940*x^5 - 90501120*x^4 + 116274240*
x^3 + 70118784*x^2 - 70118784*x) - 391744512*x))*sqrt(2*x^2 - x + 3)*sqrt(1
12285869463*sqrt(2) + 158798761480) - 431759125243238662111436958 50*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762
88*x) - 196254147837835755505198617 5*sqrt(31)*(254591*x^8 - 4815126*x^7 + 3
2303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 -
 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 +
 2268*x^2 - 1944*x) + 144820224*x))*sqrt((788032707736935368450^(1/4)*sqrt(
39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(12053*x + 513
8) - 17191*x - 6915)*sqrt(112285869463*sqrt(2) + 158798761480) + 1501825569
85858180945*x^2 + 134857806273015509420*sqrt(2)*(2*x^2 - x + 3) - 462807471
527848680055*x + 612990028513706861000)/x^2) - 277757310391603641158405696 6
2963856288375*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*s
qrt(31)*(3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285
869463*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63
5195045920)*sqrt(112285869463*sqrt(2) + 158798761480)*log(635195045920/1606
73*(788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2
*x^2 - x + 3)*(sqrt(2)*(12053*x + 5138) - 17191*x - 6915)*sqrt(112285869463
*sqrt(2) + 158798761480) + 150182556985858180945*x^2 + 1348578062730155094 2
0*sqrt(2)*(2*x^2 - x + 3) - 462807471527848680055*x + 612990028513706861000
)/x^2) - 788032707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*(
3969969037000*x^4 + 4763962844400*x^3 + 4605164082920*x^2 - 112285869463*sq
rt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 1905585137760*x + 63519504592
0)*sqrt(112285869463*sqrt(2) + 158798761480)*log(-635195045920/160673*(7880
```

```
32707736935368450^(1/4)*sqrt(39699690370)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(12053*x + 5138) - 17191*x - 6915)*sqrt(112285869463*sqrt(2)
 + 158798761480) - 150182556985858180945*x^2 - 134857806273015509420*sqrt(2
)*(2*x^2 - x + 3) + 462807471527848680055*x - 612990028513706861000)/x^2) +
 769228926981280465731680*(68325*x^3 + 58315*x^2 + 51362*x + 11020)*sqrt(2*
x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{2x^2 - x + 3}}{\left(5x^2 + 3x + 2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(1/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(sqrt(2*x**2 - x + 3)/(5*x**2 + 3*x + 2)**3, x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(1/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.65 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right)^4 \, dx$

**Optimal**. Leaf size=231

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7+\frac{7625}{96}\left(2x^2-x+3\right)^{5/2}x^6+\frac{95165}{768}\left(2x^2-x+3\right)^{5/2}x^5+\frac{941905\left(2x^2-x+3\right)^{5/2}x^4}{9216}+\frac{10444117}{}$$

```
[Out] (-26366414481*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2147483648 - (382121949*(1 - 4
*x)*(3 - x + 2*x^2)^(3/2))/134217728 + (2124689283*(3 - x + 2*x^2)^(5/2))/1
46800640 + (48669967*x*(3 - x + 2*x^2)^(5/2))/22020096 - (56422489*x^2*(3 -
 x + 2*x^2)^(5/2))/8257536 + (10444117*x^3*(3 - x + 2*x^2)^(5/2))/294912 +
(941905*x^4*(3 - x + 2*x^2)^(5/2))/9216 + (95165*x^5*(3 - x + 2*x^2)^(5/2))
/768 + (7625*x^6*(3 - x + 2*x^2)^(5/2))/96 + (625*x^7*(3 - x + 2*x^2)^(5/2)
)/24 - (606427533063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4294967296*Sqrt[2])
```

---

**Rubi [A]** time = 0.342448, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{24}\left(2x^2-x+3\right)^{5/2}x^7+\frac{7625}{96}\left(2x^2-x+3\right)^{5/2}x^6+\frac{95165}{768}\left(2x^2-x+3\right)^{5/2}x^5+\frac{941905\left(2x^2-x+3\right)^{5/2}x^4}{9216}+\frac{10444117}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```


```
[Out] (-26366414481*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/2147483648 - (382121949*(1 - 4
*x)*(3 - x + 2*x^2)^(3/2))/134217728 + (2124689283*(3 - x + 2*x^2)^(5/2))/1
46800640 + (48669967*x*(3 - x + 2*x^2)^(5/2))/22020096 - (56422489*x^2*(3 -
 x + 2*x^2)^(5/2))/8257536 + (10444117*x^3*(3 - x + 2*x^2)^(5/2))/294912 +
(941905*x^4*(3 - x + 2*x^2)^(5/2))/9216 + (95165*x^5*(3 - x + 2*x^2)^(5/2))
/768 + (7625*x^6*(3 - x + 2*x^2)^(5/2))/96 + (625*x^7*(3 - x + 2*x^2)^(5/2)
)/24 - (606427533063*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4294967296*Sqrt[2])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^4\,dx = \frac{625}{24}x^7\left(3-x+2x^2\right)^{5/2}+\frac{1}{24}\int\left(3-x+2x^2\right)^{3/2}\left(384+2304x+9024x^2+2\right.$$

$$=\frac{7625}{96}x^6\left(3-x+2x^2\right)^{5/2}+\frac{625}{24}x^7\left(3-x+2x^2\right)^{5/2}+\frac{1}{528}\int\left(3-x+2x^2\right)^{3/2}$$

$$=\frac{95165}{768}x^5\left(3-x+2x^2\right)^{5/2}+\frac{7625}{96}x^6\left(3-x+2x^2\right)^{5/2}+\frac{625}{24}x^7\left(3-x+2x^2\right)^{5/}$$

$$=\frac{941905x^4\left(3-x+2x^2\right)^{5/2}}{9216}+\frac{95165}{768}x^5\left(3-x+2x^2\right)^{5/2}+\frac{7625}{96}x^6\left(3-x+2x\right.$$

$$=\frac{10444117x^3\left(3-x+2x^2\right)^{5/2}}{294912}+\frac{941905x^4\left(3-x+2x^2\right)^{5/2}}{9216}+\frac{95165}{768}x^5\left(3-x\right.$$

$$=-\frac{56422489x^2\left(3-x+2x^2\right)^{5/2}}{8257536}+\frac{10444117x^3\left(3-x+2x^2\right)^{5/2}}{294912}+\frac{941905x^4\left(\right.}{9}$$

$$=\frac{48669967x\left(3-x+2x^2\right)^{5/2}}{22020096}-\frac{56422489x^2\left(3-x+2x^2\right)^{5/2}}{8257536}+\frac{10444117x^3\left(\right.}{294}$$

$$=\frac{2124689283\left(3-x+2x^2\right)^{5/2}}{146800640}+\frac{48669967x\left(3-x+2x^2\right)^{5/2}}{22020096}-\frac{56422489x^2\left(\right.}{825}$$

$$=-\frac{382121949(1-4x)\left(3-x+2x^2\right)^{3/2}}{134217728}+\frac{2124689283\left(3-x+2x^2\right)^{5/2}}{146800640}+\frac{4866}{}$$

$$=-\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+2x^2\right)^{3/2}}{134217728}+$$

$$=-\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+2x^2\right)^{3/2}}{134217728}+$$

$$=-\frac{26366414481(1-4x)\sqrt{3-x+2x^2}}{2147483648}-\frac{382121949(1-4x)\left(3-x+2x^2\right)^{3/2}}{134217728}+$$

**Mathematica [A]** time = 0.366561, size = 95, normalized size = 0.41

$$4\sqrt{2x^2-x+3}\left(70464307200000x^{11}+144451829760000x^{10}+349379651174400x^9+534038708224000x^8+745133\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(74032009514181 + 12971175524316*x + 65151998063712*
x^2 + 239021184223104*x^3 + 451581382260736*x^4 + 675479464714240*x^5 + 765
087080448000*x^6 + 745133229998080*x^7 + 534038708224000*x^8 + 349379651174
400*x^9 + 144451829760000*x^10 + 70464307200000*x^11) - 191024672914845*Sqr
t[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/2705829396480
```

---

**Maple [A]** time = 0.072, size = 185, normalized size = 0.8

$$\frac{7625\,x^6}{96}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{95165\,x^5}{768}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{941905\,x^4}{9216}\left(2\,x^2-x+3\right)^{\frac{5}{2}}+\frac{10444117\,x^3}{294912}\left(2\,x^2-x+3\right)^{\frac{5}{2}}-\frac{56422\ldots}{8257\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 7625/96*x^6*(2*x^2-x+3)^(5/2)+95165/768*x^5*(2*x^2-x+3)^(5/2)+941905/9216*x
^4*(2*x^2-x+3)^(5/2)+10444117/294912*x^3*(2*x^2-x+3)^(5/2)-56422489/8257536
*x^2*(2*x^2-x+3)^(5/2)+48669967/22020096*x*(2*x^2-x+3)^(5/2)+26366414481/21
47483648*(-1+4*x)*(2*x^2-x+3)^(1/2)+606427533063/8589934592*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+382121949/134217728*(-1+4*x)*(2*x^2-x+3)^(3/2)+2124
689283/146800640*(2*x^2-x+3)^(5/2)+625/24*x^7*(2*x^2-x+3)^(5/2)
```

---

**Maxima [A]** time = 1.54572, size = 278, normalized size = 1.2

$$\frac{625}{24}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^7+\frac{7625}{96}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^6+\frac{95165}{768}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^5+\frac{941905}{9216}\left(2\,x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{10444117}{294912}\left(2\,x\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/24*(2*x^2 - x + 3)^(5/2)*x^7 + 7625/96*(2*x^2 - x + 3)^(5/2)*x^6 + 9516
5/768*(2*x^2 - x + 3)^(5/2)*x^5 + 941905/9216*(2*x^2 - x + 3)^(5/2)*x^4 + 1
0444117/294912*(2*x^2 - x + 3)^(5/2)*x^3 - 56422489/8257536*(2*x^2 - x + 3)
^(5/2)*x^2 + 48669967/22020096*(2*x^2 - x + 3)^(5/2)*x + 2124689283/1468006
40*(2*x^2 - x + 3)^(5/2) + 382121949/33554432*(2*x^2 - x + 3)^(3/2)*x - 382
121949/134217728*(2*x^2 - x + 3)^(3/2) + 26366414481/536870912*sqrt(2*x^2 -
 x + 3)*x + 606427533063/8589934592*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)
) - 26366414481/2147483648*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.58455, size = 532, normalized size = 2.3

$$\frac{1}{676457349120}\left(70464307200000\,x^{11}+144451829760000\,x^{10}+349379651174400\,x^9+534038708224000\,x^8+745133\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/676457349120*(70464307200000*x^11 + 144451829760000*x^10 + 34937965117440
0*x^9 + 534038708224000*x^8 + 745133229998080*x^7 + 765087080448000*x^6 + 6
75479464714240*x^5 + 451581382260736*x^4 + 239021184223104*x^3 + 6515199806
3712*x^2 + 12971175524316*x + 74032009514181)*sqrt(2*x^2 - x + 3) + 6064275
33063/17179869184*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32
*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(2x^2 - x + 3\right)^{\frac{3}{2}} \left(5x^2 + 3x + 2\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**4, x)
```

---

**Giac [A]** time = 1.17427, size = 139, normalized size = 0.6

$$\frac{1}{676457349120} \left(4 \left(8 \left(4 \left(16 \left(20 \left(8 \left(28 \left(160 \left(12 \left(200 \left(20 x + 41\right) x + 19833\right) x + 363785\right) x + 81213077\right) x + 2334860475\right) x\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/676457349120*(4*(8*(4*(16*(20*(8*(28*(160*(12*(200*(20*x + 41)*x + 19833)
*x + 363785)*x + 81213077)*x + 2334860475)*x + 16491197869)*x + 22049872180
7)*x + 1867353001743)*x + 2035999939491)*x + 3242793881079)*x + 74032009514
181)*sqrt(2*x^2 - x + 3) - 606427533063/8589934592*sqrt(2)*log(-2*sqrt(2)*(
sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.66 $\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx$

**Optimal**. Leaf size=189

$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5+\frac{725}{48}\left(2x^2-x+3\right)^{5/2}x^4+\frac{27785\left(2x^2-x+3\right)^{5/2}x^3}{1536}+\frac{384739\left(2x^2-x+3\right)^{5/2}x^2}{43008}-\frac{81685\left(2x^2\right.}{114}$$

```
[Out] (-46077855*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/33554432 - (667795*(1 - 4*x)*(3 -
 x + 2*x^2)^(3/2))/2097152 - (4625907*(3 - x + 2*x^2)^(5/2))/2293760 - (816
85*x*(3 - x + 2*x^2)^(5/2))/114688 + (384739*x^2*(3 - x + 2*x^2)^(5/2))/430
08 + (27785*x^3*(3 - x + 2*x^2)^(5/2))/1536 + (725*x^4*(3 - x + 2*x^2)^(5/2
))/48 + (25*x^5*(3 - x + 2*x^2)^(5/2))/4 - (1059790665*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(67108864*Sqrt[2])
```

---

**Rubi [A]** time = 0.18971, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{4}\left(2x^2-x+3\right)^{5/2}x^5+\frac{725}{48}\left(2x^2-x+3\right)^{5/2}x^4+\frac{27785\left(2x^2-x+3\right)^{5/2}x^3}{1536}+\frac{384739\left(2x^2-x+3\right)^{5/2}x^2}{43008}-\frac{81685\left(2x^2\right.}{114}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (-46077855*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/33554432 - (667795*(1 - 4*x)*(3 -
 x + 2*x^2)^(3/2))/2097152 - (4625907*(3 - x + 2*x^2)^(5/2))/2293760 - (816
85*x*(3 - x + 2*x^2)^(5/2))/114688 + (384739*x^2*(3 - x + 2*x^2)^(5/2))/430
08 + (27785*x^3*(3 - x + 2*x^2)^(5/2))/1536 + (725*x^4*(3 - x + 2*x^2)^(5/2
))/48 + (25*x^5*(3 - x + 2*x^2)^(5/2))/4 - (1059790665*ArcSinh[(1 - 4*x)/Sq
rt[23]])/(67108864*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3\,dx = \frac{25}{4}x^5\left(3-x+2x^2\right)^{5/2}+\frac{1}{20}\int \left(3-x+2x^2\right)^{3/2}\left(160+720x+2280x^2+414\right.$$

$$=\frac{725}{48}x^4\left(3-x+2x^2\right)^{5/2}+\frac{25}{4}x^5\left(3-x+2x^2\right)^{5/2}+\frac{1}{360}\int \left(3-x+2x^2\right)^{3/2}\left(2\right.$$

$$=\frac{27785x^3\left(3-x+2x^2\right)^{5/2}}{1536}+\frac{725}{48}x^4\left(3-x+2x^2\right)^{5/2}+\frac{25}{4}x^5\left(3-x+2x^2\right)^{5/2}$$

$$=\frac{384739x^2\left(3-x+2x^2\right)^{5/2}}{43008}+\frac{27785x^3\left(3-x+2x^2\right)^{5/2}}{1536}+\frac{725}{48}x^4\left(3-x+2x^2\right.$$

$$=-\frac{81685x\left(3-x+2x^2\right)^{5/2}}{114688}+\frac{384739x^2\left(3-x+2x^2\right)^{5/2}}{43008}+\frac{27785x^3\left(3-x+\right.}{1536}$$

$$=-\frac{4625907\left(3-x+2x^2\right)^{5/2}}{2293760}-\frac{81685x\left(3-x+2x^2\right)^{5/2}}{114688}+\frac{384739x^2\left(3-x+\right.}{43008}$$

$$=-\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}-\frac{4625907\left(3-x+2x^2\right)^{5/2}}{2293760}-\frac{81685x\left(3-\right.}{114}$$

$$=-\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}-\frac{4625}{}$$

$$=-\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}-\frac{4625}{}$$

$$=-\frac{46077855(1-4x)\sqrt{3-x+2x^2}}{33554432}-\frac{667795(1-4x)\left(3-x+2x^2\right)^{3/2}}{2097152}-\frac{4625}{}$$

**Mathematica [A]** time = 0.2339, size = 85, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+487\right.}{1409}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-72152399943 + 53985432012*x + 199615064544*x^2 + 3
89257196928*x^3 + 487891884032*x^4 + 571298324480*x^5 + 430820229120*x^6 +
328328806400*x^7 + 124780544000*x^8 + 88080384000*x^9) - 111278019825*Sqrt[
2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/14092861440
```

**Maple [A]** time = 0.059, size = 151, normalized size = 0.8

$$\frac{25x^5}{4}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{725x^4}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{27785x^3}{1536}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{384739x^2}{43008}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{81685x}{114688}\left(2x^2-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x)
```

```
[Out] 25/4*x^5*(2*x^2-x+3)^(5/2)+725/48*x^4*(2*x^2-x+3)^(5/2)+27785/1536*x^3*(2*x
^2-x+3)^(5/2)+384739/43008*x^2*(2*x^2-x+3)^(5/2)-81685/114688*x*(2*x^2-x+3)
^(5/2)+46077855/33554432*(-1+4*x)*(2*x^2-x+3)^(1/2)+1059790665/134217728*2^
(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+667795/2097152*(-1+4*x)*(2*x^2-x+3)^(3
/2)-4625907/2293760*(2*x^2-x+3)^(5/2)
```

**Maxima [A]** time = 1.50344, size = 232, normalized size = 1.23

$$\frac{25}{4}\left(2x^2-x+3\right)^{\frac{5}{2}}x^5+\frac{725}{48}\left(2x^2-x+3\right)^{\frac{5}{2}}x^4+\frac{27785}{1536}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{384739}{43008}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2-\frac{81685}{114688}\left(2x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 25/4*(2*x^2 - x + 3)^(5/2)*x^5 + 725/48*(2*x^2 - x + 3)^(5/2)*x^4 + 27785/1
536*(2*x^2 - x + 3)^(5/2)*x^3 + 384739/43008*(2*x^2 - x + 3)^(5/2)*x^2 - 81
685/114688*(2*x^2 - x + 3)^(5/2)*x - 4625907/2293760*(2*x^2 - x + 3)^(5/2)
+ 667795/524288*(2*x^2 - x + 3)^(3/2)*x - 667795/2097152*(2*x^2 - x + 3)^(3
/2) + 46077855/8388608*sqrt(2*x^2 - x + 3)*x + 1059790665/134217728*sqrt(2)
*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 46077855/33554432*sqrt(2*x^2 - x + 3)
```

**Fricas [A]** time = 1.59326, size = 423, normalized size = 2.24

$$\frac{1}{3523215360}\left(88080384000x^9+124780544000x^8+328328806400x^7+430820229120x^6+571298324480x^5+48789\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/3523215360*(88080384000*x^9 + 124780544000*x^8 + 328328806400*x^7 + 43082
0229120*x^6 + 571298324480*x^5 + 487891884032*x^4 + 389257196928*x^3 + 1996
15064544*x^2 + 53985432012*x - 72152399943)*sqrt(2*x^2 - x + 3) + 105979066
5/268435456*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 +
 16*x - 25)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3, x)
```

---

**Giac [A]** time = 1.16042, size = 126, normalized size = 0.67

$$\frac{1}{3523215360}(4(8(4(16(20(8(140(160(12x+17)x+7157)x+1314759)x+13947713)x+238228459)x+30410718$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/3523215360*(4*(8*(4*(16*(20*(8*(140*(160*(12*x + 17)*x + 7157)*x + 131475
9)*x + 13947713)*x + 238228459)*x + 3041071851)*x + 6237970767)*x + 1349635
8003)*x - 72152399943)*sqrt(2*x^2 - x + 3) - 1059790665/134217728*sqrt(2)*l
og(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.67 $\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2\,dx$

**Optimal**. Leaf size=147

$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{1235}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{24499\left(2x^2-x+3\right)^{5/2}x}{10752}+\frac{73861\left(2x^2-x+3\right)^{5/2}}{215040}+\frac{24293(1-4x)\left(\right.}{1966}$$

```
[Out] (558739*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 + (24293*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/196608 + (73861*(3 - x + 2*x^2)^(5/2))/215040 + (24499*x*(3 -
 x + 2*x^2)^(5/2))/10752 + (1235*x^2*(3 - x + 2*x^2)^(5/2))/448 + (25*x^3*(
3 - x + 2*x^2)^(5/2))/16 + (12850997*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*
Sqrt[2])
```

---

**Rubi [A]** time = 0.122099, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{25}{16}\left(2x^2-x+3\right)^{5/2}x^3+\frac{1235}{448}\left(2x^2-x+3\right)^{5/2}x^2+\frac{24499\left(2x^2-x+3\right)^{5/2}x}{10752}+\frac{73861\left(2x^2-x+3\right)^{5/2}}{215040}+\frac{24293(1-4x)\left(\right.}{1966}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```


```
[Out] (558739*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1048576 + (24293*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/196608 + (73861*(3 - x + 2*x^2)^(5/2))/215040 + (24499*x*(3 -
 x + 2*x^2)^(5/2))/10752 + (1235*x^2*(3 - x + 2*x^2)^(5/2))/448 + (25*x^3*(
3 - x + 2*x^2)^(5/2))/16 + (12850997*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2097152*
Sqrt[2])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2\,dx = \frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{16}\int \left(3-x+2x^2\right)^{3/2}\left(64+192x+239x^2+\frac{1235x^3}{2}\right.$$

$$= \frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2}+\frac{1}{224}\int \left(3-x+2x^2\right)^{3/2}\Big($$

$$= \frac{24499x\left(3-x+2x^2\right)^{5/2}}{10752}+\frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2}+\frac{25}{16}x^3\left(3-x+2x^2\right)^{5/2}$$

$$= \frac{73861\left(3-x+2x^2\right)^{5/2}}{215040}+\frac{24499x\left(3-x+2x^2\right)^{5/2}}{10752}+\frac{1235}{448}x^2\left(3-x+2x^2\right)^{5/2}$$

$$= \frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}+\frac{73861\left(3-x+2x^2\right)^{5/2}}{215040}+\frac{24499x\left(3-x+\right.}{10752}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576}+\frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}+\frac{73861\left(3-\right.}{215}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576}+\frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}+\frac{73861\left(3-\right.}{215}$$

$$= \frac{558739(1-4x)\sqrt{3-x+2x^2}}{1048576}+\frac{24293(1-4x)\left(3-x+2x^2\right)^{3/2}}{196608}+\frac{73861\left(3-\right.}{215}$$

**Mathematica [A]** time = 0.14373, size = 75, normalized size = 0.51

$$\frac{4\sqrt{2x^2-x+3}\left(688128000x^7+525926400x^6+2025840640x^5+2061273088x^4+2728413312x^3+1799647136x^2+\right.}{440401920}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(439831323 + 1619403428*x + 1799647136*x^2 + 2728413
312*x^3 + 2061273088*x^4 + 2025840640*x^5 + 525926400*x^6 + 688128000*x^7)
+ 1349354685*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/440401920
```

**Maple [A]** time = 0.055, size = 117, normalized size = 0.8

$$\frac{25x^3}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{1235x^2}{448}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{24499x}{10752}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{-558739+2234956x}{1048576}\sqrt{2x^2-x+3}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 25/16*x^3*(2*x^2-x+3)^(5/2)+1235/448*x^2*(2*x^2-x+3)^(5/2)+24499/10752*x*(2
*x^2-x+3)^(5/2)-558739/1048576*(-1+4*x)*(2*x^2-x+3)^(1/2)-12850997/4194304*
2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-24293/196608*(-1+4*x)*(2*x^2-x+3)^(3
/2)+73861/215040*(2*x^2-x+3)^(5/2)
```

---

**Maxima [A]** time = 1.49987, size = 186, normalized size = 1.27

$$\frac{25}{16}\left(2x^2-x+3\right)^{\frac{5}{2}}x^3+\frac{1235}{448}\left(2x^2-x+3\right)^{\frac{5}{2}}x^2+\frac{24499}{10752}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{73861}{215040}\left(2x^2-x+3\right)^{\frac{5}{2}}-\frac{24293}{49152}\left(2x^2-x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 25/16*(2*x^2 - x + 3)^(5/2)*x^3 + 1235/448*(2*x^2 - x + 3)^(5/2)*x^2 + 2449
9/10752*(2*x^2 - x + 3)^(5/2)*x + 73861/215040*(2*x^2 - x + 3)^(5/2) - 2429
3/49152*(2*x^2 - x + 3)^(3/2)*x + 24293/196608*(2*x^2 - x + 3)^(3/2) - 5587
39/262144*sqrt(2*x^2 - x + 3)*x - 12850997/4194304*sqrt(2)*arcsinh(1/23*sqr
t(23)*(4*x - 1)) + 558739/1048576*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.51727, size = 342, normalized size = 2.33

$$\frac{1}{110100480}\left(688128000\,x^7+525926400\,x^6+2025840640\,x^5+2061273088\,x^4+2728413312\,x^3+1799647136\,x^2+1619\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/110100480*(688128000*x^7 + 525926400*x^6 + 2025840640*x^5 + 2061273088*x^
4 + 2728413312*x^3 + 1799647136*x^2 + 1619403428*x + 439831323)*sqrt(2*x^2
- x + 3) + 12850997/8388608*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2, x)
```

---

**Giac [A]** time = 1.15408, size = 112, normalized size = 0.76

$$\frac{1}{110100480}\left(4\left(8\left(4\left(16\left(20\left(120\left(140\,x+107\right)x+49459\right)x+1006481\right)x+21315729\right)x+56238973\right)x+404850857\right)x+43\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/110100480*(4*(8*(4*(16*(20*(120*(140*x + 107)*x + 49459)*x + 1006481)*x +
 21315729)*x + 56238973)*x + 404850857)*x + 439831323)*sqrt(2*x^2 - x + 3)
+ 12850997/4194304*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
 + 1)
```

# 3.68 $\int \left(3-x+2x^2\right)^{3/2} \left(2+3x+5x^2\right) dx$

**Optimal**. Leaf size=105

$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}+\frac{107}{240}\left(2x^2-x+3\right)^{5/2}-\frac{179(1-4x)\left(2x^2-x+3\right)^{3/2}}{1536}-\frac{4117(1-4x)\sqrt{2x^2-x+3}}{8192}-\frac{94691\sinh^{-1}\left(\right.}{16384\sqrt{2}}$$

```
[Out] (-4117*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/8192 - (179*(1 - 4*x)*(3 - x + 2*x^2)
^(3/2))/1536 + (107*(3 - x + 2*x^2)^(5/2))/240 + (5*x*(3 - x + 2*x^2)^(5/2)
)/12 - (94691*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2])
```

---

**Rubi [A]** time = 0.0504904, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{12}x\left(2x^2-x+3\right)^{5/2}+\frac{107}{240}\left(2x^2-x+3\right)^{5/2}-\frac{179(1-4x)\left(2x^2-x+3\right)^{3/2}}{1536}-\frac{4117(1-4x)\sqrt{2x^2-x+3}}{8192}-\frac{94691\sinh^{-1}\left(\right.}{16384\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (-4117*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/8192 - (179*(1 - 4*x)*(3 - x + 2*x^2)
^(3/2))/1536 + (107*(3 - x + 2*x^2)^(5/2))/240 + (5*x*(3 - x + 2*x^2)^(5/2)
)/12 - (94691*ArcSinh[(1 - 4*x)/Sqrt[23]])/(16384*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)dx = \frac{5}{12}x\left(3-x+2x^2\right)^{5/2}+\frac{1}{12}\int\left(9+\frac{107x}{2}\right)\left(3-x+2x^2\right)^{3/2}dx$$

$$= \frac{107}{240}\left(3-x+2x^2\right)^{5/2}+\frac{5}{12}x\left(3-x+2x^2\right)^{5/2}+\frac{179}{96}\int\left(3-x+2x^2\right)^{3/2}dx$$

$$= -\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x+2x^2\right)^{5/2}+\frac{5}{12}x\left(3-x+2x^2\right)^{5/2}$$

$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x+2x\right.$$

$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x+2x\right.$$

$$= -\frac{4117(1-4x)\sqrt{3-x+2x^2}}{8192}-\frac{179(1-4x)\left(3-x+2x^2\right)^{3/2}}{1536}+\frac{107}{240}\left(3-x+2x\right.$$

**Mathematica [A]** time = 0.0837233, size = 65, normalized size = 0.62

$$\frac{4\sqrt{2x^2-x+3}\left(204800x^5+14336x^4+561024x^3+319072x^2+565276x+388341\right)-1420365\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{491520}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(388341 + 565276*x + 319072*x^2 + 561024*x^3 + 14336
*x^4 + 204800*x^5) - 1420365*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/491520
```

---

**Maple [A]** time = 0.058, size = 83, normalized size = 0.8

$$\frac{5x}{12}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{107}{240}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{-179+716x}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{-4117+16468x}{8192}\sqrt{2x^2-x+3}+\frac{94691}{3276}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x)
```

```
[Out] 5/12*x*(2*x^2-x+3)^(5/2)+107/240*(2*x^2-x+3)^(5/2)+179/1536*(-1+4*x)*(2*x^2
-x+3)^(3/2)+4117/8192*(-1+4*x)*(2*x^2-x+3)^(1/2)+94691/32768*2^(1/2)*arcsin
h(4/23*23^(1/2)*(x-1/4))
```

---

**Maxima [A]** time = 1.49874, size = 140, normalized size = 1.33

$$\frac{5}{12}\left(2x^2-x+3\right)^{\frac{5}{2}}x+\frac{107}{240}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{179}{384}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{179}{1536}\left(2x^2-x+3\right)^{\frac{3}{2}}+\frac{4117}{2048}\sqrt{2x^2-x+3}x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 5/12*(2*x^2 - x + 3)^(5/2)*x + 107/240*(2*x^2 - x + 3)^(5/2) + 179/384*(2*x
^2 - x + 3)^(3/2)*x - 179/1536*(2*x^2 - x + 3)^(3/2) + 4117/2048*sqrt(2*x^2
 - x + 3)*x + 94691/32768*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 4117/8
192*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.60156, size = 257, normalized size = 2.45

$$\frac{1}{122880}\left(204800\,x^5+14336\,x^4+561024\,x^3+319072\,x^2+565276\,x+388341\right)\sqrt{2\,x^2-x+3}+\frac{94691}{65536}\,\sqrt{2}\log\left(-4\,\sqrt{2}\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/122880*(204800*x^5 + 14336*x^4 + 561024*x^3 + 319072*x^2 + 565276*x + 388
341)*sqrt(2*x^2 - x + 3) + 94691/65536*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 -
x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)*(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2), x)
```

---

**Giac [A]** time = 1.24798, size = 99, normalized size = 0.94

$$\frac{1}{122880}\left(4\left(8\left(4\left(16\left(100\,x+7\right)x+4383\right)x+9971\right)x+141319\right)x+388341\right)\sqrt{2\,x^2-x+3}-\frac{94691}{32768}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 1/122880*(4*(8*(4*(16*(100*x + 7)*x + 4383)*x + 9971)*x + 141319)*x + 38834
1)*sqrt(2*x^2 - x + 3) - 94691/32768*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sq
rt(2*x^2 - x + 3)) + 1)
```

## 3.69 $\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx$

**Optimal**. Leaf size=197

$$-\frac{1}{100}\sqrt{2x^2-x+3}(49-20x)+\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right)-\frac{11}{12}$$

```
[Out] -((49 - 20*x)*Sqrt[3 - x + 2*x^2])/100 - (2203*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(1000*Sqrt[2]) + (11*Sqrt[(11*(247 + 500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62
*(247 + 500*Sqrt[2]))]*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/125 - (11*Sqrt[(11*(-247 + 500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(6
2*(-247 + 500*Sqrt[2]))]*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x))/Sqrt[3 -
x + 2*x^2]])/125
```

---

**Rubi [A]** time = 0.487911, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {977, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{100}\sqrt{2x^2-x+3}(49-20x)+\frac{11}{125}\sqrt{\frac{11}{31}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(247+500\sqrt{2}\right)}}\left(\left(130+69\sqrt{2}\right)x+61\sqrt{2}+8\right)}{\sqrt{2x^2-x+3}}\right)-\frac{11}{12}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] -((49 - 20*x)*Sqrt[3 - x + 2*x^2])/100 - (2203*ArcSinh[(1 - 4*x)/Sqrt[23]])
/(1000*Sqrt[2]) + (11*Sqrt[(11*(247 + 500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62
*(247 + 500*Sqrt[2]))]*(8 + 61*Sqrt[2] + (130 + 69*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/125 - (11*Sqrt[(11*(-247 + 500*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/(6
2*(-247 + 500*Sqrt[2]))]*(8 - 61*Sqrt[2] + (130 - 69*Sqrt[2])*x))/Sqrt[3 -
x + 2*x^2]])/125
```

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
```

```
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{3/2}}{2+3x+5x^2}\,dx = -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{50}\int \frac{-\frac{731}{2}+\frac{1195x}{4}-\frac{2203x^2}{4}}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{1}{250}\int \frac{-726+3146x}{\sqrt{3-x+2x^2}\,(2+3x+5x^2)}\,dx + \frac{2203\int \frac{1}{\sqrt{3-x+2x^2}}\,d}{1000}$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} + \frac{\int \frac{2662(16+3\sqrt{2})+2662(10-13\sqrt{2})x}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx}{5500\sqrt{2}} - \frac{\int \frac{2662(16-3\sqrt{2})+2662(10+13\sqrt{}}{\sqrt{3-x+2x^2}(2+3x+5x^2)}}{5500\sqrt{2}}$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} - \frac{1}{125}\left(322102\left(1000-247\sqrt{2}\right)\right)\text{Subst}$$

$$= -\frac{1}{100}(49-20x)\sqrt{3-x+2x^2} - \frac{2203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{1000\sqrt{2}} + \frac{11}{125}\sqrt{\frac{11}{31}}\left(247+500\sqrt{2}\right)\tan^{-1}\left(\sqrt{\frac{}{62}}\right.$$

**Mathematica [C]** time = 0.695284, size = 310, normalized size = 1.57

$$\frac{400\sqrt{31}\sqrt{2x^2-x+3}\,x - 980\sqrt{31}\sqrt{2x^2-x+3} + 44\sqrt{286+22i\sqrt{31}}\left(\sqrt{31}-13i\right)\tanh^{-1}\left(\frac{-4i\sqrt{31}x-22x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) + 572}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (-980*Sqrt[31]*Sqrt[3 - x + 2*x^2] + 400*Sqrt[31]*x*Sqrt[3 - x + 2*x^2] - 2
203*Sqrt[62]*ArcSinh[(1 - 4*x)/Sqrt[23]] + 44*Sqrt[286 + (22*I)*Sqrt[31]]*(
-13*I + Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sq
rt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 44*Sqrt[682*(13 - I*Sqrt[
31])]*ArcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (572*I)*Sqrt[286 - (22*I)*Sqrt[31]]*A
rcTanh[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqr
t[31]]*Sqrt[3 - x + 2*x^2])])/(2000*Sqrt[31])
```

---

**Maple [B]** time = 0.19, size = 3460, normalized size = 17.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x*(2*x^2-x+3)^(1/2)-49/100*(2*x^2-x+3)^(1/2)+2203/2000*2^(1/2)*arcsinh(
4/23*23^(1/2)*(x-1/4))-2/1321375*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(4245*(-77568
7+549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/1169248
7*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2
^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1
```

```
/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1
/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+6154*(-775687+549362*2^(1/2))^(1/2)*(-886
6+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23
*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(
6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+123
25786*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^
(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)
-359414*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*
(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8
-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1
/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-2/264275*(8*(2^(1/2)-1+x
)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^
(1/2)*2^(1/2)*(2365*(-775687+549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1
/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/
2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/
2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+223
79*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+3338*(-775687+
549362*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687
+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x
)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+103
68*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4
/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1
/2)-1+x)/(2^(1/2)+1-x))+3192442*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-
x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+68
20*2^(1/2))^(1/2))*2^(1/2)-5264358*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866
+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^
2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(
1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/
2)-13/105710*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+
1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(285*(-775687+549362*2^(1/2))^(1/
2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(
1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1
/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)
-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1
-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2
^(1/2)+1-x))+386*(-775687+549362*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*
arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+
32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)
^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))-274846*arctanh(31/2*(8*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3
*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-1543366*arctanh(31/2*(8
*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+
8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2
)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)
-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))
/(-8866+6820*2^(1/2))^(1/2)+3/10571*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*(-775
687+549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692
487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2
*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^
(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^
```

```
(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+218*(-775687+549362*2^(1/2))^(1/2)*(-88
66+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-2
3*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*
(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2
)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/
2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+40
1698*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(
1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-
63426*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^
(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3
*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2
)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+9/21142*(8*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/
2)*2^(1/2)*(369*(-775687+549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))
^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*
(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1
+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2
^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+520*(-775687+54936
2*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+5493
62*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+2
4*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(
1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1
+x)/(2^(1/2)+1-x))+465124*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3
*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(
1/2))^(1/2))*2^(1/2)-866822*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2
+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2
^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1
/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1
+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^{2}-x+3\right)^{\frac{3}{2}}}{5\,x^{2}+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2), x)
```

---

**Fricas [B]** time = 4.32353, size = 7468, normalized size = 37.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arc
tan(1/10605875*(230*sqrt(10)*(2*24200^(3/4)*sqrt(31)*(20846*x^7 - 109153*x^
```

6 + 215386*x^5 - 427391*x^4 + 234360*x^3 - 156600*x^2 - sqrt(2)*(28854*x^7
- 90639*x^6 + 200187*x^5 - 262838*x^4 + 117544*x^3 - 23472*x^2 - 186624*x +
86400) - 172800*x + 186624) + 5*24200^(1/4)*sqrt(31)*(112238*x^7 - 1817988
*x^6 + 10351960*x^5 - 25791248*x^4 + 34522560*x^3 - 28368000*x^2 - sqrt(2)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - 19989504*x + 10533888) - 21067776*x + 19989504))*sqrt(2*x^2 - x
+ 3)*sqrt(247*sqrt(2) + 1000) + 30107000*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(5/
119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)*(46522*x^7 - 71117*x^6 + 257247*x^5
- 273360*x^4 + 484920*x^3 - 269568*x^2 - 16*sqrt(2)*(7714*x^7 - 10881*x^6 +
33771*x^5 - 5576*x^4 - 576*x^3 + 32184*x^2 - 32184*x) + 269568*x) + 5*2420
0^(1/4)*sqrt(31)*(309512*x^7 - 4017952*x^6 + 15741280*x^5 - 22625280*x^4 +
37693440*x^3 - 13519872*x^2 - sqrt(2)*(516957*x^7 - 6676948*x^6 + 25569820*
x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 46199808*x) + 13519872*x
))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) + 130900*sqrt(31)*sqrt(2)*(
123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x
^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^
5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 595
0*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781
920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 51
7*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))
*sqrt((24200^(1/4)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x -
76)*sqrt(247*sqrt(2) + 1000) + 58310*x^2 + 52360*sqrt(2)*(2*x^2 - x + 3) -
179690*x + 238000)/x^2) + 342125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 5338
5560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7
744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*
x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4
661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3461
5296*x^2 - 24772608*x + 18579456)) + 11/77500*24200^(1/4)*sqrt(31)*sqrt(10)
*sqrt(2)*sqrt(247*sqrt(2) + 1000)*arctan(1/10605875*(230*sqrt(10)*(2*24200^
(3/4)*sqrt(31)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 234360*x
^3 - 156600*x^2 - sqrt(2)*(28854*x^7 - 90639*x^6 + 200187*x^5 - 262838*x^4
+ 117544*x^3 - 23472*x^2 - 186624*x + 86400) - 172800*x + 186624) + 5*24200
^(1/4)*sqrt(31)*(112238*x^7 - 1817988*x^6 + 10351960*x^5 - 25791248*x^4 + 3
4522560*x^3 - 28368000*x^2 - sqrt(2)*(125839*x^7 - 1864281*x^6 + 9323336*x^
5 - 19725020*x^4 + 24624288*x^3 - 10862496*x^2 - 19989504*x + 10533888) - 2
1067776*x + 19989504))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2) + 1000) - 30107
000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1
549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 3961
04*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53
9136) + 1154304*x - 456192) - sqrt(5/119)*(sqrt(10)*(2*24200^(3/4)*sqrt(31)
*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2
- 16*sqrt(2)*(7714*x^7 - 10881*x^6 + 33771*x^5 - 5576*x^4 - 576*x^3 + 3218
4*x^2 - 32184*x) + 269568*x) + 5*24200^(1/4)*sqrt(31)*(309512*x^7 - 4017952
*x^6 + 15741280*x^5 - 22625280*x^4 + 37693440*x^3 - 13519872*x^2 - sqrt(2)*
(516957*x^7 - 6676948*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46
199808*x^2 - 46199808*x) + 13519872*x))*sqrt(2*x^2 - x + 3)*sqrt(247*sqrt(2
) + 1000) - 130900*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6
- 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8
- 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 120960
0*x^2 - 1036800*x) + 3276288*x) - 5950*sqrt(31)*(254591*x^8 - 4815126*x^7 +
32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
- 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
+ 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(24200^(1/4)*sqrt(10)*sqrt(2*x^
2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2) + 1000) - 58310*
x^2 - 52360*sqrt(2)*(2*x^2 - x + 3) + 179690*x - 238000)/x^2) - 342125*sqrt
(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*

```
x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 97
89*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223
064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^
5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) +
11/36890000*24200^(1/4)*sqrt(10)*sqrt(247*sqrt(2) + 1000)*(247*sqrt(2) - 10
00)*log(1512500/119*(24200^(1/4)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x -
 75) + 74*x - 76)*sqrt(247*sqrt(2) + 1000) + 58310*x^2 + 52360*sqrt(2)*(2*x
^2 - x + 3) - 179690*x + 238000)/x^2) - 11/36890000*24200^(1/4)*sqrt(10)*sq
rt(247*sqrt(2) + 1000)*(247*sqrt(2) - 1000)*log(-1512500/119*(24200^(1/4)*s
qrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(x - 75) + 74*x - 76)*sqrt(247*sqrt(2)
 + 1000) - 58310*x^2 - 52360*sqrt(2)*(2*x^2 - x + 3) + 179690*x - 238000)/x
^2) + 1/100*sqrt(2*x^2 - x + 3)*(20*x - 49) + 2203/4000*sqrt(2)*log(-4*sqrt
(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}{5x^2 + 3x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.70 $\int \frac{(3-x+2x^2)^{3/2}}{(2+3x+5x^2)^2}\,dx$

**Optimal**. Leaf size=232

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\sqrt{2x^2-x+3}+\frac{\sqrt{\frac{11}{31}\left(3169333+2265350\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(3169333+2265350\sqrt{2}\right)}}\left((9440+6\right.}{\sqrt{2x^2-x+}}\right.}{1550}$$

```
[Out] (4*(4 - 5*x)*Sqrt[3 - x + 2*x^2])/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/
(31*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/25 + (Sqrt
[(11*(3169333 + 2265350*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(3169333 + 226535
0*Sqrt[2]))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3 - x +
2*x^2]])/1550 - (Sqrt[(11*(-3169333 + 2265350*Sqrt[2]))/31]*ArcTanh[(Sqrt[1
1/(62*(-3169333 + 2265350*Sqrt[2]))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/1550
```

---

**Rubi [A]** time = 0.575455, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{31\left(5x^2+3x+2\right)}+\frac{4}{155}(4-5x)\sqrt{2x^2-x+3}+\frac{\sqrt{\frac{11}{31}\left(3169333+2265350\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{62\left(3169333+2265350\sqrt{2}\right)}}\left((9440+6\right.}{\sqrt{2x^2-x+}}\right.}{1550}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*(4 - 5*x)*Sqrt[3 - x + 2*x^2])/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/
(31*(2 + 3*x + 5*x^2)) - (2*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/25 + (Sqrt
[(11*(3169333 + 2265350*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(3169333 + 226535
0*Sqrt[2]))]*(3514 + 2963*Sqrt[2] + (9440 + 6477*Sqrt[2])*x))/Sqrt[3 - x +
2*x^2]])/1550 - (Sqrt[(11*(-3169333 + 2265350*Sqrt[2]))/31]*ArcTanh[(Sqrt[1
1/(62*(-3169333 + 2265350*Sqrt[2]))]*(3514 - 2963*Sqrt[2] + (9440 - 6477*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/1550
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
```

```
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^2}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{1}{31}\int \frac{\sqrt{3-x+2x^2}\left(-\frac{69}{2}+13x+40x^2\right)}{2+3x+5x^2}\,dx$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{13070-5750x+2480x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{3100}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{\int \frac{60390-36190x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{15500} + \frac{4}{25}\int \frac{}{\sqrt{3-}}$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} + \frac{1}{25}\left(2\sqrt{\frac{2}{23}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x,\right.$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{1}{155}\left(1331\left(4\right.\right.$$

$$= \frac{4}{155}(4-5x)\sqrt{3-x+2x^2} + \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{31\left(2+3x+5x^2\right)} - \frac{2}{25}\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right) + \frac{\sqrt{\frac{11}{31}}\left(316933\right.}{}$$

**Mathematica [C]** time = 2.58377, size = 530, normalized size = 2.28

$$\frac{\frac{62000\sqrt{2x^2-x+3}x^2}{10x-i\sqrt{31}+3} + \frac{62000\sqrt{2x^2-x+3}x^2}{10x+i\sqrt{31}+3} - \frac{31000\sqrt{2x^2-x+3}x}{10x-i\sqrt{31}+3} - \frac{31000\sqrt{2x^2-x+3}x}{10x+i\sqrt{31}+3} - 12400\sqrt{2x^2-x+3}x + \frac{93000\sqrt{2x^2-x+3}}{10x-i\sqrt{31}+3} + \frac{93000\sqrt{2x^2-x+}}{10x+i\sqrt{31}+3}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (9920*Sqrt[3 - x + 2*x^2] - 12400*x*Sqrt[3 - x + 2*x^2] + (93000*Sqrt[3 - x
 + 2*x^2])/(3 - I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x + 2*x^2])/(3 - I*S
qrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 - I*Sqrt[31] + 10*x) +
 (93000*Sqrt[3 - x + 2*x^2])/(3 + I*Sqrt[31] + 10*x) - (31000*x*Sqrt[3 - x
+ 2*x^2])/(3 + I*Sqrt[31] + 10*x) + (62000*x^2*Sqrt[3 - x + 2*x^2])/(3 + I*
Sqrt[31] + 10*x) - 7688*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]] - (Sqrt[286 + (
22*I)*Sqrt[31]]*(10199*I + 6477*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 22*x -
 (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-
13*I + Sqrt[31]) - (6477*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*Sqrt[3
1] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])])/(13*I + Sqrt[31]) + ((10199*I)*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh
[(63 - I*Sqrt[31] - 22*x + (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]
*Sqrt[3 - x + 2*x^2])])/(13*I + Sqrt[31]))/96100
```

**Maple [B]** time = 0.355, size = 28185, normalized size = 121.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2-x+3\right)^{\frac{3}{2}}}{\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^2, x)
```

**Fricas [B]** time = 5.10758, size = 8892, normalized size = 38.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/90746855745853600*(10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(2)*(5*x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/172758074198
807633719789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118
*x^7 - 9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x
^2 - sqrt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 113
67152*x^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) +
1404517*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x
^5 - 70476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 423
2733*x^6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 4669
7472*x + 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(31693
33*sqrt(2) + 4530700) + 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^
8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 9849
6*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*
x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
sqrt(45307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7
- 12210811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2
- sqrt(2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 121140
72*x^3 + 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(
1/4)*sqrt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 1
12374720*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 5039126
0*x^5 - 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304
*x))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) + 7590571938849196
```

```
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
 + 3276288*x) + 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 323035
80*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 1548
8*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268
*x^2 - 1944*x) + 144820224*x))*sqrt(-(1987037073032^(1/4)*sqrt(45307)*sqrt(
62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*s
qrt(3169333*sqrt(2) + 4530700) - 11567627293306*x^2 - 10387257161336*sqrt(2
)*(2*x^2 - x + 3) + 35647177985494*x - 47214805278800)/x^2) + 5572841103187
343023219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^
5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 -
 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x
 - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x
^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x +
 18579456)) + 10421084*1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(2)*(5*
x^2 + 3*x + 2)*sqrt(3169333*sqrt(2) + 4530700)*arctan(1/1727580741988076337
19789*(64607782*sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(2433118*x^7 -
9616349*x^6 + 20077988*x^5 - 32895253*x^4 + 16664280*x^3 - 8289000*x^2 - sq
rt(2)*(1842432*x^7 - 6916062*x^6 + 14611071*x^5 - 22920229*x^4 + 11367152*x
^3 - 5107176*x^2 - 12897792*x + 8726400) - 17452800*x + 12897792) + 1404517
*1987037073032^(1/4)*sqrt(62)*(373384*x^7 - 5757834*x^6 + 30631880*x^5 - 70
476664*x^4 + 91370880*x^3 - 59457600*x^2 - sqrt(2)*(276977*x^7 - 4232733*x^
6 + 22218448*x^5 - 50249260*x^4 + 64668384*x^3 - 39479328*x^2 - 46697472*x
+ 32016384) - 64032768*x + 46697472))*sqrt(2*x^2 - x + 3)*sqrt(3169333*sqrt
(2) + 4530700) - 490410017080486186043272*sqrt(31)*sqrt(2)*(28180*x^8 - 254
666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 -
 sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 7
52088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(45
307/2711)*(sqrt(45307)*(2*1987037073032^(3/4)*sqrt(62)*(8480726*x^7 - 12210
811*x^6 + 39548601*x^5 - 16962480*x^4 + 21434760*x^3 + 14432256*x^2 - sqrt(
2)*(6779042*x^7 - 9704193*x^6 + 31062363*x^5 - 11094928*x^4 + 12114072*x^3
+ 16301952*x^2 - 16301952*x) - 14432256*x) + 1404517*1987037073032^(1/4)*sq
rt(62)*(1312966*x^7 - 16987736*x^6 + 65572040*x^5 - 85530240*x^4 + 11237472
0*x^3 + 57314304*x^2 - sqrt(2)*(1011501*x^7 - 13081364*x^6 + 50391260*x^5 -
 64806336*x^4 + 81634464*x^3 + 56070144*x^2 - 56070144*x) - 57314304*x))*sq
rt(2*x^2 - x + 3)*sqrt(3169333*sqrt(2) + 4530700) - 7590571938849196*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) - 345025997220418*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6
- 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(
2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 -
1944*x) + 144820224*x))*sqrt((1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169
333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x^2
 - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 557284110318734302321
9*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 2541
46592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^
7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184)
 + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490
880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1857945
6)) + 1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 316
9333*sqrt(2)*(5*x^2 + 3*x + 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2)
 + 4530700)*log(113267500/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sq
rt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(31
69333*sqrt(2) + 4530700) + 11567627293306*x^2 + 10387257161336*sqrt(2)*(2*x
^2 - x + 3) - 35647177985494*x + 47214805278800)/x^2) - 1987037073032^(1/4)
*sqrt(45307)*sqrt(62)*sqrt(31)*(22653500*x^2 - 3169333*sqrt(2)*(5*x^2 + 3*x
```

```
 + 2) + 13592100*x + 9061400)*sqrt(3169333*sqrt(2) + 4530700)*log(-11326750
0/2711*(1987037073032^(1/4)*sqrt(45307)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x +
3)*(sqrt(2)*(1867*x + 1425) - 3292*x - 442)*sqrt(3169333*sqrt(2) + 4530700)
 - 11567627293306*x^2 - 10387257161336*sqrt(2)*(2*x^2 - x + 3) + 3564717798
5494*x - 47214805278800)/x^2) + 3629874229834144*sqrt(2)*(5*x^2 + 3*x + 2)*
log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 644009
9440028320*sqrt(2*x^2 - x + 3)*(13*x + 7))/(5*x^2 + 3*x + 2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}{\left(5x^2 + 3x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**2, x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.71 $\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx$

**Optimal**. Leaf size=223

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(696x+277)\sqrt{2x^2-x+3}}{3844\left(5x^2+3x+2\right)}+\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{11}{31(366990269+25950}}\right.}{7688}$$

```
[Out] ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(277 + 696
*x)*Sqrt[3 - x + 2*x^2])/(3844*(2 + 3*x + 5*x^2)) + (3*Sqrt[(366990269 + 25
9509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))]
*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])
/7688 - (3*Sqrt[(-366990269 + 259509026*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*
(-366990269 + 259509026*Sqrt[2]))]*(29367 - 20575*Sqrt[2] + (70517 - 49942*
Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/7688
```

---

**Rubi [A]** time = 0.432945, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {971, 1013, 1035, 1029, 206, 204}

$$\frac{(10x+3)\left(2x^2-x+3\right)^{3/2}}{62\left(5x^2+3x+2\right)^2}+\frac{3(696x+277)\sqrt{2x^2-x+3}}{3844\left(5x^2+3x+2\right)}+\frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{11}{31(366990269+25950}}\right.}{7688}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] ((3 + 10*x)*(3 - x + 2*x^2)^(3/2))/(62*(2 + 3*x + 5*x^2)^2) + (3*(277 + 696
*x)*Sqrt[3 - x + 2*x^2])/(3844*(2 + 3*x + 5*x^2)) + (3*Sqrt[(366990269 + 25
9509026*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(366990269 + 259509026*Sqrt[2]))]
*(29367 + 20575*Sqrt[2] + (70517 + 49942*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])
/7688 - (3*Sqrt[(-366990269 + 259509026*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*
(-366990269 + 259509026*Sqrt[2]))]*(29367 - 20575*Sqrt[2] + (70517 - 49942*
Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/7688
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1013

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((g*b - 2*a*h - (b*h - 2*g
*c)*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1
```

```
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{3/2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{\left(-\frac{189}{2}+33x\right)\sqrt{3-x+2x^2}}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{\frac{13359}{4}-1353x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{1922}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{-\frac{33}{4}\left(6257-4453\sqrt{2}\right)+\frac{33}{4}\left(2649-1804\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,d}{42284\sqrt{2}}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{\left(99\left(519018052-366990269\sqrt{2}\right)\right)}{}$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{3/2}}{62\left(2+3x+5x^2\right)^2} + \frac{3(277+696x)\sqrt{3-x+2x^2}}{3844\left(2+3x+5x^2\right)} + \frac{3\sqrt{\frac{1}{682}\left(366990269+259509026\sqrt{}\right.}}{}$$

<u>**Mathematica**</u> **[C]** time = 5.38845, size = 1262, normalized size = 5.66

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(3/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (((248000*I)*Sqrt[31]*(3 - x + 2*x^2)^(3/2))/(3*I + Sqrt[31] + (10*I)*x)^2
+ (744000*(3 - x + 2*x^2)^(3/2))/(3 - I*Sqrt[31] + 10*x) + ((248000*I)*Sqrt
[31]*(3 - x + 2*x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x)^2 + (744000*(3 - x + 2*
x^2)^(3/2))/(3 + I*Sqrt[31] + 10*x) + (3*I)*Sqrt[31]*(20*(1199 + (98*I)*Sqr
t[31] - 20*(11 + (2*I)*Sqrt[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(13453 +
(4406*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (352*Sqrt[286 + (22*I)*Sqr
t[31]]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[3
1])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sqrt[
31])) + 558*(20*(27 + (4*I)*Sqrt[31] - 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*
(569 + (88*I)*Sqrt[31])*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 + (22*I)*
Sqrt[31]]*(-81*I + 37*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqr
t[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(-13*I + Sq
rt[31])) + (744*Sqrt[31]*(220*(-439 + (497*I)*Sqrt[31] + 20*(69 + (13*I)*Sq
rt[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 - (398*I)*Sqrt[31] + 5*(4
7 - (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 + (22*I)*S
qrt[31]]*(19548 - (4904*I)*Sqrt[31] + (-23345 - (8565*I)*Sqrt[31])*x)*ArcTa
nh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])]))/(11*(-13*I + Sqrt[31])^2*(-3*I + Sqrt[31] - (10
*I)*x)) + (744*Sqrt[31]*(220*(-439 - (497*I)*Sqrt[31] + 20*(69 - (13*I)*Sqr
t[31])*x)*Sqrt[3 - x + 2*x^2] + 88*Sqrt[2]*(4426 + (398*I)*Sqrt[31] + 5*(47
 + (281*I)*Sqrt[31])*x)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + Sqrt[286 - (22*I)*Sq
rt[31]]*(-19548 - (4904*I)*Sqrt[31] + 5*(4669 - (1713*I)*Sqrt[31])*x)*ArcTa
nh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[3
1]]*Sqrt[3 - x + 2*x^2])]))/(11*(13*I + Sqrt[31])^2*(3*I + Sqrt[31] + (10*I
)*x)) + (3*Sqrt[31]*(20*(12549 - (2473*I)*Sqrt[31] + (20*I)*(81*I + 37*Sqrt
[31])*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(38303 - (70731*I)*Sqrt[31])*ArcSinh
[(1 - 4*x)/Sqrt[23]] + (352*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(69*I + 13*Sqrt[
```

```
31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/(13*I + Sqrt[31]) + 558*(-20*(-27 + (
4*I)*Sqrt[31] + 20*x)*Sqrt[3 - x + 2*x^2] + Sqrt[2]*(569 - (88*I)*Sqrt[31])
*ArcSinh[(1 - 4*x)/Sqrt[23]] - (4*Sqrt[286 - (22*I)*Sqrt[31]]*(81*I + 37*Sq
rt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 -
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(13*I + Sqrt[31])))/4766560
```

---

**Maple [B]** time = 0.607, size = 81552, normalized size = 365.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}{\left(5\,x^2+3\,x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(3/2)/(5*x^2 + 3*x + 2)^3, x)
```

---

**Fricas [B]** time = 4.94277, size = 9586, normalized size = 42.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] -1/85773071417697924109696*(189113268*134689869150937352^(1/4)*sqrt(1297545
13)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(366990269*
sqrt(2) + 519018052)*arctan(1/1067259092343193675559267622545473*(160895596
12*sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)*(38305160*x^7 - 1
47261352*x^6 + 309398878*x^5 - 495410374*x^4 + 248212864*x^3 - 117285552*x^
2 - sqrt(2)*(26988622*x^7 - 104036813*x^6 + 218448200*x^5 - 350579241*x^4 +
 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585792) - 271171584*x + 19
1303424) + 4022389903*134689869150937352^(1/4)*sqrt(341)*(2906601*x^7 - 446
04657*x^6 + 235604928*x^5 - 537156764*x^4 + 693706464*x^3 - 436717728*x^2 -
 sqrt(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5 - 379661892*x^4 + 4905
00864*x^3 - 309827808*x^2 - 348696576*x + 246965760) - 493931520*x + 348696
576))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) + 30296387137
48420756426308089806504*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x
^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8
 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144
```

```
*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(259509026/713)*(sq
rt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)*(5980372*x^7 - 8582986
*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 + 12517632*x^2 - sqrt(2)*
(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*x^4 + 9050328*x^3 + 864
0000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134689869150937352^(1/4)*s
qrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 - 29282112*x^4 + 37610208
*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*x^6 + 15927060*x^5 - 20
630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*x) - 22726656*x))*sqrt(
2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) + 8186887989068712800954
*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 39
6480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244
047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x)
 + 3276288*x) + 372131272230396036407*sqrt(31)*(254591*x^8 - 4815126*x^7 +
32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
- 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
+ 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(134689869150937352^(1/4)*sqrt(1
29754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 9
73*x - 419)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39
11902897144396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 177813768052
01800)/x^2) + 34427712656232054050298955565983*sqrt(31)*(2828123*x^8 - 9696
916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37
981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 155
69*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2
585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249
088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 189113268*13468986915093
7352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12
*x + 4)*sqrt(366990269*sqrt(2) + 519018052)*arctan(1/1067259092343193675559
267622545473*(16089559612*sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt
(341)*(38305160*x^7 - 147261352*x^6 + 309398878*x^5 - 495410374*x^4 + 24821
2864*x^3 - 117285552*x^2 - sqrt(2)*(26988622*x^7 - 104036813*x^6 + 21844820
0*x^5 - 350579241*x^4 + 175844824*x^3 - 83534472*x^2 - 191303424*x + 135585
792) - 271171584*x + 191303424) + 4022389903*134689869150937352^(1/4)*sqrt(
341)*(2906601*x^7 - 44604657*x^6 + 235604928*x^5 - 537156764*x^4 + 69370646
4*x^3 - 436717728*x^2 - sqrt(2)*(2050114*x^7 - 31475955*x^6 + 166375268*x^5
 - 379661892*x^4 + 490500864*x^3 - 309827808*x^2 - 348696576*x + 246965760)
 - 493931520*x + 348696576))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 5
19018052) - 3029638713748420756426308089806504*sqrt(31)*sqrt(2)*(28180*x^8
- 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^
4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*
sqrt(259509026/713)*(sqrt(129754513)*(11*134689869150937352^(3/4)*sqrt(341)
*(5980372*x^7 - 8582986*x^6 + 27618126*x^5 - 10751392*x^4 + 12649968*x^3 +
12517632*x^2 - sqrt(2)*(4201650*x^7 - 6032009*x^6 + 19421619*x^5 - 7633552*
x^4 + 9050328*x^3 + 8640000*x^2 - 8640000*x) - 12517632*x) + 4022389903*134
689869150937352^(1/4)*sqrt(341)*(453599*x^7 - 5867420*x^6 + 22622900*x^5 -
29282112*x^4 + 37610208*x^3 + 22726656*x^2 - sqrt(2)*(319303*x^7 - 4130364*
x^6 + 15927060*x^5 - 20630592*x^4 + 26556768*x^3 + 15800832*x^2 - 15800832*
x) - 22726656*x))*sqrt(2*x^2 - x + 3)*sqrt(366990269*sqrt(2) + 519018052) -
 8186887989068712800954*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888
*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550
*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1
209600*x^2 - 1036800*x) + 3276288*x) - 372131272230396036407*sqrt(31)*(2545
91*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742193
28*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5
 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((134689869
150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqr
t(2)*(696*x + 277) - 973*x - 419)*sqrt(366990269*sqrt(2) + 519018052) + 435
6437317274441*x^2 + 3911902897144396*sqrt(2)*(2*x^2 - x + 3) - 134249394879
27359*x + 17781376805201800)/x^2) - 34427712656232054050298955565983*sqrt(3
```

```
1)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^
4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789
*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306
4064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5
- 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 3*
134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300*x^
4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2) +
519018052)*log(9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)*sqr
t(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 419)*s
qrt(366990269*sqrt(2) + 519018052) + 4356437317274441*x^2 + 391190289714439
6*sqrt(2)*(2*x^2 - x + 3) - 13424939487927359*x + 17781376805201800)/x^2) +
 3*134689869150937352^(1/4)*sqrt(129754513)*sqrt(341)*sqrt(31)*(12975451300
*x^4 + 15570541560*x^3 + 15051523508*x^2 - 366990269*sqrt(2)*(25*x^4 + 30*x
^3 + 29*x^2 + 12*x + 4) + 6228216624*x + 2076072208)*sqrt(366990269*sqrt(2)
 + 519018052)*log(-9342324936/713*(134689869150937352^(1/4)*sqrt(129754513)
*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(696*x + 277) - 973*x - 41
9)*sqrt(366990269*sqrt(2) + 519018052) - 4356437317274441*x^2 - 39119028971
44396*sqrt(2)*(2*x^2 - x + 3) + 13424939487927359*x - 17781376805201800)/x^
2) - 22313494125311634784*(11680*x^3 + 10171*x^2 + 8343*x + 2220)*sqrt(2*x^
2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{3}{2}}}{\left(5x^2 + 3x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(3/2)/(5*x**2 + 3*x + 2)**3, x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.72 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^4 \, dx$

**Optimal**. Leaf size=254

$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7+\frac{13875}{208}\left(2x^2-x+3\right)^{7/2}x^6+\frac{1046225\left(2x^2-x+3\right)^{7/2}x^5}{9984}+\frac{3684995\left(2x^2-x+3\right)^{7/2}x^4}{39936}+\frac{2346}{}$$

```
[Out] (-636602271789*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/34359738368 - (9226119881*(1
- 4*x)*(3 - x + 2*x^2)^(3/2))/2147483648 - (401135647*(1 - 4*x)*(3 - x + 2*
x^2)^(5/2))/335544320 + (25250178739*(3 - x + 2*x^2)^(7/2))/5725224960 + (1
12244125*x*(3 - x + 2*x^2)^(7/2))/122683392 + (122595067*x^2*(3 - x + 2*x^2
)^(7/2))/19169280 + (23460839*x^3*(3 - x + 2*x^2)^(7/2))/532480 + (3684995*
x^4*(3 - x + 2*x^2)^(7/2))/39936 + (1046225*x^5*(3 - x + 2*x^2)^(7/2))/9984
 + (13875*x^6*(3 - x + 2*x^2)^(7/2))/208 + (625*x^7*(3 - x + 2*x^2)^(7/2))/
28 - (14641852251147*ArcSinh[(1 - 4*x)/Sqrt[23]])/(68719476736*Sqrt[2])
```

---

**Rubi [A]** time = 0.372556, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{625}{28}\left(2x^2-x+3\right)^{7/2}x^7+\frac{13875}{208}\left(2x^2-x+3\right)^{7/2}x^6+\frac{1046225\left(2x^2-x+3\right)^{7/2}x^5}{9984}+\frac{3684995\left(2x^2-x+3\right)^{7/2}x^4}{39936}+\frac{2346}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```


```
[Out] (-636602271789*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/34359738368 - (9226119881*(1
- 4*x)*(3 - x + 2*x^2)^(3/2))/2147483648 - (401135647*(1 - 4*x)*(3 - x + 2*
x^2)^(5/2))/335544320 + (25250178739*(3 - x + 2*x^2)^(7/2))/5725224960 + (1
12244125*x*(3 - x + 2*x^2)^(7/2))/122683392 + (122595067*x^2*(3 - x + 2*x^2
)^(7/2))/19169280 + (23460839*x^3*(3 - x + 2*x^2)^(7/2))/532480 + (3684995*
x^4*(3 - x + 2*x^2)^(7/2))/39936 + (1046225*x^5*(3 - x + 2*x^2)^(7/2))/9984
 + (13875*x^6*(3 - x + 2*x^2)^(7/2))/208 + (625*x^7*(3 - x + 2*x^2)^(7/2))/
28 - (14641852251147*ArcSinh[(1 - 4*x)/Sqrt[23]])/(68719476736*Sqrt[2])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^4 dx = \frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2}+\frac{1}{28}\int \left(3-x+2x^2\right)^{5/2}\left(448+2688x+10528x^2+\right.$$

$$= \frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2}+\frac{625}{28}x^7\left(3-x+2x^2\right)^{7/2}+\frac{1}{728}\int \left(3-x+2x^2\right)^{5/2}$$

$$= \frac{1046225x^5\left(3-x+2x^2\right)^{7/2}}{9984}+\frac{13875}{208}x^6\left(3-x+2x^2\right)^{7/2}+\frac{625}{28}x^7\left(3-x+2x\right.$$

$$= \frac{3684995x^4\left(3-x+2x^2\right)^{7/2}}{39936}+\frac{1046225x^5\left(3-x+2x^2\right)^{7/2}}{9984}+\frac{13875}{208}x^6\left(3-x\right.$$

$$= \frac{23460839x^3\left(3-x+2x^2\right)^{7/2}}{532480}+\frac{3684995x^4\left(3-x+2x^2\right)^{7/2}}{39936}+\frac{1046225x^5\left(3\right.}{99}$$

$$= \frac{122595067x^2\left(3-x+2x^2\right)^{7/2}}{19169280}+\frac{23460839x^3\left(3-x+2x^2\right)^{7/2}}{532480}+\frac{3684995x^4}{3}$$

$$= \frac{112244125x\left(3-x+2x^2\right)^{7/2}}{122683392}+\frac{122595067x^2\left(3-x+2x^2\right)^{7/2}}{19169280}+\frac{23460839x^3}{5}$$

$$= \frac{25250178739\left(3-x+2x^2\right)^{7/2}}{5725224960}+\frac{112244125x\left(3-x+2x^2\right)^{7/2}}{122683392}+\frac{12259506}{19}$$

$$= -\frac{401135647(1-4x)\left(3-x+2x^2\right)^{5/2}}{335544320}+\frac{25250178739\left(3-x+2x^2\right)^{7/2}}{5725224960}+\frac{112}{}$$

$$= -\frac{9226119881(1-4x)\left(3-x+2x^2\right)^{3/2}}{2147483648}-\frac{401135647(1-4x)\left(3-x+2x^2\right)^{5/2}}{335544320}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2x^2\right)^{3/2}}{2147483648}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2x^2\right)^{3/2}}{2147483648}$$

$$= -\frac{636602271789(1-4x)\sqrt{3-x+2x^2}}{34359738368}-\frac{9226119881(1-4x)\left(3-x+2x^2\right)^{3/2}}{2147483648}$$

**Mathematica [A]** time = 0.453735, size = 105, normalized size = 0.41

$$4\sqrt{2x^2-x+3}\left(25125558681600000x^{13}+37398427729920000x^{12}+137233466130432000x^{11}+204932411660697600\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^4,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(10820567498568669 + 12071614275862524*x + 500641740
38215008*x^2 + 142490931553577856*x^3 + 257786732552566784*x^4 + 4054683822
84161024*x^5 + 485091164642279424*x^6 + 530502956133122048*x^7 + 4390645588
46345216*x^8 + 363646430503501824*x^9 + 204932411660697600*x^10 + 137233466
130432000*x^11 + 37398427729920000*x^12 + 25125558681600000*x^13) - 5995838
4968446965*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/562812514467840
```

---

**Maple [A]** time = 0.08, size = 204, normalized size = 0.8

$$\frac{1046225\,x^5}{9984}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{3684995\,x^4}{39936}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{23460839\,x^3}{532480}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{122595067\,x^2}{19169280}\left(2\,x^2-x+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x)
```

```
[Out] 1046225/9984*x^5*(2*x^2-x+3)^(7/2)+3684995/39936*x^4*(2*x^2-x+3)^(7/2)+2346
0839/532480*x^3*(2*x^2-x+3)^(7/2)+122595067/19169280*x^2*(2*x^2-x+3)^(7/2)+
112244125/122683392*x*(2*x^2-x+3)^(7/2)+636602271789/34359738368*(-1+4*x)*(
2*x^2-x+3)^(1/2)+14641852251147/137438953472*2^(1/2)*arcsinh(4/23*23^(1/2)*
(x-1/4))+401135647/335544320*(-1+4*x)*(2*x^2-x+3)^(5/2)+9226119881/21474836
48*(-1+4*x)*(2*x^2-x+3)^(3/2)+25250178739/5725224960*(2*x^2-x+3)^(7/2)+1387
5/208*x^6*(2*x^2-x+3)^(7/2)+625/28*x^7*(2*x^2-x+3)^(7/2)
```

---

**Maxima [A]** time = 1.53405, size = 317, normalized size = 1.25

$$\frac{625}{28}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^7+\frac{13875}{208}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^6+\frac{1046225}{9984}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^5+\frac{3684995}{39936}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^4+\frac{2346083}{532480}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="maxima")
```

```
[Out] 625/28*(2*x^2 - x + 3)^(7/2)*x^7 + 13875/208*(2*x^2 - x + 3)^(7/2)*x^6 + 10
46225/9984*(2*x^2 - x + 3)^(7/2)*x^5 + 3684995/39936*(2*x^2 - x + 3)^(7/2)*
x^4 + 23460839/532480*(2*x^2 - x + 3)^(7/2)*x^3 + 122595067/19169280*(2*x^2
 - x + 3)^(7/2)*x^2 + 112244125/122683392*(2*x^2 - x + 3)^(7/2)*x + 2525017
8739/5725224960*(2*x^2 - x + 3)^(7/2) + 401135647/83886080*(2*x^2 - x + 3)^
(5/2)*x - 401135647/335544320*(2*x^2 - x + 3)^(5/2) + 9226119881/536870912*
(2*x^2 - x + 3)^(3/2)*x - 9226119881/2147483648*(2*x^2 - x + 3)^(3/2) + 636
602271789/8589934592*sqrt(2*x^2 - x + 3)*x + 14641852251147/137438953472*sq
rt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 636602271789/34359738368*sqrt(2*x^
2 - x + 3)
```

**Fricas [A]** time = 1.40691, size = 657, normalized size = 2.59

$$\frac{1}{140703128616960}\left(25125558681600000\,x^{13} + 37398427729920000\,x^{12} + 137233466130432000\,x^{11} + 204932411660\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="fricas")
```

```
[Out] 1/140703128616960*(25125558681600000*x^13 + 37398427729920000*x^12 + 137233
466130432000*x^11 + 204932411660697600*x^10 + 363646430503501824*x^9 + 4390
64558846345216*x^8 + 530502956133122048*x^7 + 485091164642279424*x^6 + 4054
68382284161024*x^5 + 257786732552566784*x^4 + 142490931553577856*x^3 + 5006
4174038215008*x^2 + 12071614275862524*x + 10820567498568669)*sqrt(2*x^2 - x
 + 3) + 14641852251147/274877906944*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x +
 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(2x^2 - x + 3\right)^{\frac{5}{2}} \left(5x^2 + 3x + 2\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**4,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**4, x)
```

**Giac [A]** time = 1.1553, size = 153, normalized size = 0.6

$$\frac{1}{140703128616960}\,(4\,(8\,(4\,(16\,(4\,(8\,(4\,(32\,(12\,(200\,(20\,(240\,(260\,x + 387)x + 340823)x + 10179103)x + 3612502719)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^4,x, algorithm="giac")
```

```
[Out] 1/140703128616960*(4*(8*(4*(16*(4*(8*(4*(32*(12*(200*(20*(240*(260*x + 387)
*x + 340823)*x + 10179103)*x + 3612502719)*x + 52340574127)*x + 20237081761
67)*x + 7401903757359)*x + 49495652134297)*x + 125872428004183)*x + 1113210
402762327)*x + 1564505438694219)*x + 3017903568965631)*x + 1082056749856866
9)*sqrt(2*x^2 - x + 3) - 14641852251147/137438953472*sqrt(2)*log(-2*sqrt(2)
*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

# 3.73 $\int (3 - x + 2x^2)^{5/2} (2 + 3x + 5x^2)^3 \, dx$

**Optimal**. Leaf size=212

$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5+\frac{1175}{96}\left(2x^2-x+3\right)^{7/2}x^4+\frac{3823}{256}\left(2x^2-x+3\right)^{7/2}x^3+\frac{80483\left(2x^2-x+3\right)^{7/2}x^2}{9216}+\frac{509257\left(2x^2\right.}{294}$$

```
[Out] (-459555525*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1073741824 - (6660225*(1 - 4*x)*
(3 - x + 2*x^2)^(3/2))/67108864 - (57915*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/2
097152 - (1696165*(3 - x + 2*x^2)^(7/2))/2752512 + (509257*x*(3 - x + 2*x^2
)^(7/2))/294912 + (80483*x^2*(3 - x + 2*x^2)^(7/2))/9216 + (3823*x^3*(3 - x
 + 2*x^2)^(7/2))/256 + (1175*x^4*(3 - x + 2*x^2)^(7/2))/96 + (125*x^5*(3 -
x + 2*x^2)^(7/2))/24 - (10569777075*ArcSinh[(1 - 4*x)/Sqrt[23]])/(214748364
8*Sqrt[2])
```

---

**Rubi [A]** time = 0.219527, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{125}{24}\left(2x^2-x+3\right)^{7/2}x^5+\frac{1175}{96}\left(2x^2-x+3\right)^{7/2}x^4+\frac{3823}{256}\left(2x^2-x+3\right)^{7/2}x^3+\frac{80483\left(2x^2-x+3\right)^{7/2}x^2}{9216}+\frac{509257\left(2x^2\right.}{294}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```


```
[Out] (-459555525*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/1073741824 - (6660225*(1 - 4*x)*
(3 - x + 2*x^2)^(3/2))/67108864 - (57915*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/2
097152 - (1696165*(3 - x + 2*x^2)^(7/2))/2752512 + (509257*x*(3 - x + 2*x^2
)^(7/2))/294912 + (80483*x^2*(3 - x + 2*x^2)^(7/2))/9216 + (3823*x^3*(3 - x
 + 2*x^2)^(7/2))/256 + (1175*x^4*(3 - x + 2*x^2)^(7/2))/96 + (125*x^5*(3 -
x + 2*x^2)^(7/2))/24 - (10569777075*ArcSinh[(1 - 4*x)/Sqrt[23]])/(214748364
8*Sqrt[2])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right)^3 \, dx = \frac{125}{24}x^5 \left(3-x+2x^2\right)^{7/2} + \frac{1}{24}\int \left(3-x+2x^2\right)^{5/2} \left(192+864x+2736x^2+496$$

$$= \frac{1175}{96}x^4 \left(3-x+2x^2\right)^{7/2} + \frac{125}{24}x^5 \left(3-x+2x^2\right)^{7/2} + \frac{1}{528}\int \left(3-x+2x^2\right)^{5/2}$$

$$= \frac{3823}{256}x^3 \left(3-x+2x^2\right)^{7/2} + \frac{1175}{96}x^4 \left(3-x+2x^2\right)^{7/2} + \frac{125}{24}x^5 \left(3-x+2x^2\right)^{7/2}$$

$$= \frac{80483x^2 \left(3-x+2x^2\right)^{7/2}}{9216} + \frac{3823}{256}x^3 \left(3-x+2x^2\right)^{7/2} + \frac{1175}{96}x^4 \left(3-x+2x^2\right)$$

$$= \frac{509257x \left(3-x+2x^2\right)^{7/2}}{294912} + \frac{80483x^2 \left(3-x+2x^2\right)^{7/2}}{9216} + \frac{3823}{256}x^3 \left(3-x+2x^2\right.$$

$$= -\frac{1696165 \left(3-x+2x^2\right)^{7/2}}{2752512} + \frac{509257x \left(3-x+2x^2\right)^{7/2}}{294912} + \frac{80483x^2 \left(3-x+2\right.}{9216}$$

$$= -\frac{57915(1-4x) \left(3-x+2x^2\right)^{5/2}}{2097152} - \frac{1696165 \left(3-x+2x^2\right)^{7/2}}{2752512} + \frac{509257x \left(3-\right.}{2949}$$

$$= -\frac{6660225(1-4x) \left(3-x+2x^2\right)^{3/2}}{67108864} - \frac{57915(1-4x) \left(3-x+2x^2\right)^{5/2}}{2097152} - \frac{1696}{}$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x) \left(3-x+2x^2\right)^{3/2}}{67108864} - \frac{579}{}$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x) \left(3-x+2x^2\right)^{3/2}}{67108864} - \frac{579}{}$$

$$= -\frac{459555525(1-4x)\sqrt{3-x+2x^2}}{1073741824} - \frac{6660225(1-4x) \left(3-x+2x^2\right)^{3/2}}{67108864} - \frac{579}{}$$

**Mathematica [A]** time = 0.287949, size = 95, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(2818572288000x^{11}+2395786444800x^{10}+12943588589568x^9+14341894045696x^8+27835561148\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-1191399152715 + 4560943728924*x + 10060731582048*x
^2 + 20384824684416*x^3 + 26186527209472*x^4 + 34378613923840*x^5 + 2834753
8538496*x^6 + 27835561148416*x^7 + 14341894045696*x^8 + 12943588589568*x^9
+ 2395786444800*x^10 + 2818572288000*x^11) - 665895955725*Sqrt[2]*ArcSinh[(
1 - 4*x)/Sqrt[23]])/270582939648
```

**Maple [A]** time = 0.061, size = 170, normalized size = 0.8

$$\frac{125\,x^5}{24}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{1175\,x^4}{96}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{3823\,x^3}{256}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{80483\,x^2}{9216}\left(2\,x^2-x+3\right)^{\frac{7}{2}}+\frac{509257\,x}{294912}\left(2\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x)
```

```
[Out] 125/24*x^5*(2*x^2-x+3)^(7/2)+1175/96*x^4*(2*x^2-x+3)^(7/2)+3823/256*x^3*(2*
x^2-x+3)^(7/2)+80483/9216*x^2*(2*x^2-x+3)^(7/2)+509257/294912*x*(2*x^2-x+3)
^(7/2)+459555525/1073741824*(-1+4*x)*(2*x^2-x+3)^(1/2)+10569777075/42949672
96*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+57915/2097152*(-1+4*x)*(2*x^2-x+3
)^(5/2)+6660225/67108864*(-1+4*x)*(2*x^2-x+3)^(3/2)-1696165/2752512*(2*x^2-
x+3)^(7/2)
```

**Maxima [A]** time = 1.48024, size = 271, normalized size = 1.28

$$\frac{125}{24}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^5+\frac{1175}{96}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^4+\frac{3823}{256}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^3+\frac{80483}{9216}\left(2\,x^2-x+3\right)^{\frac{7}{2}}x^2+\frac{509257}{294912}\left(2\,x^2-x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] 125/24*(2*x^2 - x + 3)^(7/2)*x^5 + 1175/96*(2*x^2 - x + 3)^(7/2)*x^4 + 3823
/256*(2*x^2 - x + 3)^(7/2)*x^3 + 80483/9216*(2*x^2 - x + 3)^(7/2)*x^2 + 509
257/294912*(2*x^2 - x + 3)^(7/2)*x - 1696165/2752512*(2*x^2 - x + 3)^(7/2)
+ 57915/524288*(2*x^2 - x + 3)^(5/2)*x - 57915/2097152*(2*x^2 - x + 3)^(5/2
) + 6660225/16777216*(2*x^2 - x + 3)^(3/2)*x - 6660225/67108864*(2*x^2 - x
+ 3)^(3/2) + 459555525/268435456*sqrt(2*x^2 - x + 3)*x + 10569777075/429496
7296*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 459555525/1073741824*sqrt(2
*x^2 - x + 3)
```

**Fricas [A]** time = 1.35757, size = 512, normalized size = 2.42

$$\frac{1}{67645734912}\left(2818572288000\,x^{11}+2395786444800\,x^{10}+12943588589568\,x^9+14341894045696\,x^8+278355611484\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/67645734912*(2818572288000*x^11 + 2395786444800*x^10 + 12943588589568*x^9
 + 14341894045696*x^8 + 27835561148416*x^7 + 28347538538496*x^6 + 343786139
23840*x^5 + 26186527209472*x^4 + 20384824684416*x^3 + 10060731582048*x^2 +
```

```
4560943728924*x - 1191399152715)*sqrt(2*x^2 - x + 3) + 10569777075/85899345
92*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 2
5)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(2x^2 - x + 3\right)^{\frac{5}{2}} \left(5x^2 + 3x + 2\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3, x)
```

---

**Giac [A]** time = 1.15239, size = 139, normalized size = 0.66

$$\frac{1}{67645734912} (4 (8 (4 (16 (4 (8 (28 (32 (12 (200 (20 x + 17)x + 18369)x + 244241)x + 15169177)x + 432549111)x + 41$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] 1/67645734912*(4*(8*(4*(16*(4*(8*(28*(32*(12*(200*(20*x + 17)*x + 18369)*x
+ 244241)*x + 15169177)*x + 432549111)*x + 4196608145)*x + 12786390239)*x +
 159256442847)*x + 314397861939)*x + 1140235932231)*x - 1191399152715)*sqrt
(2*x^2 - x + 3) - 10569777075/4294967296*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x
- sqrt(2*x^2 - x + 3)) + 1)
```

## 3.74 $\int (3-x+2x^2)^{5/2} (2+3x+5x^2)^2 \, dx$

**Optimal**. Leaf size=170

$$\frac{5}{4}x^3 (2x^2-x+3)^{7/2} + \frac{305}{144}x^2 (2x^2-x+3)^{7/2} + \frac{8467x (2x^2-x+3)^{7/2}}{4608} + \frac{23225 (2x^2-x+3)^{7/2}}{43008} - \frac{1547(1-4x) (2x^2-}{98304}$$

```
[Out] (-4091815*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16777216 - (177905*(1 - 4*x)*(3 -
x + 2*x^2)^(3/2))/3145728 - (1547*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/98304 +
(23225*(3 - x + 2*x^2)^(7/2))/43008 + (8467*x*(3 - x + 2*x^2)^(7/2))/4608 +
 (305*x^2*(3 - x + 2*x^2)^(7/2))/144 + (5*x^3*(3 - x + 2*x^2)^(7/2))/4 - (9
4111745*ArcSinh[(1 - 4*x)/Sqrt[23]])/(33554432*Sqrt[2])
```

---

**Rubi [A]** time = 0.13038, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{4}x^3 (2x^2-x+3)^{7/2} + \frac{305}{144}x^2 (2x^2-x+3)^{7/2} + \frac{8467x (2x^2-x+3)^{7/2}}{4608} + \frac{23225 (2x^2-x+3)^{7/2}}{43008} - \frac{1547(1-4x) (2x^2-}{98304}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (-4091815*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/16777216 - (177905*(1 - 4*x)*(3 -
x + 2*x^2)^(3/2))/3145728 - (1547*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/98304 +
(23225*(3 - x + 2*x^2)^(7/2))/43008 + (8467*x*(3 - x + 2*x^2)^(7/2))/4608 +
 (305*x^2*(3 - x + 2*x^2)^(7/2))/144 + (5*x^3*(3 - x + 2*x^2)^(7/2))/4 - (9
4111745*ArcSinh[(1 - 4*x)/Sqrt[23]])/(33554432*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2\,dx = \frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1}{20}\int\left(3-x+2x^2\right)^{5/2}\left(80+240x+355x^2+\frac{1525x^3}{2}\right.$$

$$= \frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+\frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+\frac{1}{360}\int\left(3-x+2x^2\right)^{5/2}\left(14\right.$$

$$= \frac{8467x\left(3-x+2x^2\right)^{7/2}}{4608}+\frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+\frac{5}{4}x^3\left(3-x+2x^2\right)^{7/2}+$$

$$= \frac{23225\left(3-x+2x^2\right)^{7/2}}{43008}+\frac{8467x\left(3-x+2x^2\right)^{7/2}}{4608}+\frac{305}{144}x^2\left(3-x+2x^2\right)^{7/2}+$$

$$= -\frac{1547(1-4x)\left(3-x+2x^2\right)^{5/2}}{98304}+\frac{23225\left(3-x+2x^2\right)^{7/2}}{43008}+\frac{8467x\left(3-x+2\right.}{4608}$$

$$= -\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{1547(1-4x)\left(3-x+2x^2\right)^{5/2}}{98304}+\frac{23225\left(\right.}{}$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{1547(1}{}$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{1547(1}{}$$

$$= -\frac{4091815(1-4x)\sqrt{3-x+2x^2}}{16777216}-\frac{177905(1-4x)\left(3-x+2x^2\right)^{3/2}}{3145728}-\frac{1547(1}{}$$

**Mathematica [A]** time = 0.190448, size = 85, normalized size = 0.5

$$\frac{4\sqrt{2x^2-x+3}\left(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+57147467\right.}{42278584}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(14824182519 + 39533249652*x + 42992644128*x^2 + 778
72272000*x^3 + 57147467776*x^4 + 75389820928*x^5 + 26401898496*x^6 + 441631
37536*x^7 + 2055208960*x^8 + 10569646080*x^9) - 5929039935*Sqrt[2]*ArcSinh[
(1 - 4*x)/Sqrt[23]])/4227858432
```

---

**Maple [A]** time = 0.053, size = 136, normalized size = 0.8

$$\frac{5x^3}{4}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{305x^2}{144}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{8467x}{4608}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{-4091815+16367260x}{16777216}\sqrt{2x^2-x+3}+\frac{94}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x)
```

```
[Out] 5/4*x^3*(2*x^2-x+3)^(7/2)+305/144*x^2*(2*x^2-x+3)^(7/2)+8467/4608*x*(2*x^2-
x+3)^(7/2)+4091815/16777216*(-1+4*x)*(2*x^2-x+3)^(1/2)+94111745/67108864*2^
(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+1547/98304*(-1+4*x)*(2*x^2-x+3)^(5/2)+
177905/3145728*(-1+4*x)*(2*x^2-x+3)^(3/2)+23225/43008*(2*x^2-x+3)^(7/2)
```

---

**Maxima [A]** time = 1.47394, size = 225, normalized size = 1.32

$$\frac{5}{4}\left(2x^2-x+3\right)^{\frac{7}{2}}x^3+\frac{305}{144}\left(2x^2-x+3\right)^{\frac{7}{2}}x^2+\frac{8467}{4608}\left(2x^2-x+3\right)^{\frac{7}{2}}x+\frac{23225}{43008}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{1547}{24576}\left(2x^2-x+3\right)^{\frac{5}{2}}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*(2*x^2 - x + 3)^(7/2)*x^3 + 305/144*(2*x^2 - x + 3)^(7/2)*x^2 + 8467/46
08*(2*x^2 - x + 3)^(7/2)*x + 23225/43008*(2*x^2 - x + 3)^(7/2) + 1547/24576
*(2*x^2 - x + 3)^(5/2)*x - 1547/98304*(2*x^2 - x + 3)^(5/2) + 177905/786432
*(2*x^2 - x + 3)^(3/2)*x - 177905/3145728*(2*x^2 - x + 3)^(3/2) + 4091815/4
194304*sqrt(2*x^2 - x + 3)*x + 94111745/67108864*sqrt(2)*arcsinh(1/23*sqrt(
23)*(4*x - 1)) - 4091815/16777216*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.37715, size = 409, normalized size = 2.41

$$\frac{1}{1056964608}\left(10569646080x^9+2055208960x^8+44163137536x^7+26401898496x^6+75389820928x^5+5714746777\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/1056964608*(10569646080*x^9 + 2055208960*x^8 + 44163137536*x^7 + 26401898
496*x^6 + 75389820928*x^5 + 57147467776*x^4 + 77872272000*x^3 + 42992644128
*x^2 + 39533249652*x + 14824182519)*sqrt(2*x^2 - x + 3) + 94111745/13421772
8*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25
)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2, x)
```

---

**Giac [A]** time = 1.1628, size = 126, normalized size = 0.74

$$\frac{1}{1056964608}\,(4\,(8\,(4\,(16\,(4\,(8\,(28\,(160\,(36\,x+7)x+24067)x+402861)x+9202859)x+27904037)x+608377125)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] 1/1056964608*(4*(8*(4*(16*(4*(8*(28*(160*(36*x + 7)*x + 24067)*x + 402861)*
x + 9202859)*x + 27904037)*x + 608377125)*x + 1343520129)*x + 9883312413)*x
 + 14824182519)*sqrt(2*x^2 - x + 3) - 94111745/67108864*sqrt(2)*log(-2*sqrt
(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.75 $\int \left(3-x+2x^2\right)^{5/2} \left(2+3x+5x^2\right) dx$

**Optimal**. Leaf size=128

$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}+\frac{141}{448}\left(2x^2-x+3\right)^{7/2}-\frac{277(1-4x)\left(2x^2-x+3\right)^{5/2}}{3072}-\frac{31855(1-4x)\left(2x^2-x+3\right)^{3/2}}{98304}-\frac{732665(1}{}$$

```
[Out] (-732665*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/524288 - (31855*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/98304 - (277*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/3072 + (141*(3
- x + 2*x^2)^(7/2))/448 + (5*x*(3 - x + 2*x^2)^(7/2))/16 - (16851295*ArcSin
h[(1 - 4*x)/Sqrt[23]])/(1048576*Sqrt[2])
```

---

**Rubi [A]** time = 0.0624409, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1661, 640, 612, 619, 215}

$$\frac{5}{16}x\left(2x^2-x+3\right)^{7/2}+\frac{141}{448}\left(2x^2-x+3\right)^{7/2}-\frac{277(1-4x)\left(2x^2-x+3\right)^{5/2}}{3072}-\frac{31855(1-4x)\left(2x^2-x+3\right)^{3/2}}{98304}-\frac{732665(1}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```


```
[Out] (-732665*(1 - 4*x)*Sqrt[3 - x + 2*x^2])/524288 - (31855*(1 - 4*x)*(3 - x +
2*x^2)^(3/2))/98304 - (277*(1 - 4*x)*(3 - x + 2*x^2)^(5/2))/3072 + (141*(3
- x + 2*x^2)^(7/2))/448 + (5*x*(3 - x + 2*x^2)^(7/2))/16 - (16851295*ArcSin
h[(1 - 4*x)/Sqrt[23]])/(1048576*Sqrt[2])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)dx = \frac{5}{16}x\left(3-x+2x^2\right)^{7/2}+\frac{1}{16}\int \left(17+\frac{141x}{2}\right)\left(3-x+2x^2\right)^{5/2}dx$$

$$= \frac{141}{448}\left(3-x+2x^2\right)^{7/2}+\frac{5}{16}x\left(3-x+2x^2\right)^{7/2}+\frac{277}{128}\int \left(3-x+2x^2\right)^{5/2}dx$$

$$= -\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}+\frac{141}{448}\left(3-x+2x^2\right)^{7/2}+\frac{5}{16}x\left(3-x+2x^2\right)^{7/2}$$

$$= -\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1-4x)\left(3-x+2x^2\right)^{5/2}}{3072}+\frac{141}{448}\left(3-x\right.$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1-4x}{}$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1-4x}{}$$

$$= -\frac{732665(1-4x)\sqrt{3-x+2x^2}}{524288}-\frac{31855(1-4x)\left(3-x+2x^2\right)^{3/2}}{98304}-\frac{277(1-4x}{}$$

**Mathematica [A]** time = 0.107564, size = 75, normalized size = 0.59

$$\frac{4\sqrt{2x^2-x+3}\left(27525120x^7-13565952x^6+118808576x^5-1619968x^4+172684416x^3+67272352x^2+148957444x\right.}{44040192}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2),x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(58536675 + 148957444*x + 67272352*x^2 + 172684416*x
^3 - 1619968*x^4 + 118808576*x^5 - 13565952*x^6 + 27525120*x^7) - 353877195
*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/44040192
```

---

**Maple [A]** time = 0.049, size = 102, normalized size = 0.8

$$\frac{5x}{16}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{141}{448}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{-277+1108x}{3072}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{-31855+127420x}{98304}\left(2x^2-x+3\right)^{\frac{3}{2}}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x)
```

```
[Out] 5/16*x*(2*x^2-x+3)^(7/2)+141/448*(2*x^2-x+3)^(7/2)+277/3072*(-1+4*x)*(2*x^2
-x+3)^(5/2)+31855/98304*(-1+4*x)*(2*x^2-x+3)^(3/2)+732665/524288*(-1+4*x)*(
```

```
2*x^2-x+3)^(1/2)+16851295/2097152*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

---

**Maxima [A]** time = 1.43838, size = 180, normalized size = 1.41

$$\frac{5}{16}\left(2x^2-x+3\right)^{\frac{7}{2}}x+\frac{141}{448}\left(2x^2-x+3\right)^{\frac{7}{2}}+\frac{277}{768}\left(2x^2-x+3\right)^{\frac{5}{2}}x-\frac{277}{3072}\left(2x^2-x+3\right)^{\frac{5}{2}}+\frac{31855}{24576}\left(2x^2-x+3\right)^{\frac{3}{2}}x-\frac{3}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] 5/16*(2*x^2 - x + 3)^(7/2)*x + 141/448*(2*x^2 - x + 3)^(7/2) + 277/768*(2*x
^2 - x + 3)^(5/2)*x - 277/3072*(2*x^2 - x + 3)^(5/2) + 31855/24576*(2*x^2 -
 x + 3)^(3/2)*x - 31855/98304*(2*x^2 - x + 3)^(3/2) + 732665/131072*sqrt(2*
x^2 - x + 3)*x + 16851295/2097152*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
- 732665/524288*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.38597, size = 327, normalized size = 2.55

$$\frac{1}{11010048}\left(27525120x^7-13565952x^6+118808576x^5-1619968x^4+172684416x^3+67272352x^2+148957444x+5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/11010048*(27525120*x^7 - 13565952*x^6 + 118808576*x^5 - 1619968*x^4 + 172
684416*x^3 + 67272352*x^2 + 148957444*x + 58536675)*sqrt(2*x^2 - x + 3) + 1
6851295/4194304*sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x
^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)*(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2), x)
```

---

**Giac [A]** time = 1.14499, size = 112, normalized size = 0.88

$$\frac{1}{11010048}\left(4\left(8\left(4\left(16\left(4\left(24\left(140x-69\right)x+14503\right)x-791\right)x+1349097\right)x+2102261\right)x+37239361\right)x+58536675\right)\sqrt{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)*(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] 1/11010048*(4*(8*(4*(16*(4*(24*(140*x - 69)*x + 14503)*x - 791)*x + 1349097
)*x + 2102261)*x + 37239361)*x + 58536675)*sqrt(2*x^2 - x + 3) - 16851295/2
097152*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.76 $\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx$

**Optimal**. Leaf size=222

$$-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}-\frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000}-\frac{121\sqrt{\frac{11}{31}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\sqrt{\frac{11}{62(25000\sqrt{2}-15}}\right.}{3125}$$

```
[Out] -((226249 - 99620*x)*Sqrt[3 - x + 2*x^2])/80000 - ((103 - 60*x)*(3 - x + 2*
x^2)^(3/2))/600 - (7216203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(800000*Sqrt[2]) -
(121*Sqrt[(11*(-15457 + 25000*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(-15457 + 2
5000*Sqrt[2]))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/3125 + (121*Sqrt[(11*(15457 + 25000*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/
(62*(15457 + 25000*Sqrt[2]))]*(196 + 443*Sqrt[2] - (690 - 247*Sqrt[2])*x))/
Sqrt[3 - x + 2*x^2]])/3125
```

---

**Rubi [A]** time = 0.538982, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {977, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$-\frac{1}{600}(103-60x)\left(2x^2-x+3\right)^{3/2}-\frac{(226249-99620x)\sqrt{2x^2-x+3}}{80000}-\frac{121\sqrt{\frac{11}{31}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\sqrt{\frac{11}{62(25000\sqrt{2}-15}}\right.}{3125}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```


```
[Out] -((226249 - 99620*x)*Sqrt[3 - x + 2*x^2])/80000 - ((103 - 60*x)*(3 - x + 2*
x^2)^(3/2))/600 - (7216203*ArcSinh[(1 - 4*x)/Sqrt[23]])/(800000*Sqrt[2]) -
(121*Sqrt[(11*(-15457 + 25000*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(-15457 + 2
5000*Sqrt[2]))]*(196 - 443*Sqrt[2] - (690 + 247*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/3125 + (121*Sqrt[(11*(15457 + 25000*Sqrt[2]))/31]*ArcTanh[(Sqrt[11/
(62*(15457 + 25000*Sqrt[2]))]*(196 + 443*Sqrt[2] - (690 - 247*Sqrt[2])*x))/
Sqrt[3 - x + 2*x^2]])/3125
```


Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{5/2}}{2+3x+5x^2}\,dx = -\frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{1}{300}\int \frac{\left(-\frac{4731}{2}+\frac{6135x}{4}-\frac{14943x^2}{4}\right)\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{\frac{3205293}{8}-\frac{11339385x}{16}+\frac{2164860}{16}}{\sqrt{3-x+2x^2}(2+3x+5x^2)}}{30000}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} + \frac{\int \frac{-702768-7602672x}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx}{150000}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{\int \frac{-702768(108-11\sqrt{2})-70276}{\sqrt{3-x+2x^2}(2+3x}}{3300000}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{7216203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}}$$

$$= -\frac{(226249-99620x)\sqrt{3-x+2x^2}}{80000} - \frac{1}{600}(103-60x)\left(3-x+2x^2\right)^{3/2} - \frac{7216203\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{800000\sqrt{2}}$$

**Mathematica [C]** time = 1.06988, size = 229, normalized size = 1.03

$$\frac{620\sqrt{2x^2-x+3}\left(48000x^3-106400x^2+412060x-802347\right)+46464\sqrt{286+22i\sqrt{31}}\left(403-69i\sqrt{31}\right)\tanh^{-1}\left(\frac{(-22-4i\sqrt{}}{2\sqrt{286+22}}\right.}{148800}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2),x]
```

```
[Out] (620*Sqrt[3 - x + 2*x^2]*(-802347 + 412060*x - 106400*x^2 + 48000*x^3) + 67
1106879*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23]] + 46464*Sqrt[286 + (22*I)*Sqrt
[31]]*(403 - (69*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[
31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (46464*I)*Sq
rt[286 - (22*I)*Sqrt[31]]*(-403*I + 69*Sqrt[31])*ArcTanh[(-63 + I*Sqrt[31]
+ (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^
2])])/148800000
```

---

**Maple [B]** time = 0.179, size = 4860, normalized size = 21.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/5*x^3*(2*x^2-x+3)^(1/2)-133/300*x^2*(2*x^2-x+3)^(1/2)+20603/12000*x*(2*x^
2-x+3)^(1/2)-267449/80000*(2*x^2-x+3)^(1/2)+7216203/1600000*2^(1/2)*arcsinh
(4/23*23^(1/2)*(x-1/4))+4/33034375*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^
(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(75195*(-77
5687+549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/1169
2487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^
2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2
^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2
^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+106294*(-775687+549362*2^(1/2))^(1/2)*
(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)
*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1
/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^
(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x)
)+108099046*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)
^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2
^(1/2)-158290154*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)
-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/
2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*
2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1
+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+6/6606875*(8*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3
*2^(1/2))^(1/2)*2^(1/2)*(10915*(-775687+549362*2^(1/2))^(1/2)*2^(1/2)*(-886
6+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23
*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(
6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+149
18*(-775687+549362*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692
487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/
(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2
*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^
(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^
(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))-5052938*arctanh(31/2*(8*(2^(1/2)-1+x)^2
/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/
2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-51565338*arctanh(31/2*(8*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))
^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/
2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820
*2^(1/2))^(1/2)-21/1321375*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(4245*(-775687+5493
62*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-77
5687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)
+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+
x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(
2^(1/2)-1+x)/(2^(1/2)+1-x))+6154*(-775687+549362*2^(1/2))^(1/2)*(-8866+6820
*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*
2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^
2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)
^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+12325786*
arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+
```

```
1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-35941
4*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2
)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/
2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(
1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-
x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-37/528550*(8*(2^(1/2)-1+x)^2/(
2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)
*2^(1/2)*(2365*(-775687+549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^
(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(
-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^
(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2
)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+3338*(-775687+54936
2*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+5493
62*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+2
4*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(
1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1
+x)/(2^(1/2)+1-x))+3192442*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+
3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^
(1/2))^(1/2))*2^(1/2)-5264358*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)
^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820
*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^
(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/
(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)-63
/105710*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^
2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(285*(-775687+549362*2^(1/2))^(1/2)*2^
(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))
^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-4
1))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)
^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4
+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2
)+1-x))+386*(-775687+549362*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arcta
n(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016
)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23
)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))-274846*arctanh(31/2*(8*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1
/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-1543366*arctanh(31/2*(8*(2^(
1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2
^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x
)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)
/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-88
66+6820*2^(1/2))^(1/2)+27/21142*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/
2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(151*(-775687+
549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*
(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(
1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(
1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2
)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2
))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+218*(-775687+549362*2^(1/2))^(1/2)*(-8866+6
820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8
+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(648
5*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-
x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1
+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+401698
*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-6342
```

```
6*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2
)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/
2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(
1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-
x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+27/21142*(8*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*
2^(1/2)*(369*(-775687+549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1
/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-2
3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)
^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1
/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+
1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+520*(-775687+549362*2
^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*
2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2
^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2
)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)
/(2^(1/2)+1-x))+465124*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2
^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2
))^(1/2))*2^(1/2)-866822*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*
(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1
/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2
^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^2-x+3\right)^{\frac{5}{2}}}{5\,x^2+3\,x+2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2), x)
```

---

**Fricas [B]** time = 3.90656, size = 7899, normalized size = 35.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 121/96875000*6050^(1/4)*sqrt(31)*sqrt(2)*sqrt(-772850000*sqrt(2) + 25000000
00)*arctan(1/254496437500*(722441500000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) + 2300*(4*6
050^(3/4)*sqrt(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 127
2680*x^3 - 1081800*x^2 - sqrt(2)*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1
083344*x^4 + 393672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 104
3712) + 5*6050^(1/4)*sqrt(31)*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 11
1531424*x^4 + 156761280*x^3 - 168192000*x^2 - sqrt(2)*(712757*x^7 - 1023330
```

```
3*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417
152*x + 37407744) - 74815488*x + 106417152))*sqrt(2*x^2 - x + 3)*sqrt(-7728
50000*sqrt(2) + 2500000000) - sqrt(10/5711)*(314105000*sqrt(31)*sqrt(2)*(12
3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5
+ 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - (4*60
50^(3/4)*sqrt(31)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 369
3960*x^3 + 4195584*x^2 + 22*sqrt(2)*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4
464*x^4 - 49464*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sq
rt(31)*(160956*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*
x^3 - 296027136*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 -
 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x
))*sqrt(2*x^2 - x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqr
t(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x
^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6
 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt
((6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt
(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2)*(2*x^
2 - x + 3) - 431180500*x + 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x
^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096
*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*
x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 9488
7936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^
4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 121/96875000*60
50^(1/4)*sqrt(31)*sqrt(2)*sqrt(-772850000*sqrt(2) + 2500000000)*arctan(-1/2
54496437500*(722441500000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270
*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x
^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3961
44*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2300*(4*6050^(3/4)*sqrt
(31)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 108
1800*x^2 - sqrt(2)*(173702*x^7 - 453907*x^6 + 1056481*x^5 - 1083344*x^4 + 3
93672*x^3 + 152064*x^2 - 1043712*x + 259200) - 518400*x + 1043712) + 5*6050
^(1/4)*sqrt(31)*(317294*x^7 - 5870544*x^6 + 38857480*x^5 - 111531424*x^4 +
156761280*x^3 - 168192000*x^2 - sqrt(2)*(712757*x^7 - 10233303*x^6 + 485297
68*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 - 106417152*x + 374077
44) - 74815488*x + 106417152))*sqrt(2*x^2 - x + 3)*sqrt(-772850000*sqrt(2)
+ 2500000000) - sqrt(10/5711)*(314105000*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2
 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4
- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + (4*6050^(3/4)*sqrt(
31)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419
5584*x^2 + 22*sqrt(2)*(37846*x^7 - 52859*x^6 + 160569*x^5 - 4464*x^4 - 4946
4*x^3 + 202176*x^2 - 202176*x) - 4195584*x) - 5*6050^(1/4)*sqrt(31)*(160956
*x^7 - 2232176*x^6 + 11218640*x^5 - 38096640*x^4 + 139374720*x^3 - 29602713
6*x^2 - sqrt(2)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4
 + 180496224*x^3 + 376648704*x^2 - 376648704*x) + 296027136*x))*sqrt(2*x^2
- x + 3)*sqrt(-772850000*sqrt(2) + 2500000000) + 14277500*sqrt(31)*(254591*
x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*
x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6050^(1/4)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 888)*sqrt(-772850000*s
qrt(2) + 2500000000) - 139919500*x^2 - 125642000*sqrt(2)*(2*x^2 - x + 3) +
431180500*x - 571100000)/x^2) + 8209562500*sqrt(31)*(2828123*x^8 - 9696916*
x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 379814
40*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x
^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(25851
91*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*
x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 121/2213012500000*6050^(1/4)
*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(91506250
```

```
00/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*x - 88
8)*sqrt(-772850000*sqrt(2) + 2500000000) + 139919500*x^2 + 125642000*sqrt(2
)*(2*x^2 - x + 3) - 431180500*x + 571100000)/x^2) + 121/2213012500000*6050^
(1/4)*(15457*sqrt(2) + 50000)*sqrt(-772850000*sqrt(2) + 2500000000)*log(-91
50625000/5711*(6050^(1/4)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(163*x - 725) + 562*
x - 888)*sqrt(-772850000*sqrt(2) + 2500000000) - 139919500*x^2 - 125642000*
sqrt(2)*(2*x^2 - x + 3) + 431180500*x - 571100000)/x^2) + 1/240000*(48000*x
^3 - 106400*x^2 + 412060*x - 802347)*sqrt(2*x^2 - x + 3) + 7216203/3200000*
sqrt(2)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{5}{2}}}{5x^2 + 3x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.77 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2} \, dx$

**Optimal**. Leaf size=255

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)}+\frac{4}{155}(4-5x)(2x^2-x+3)^{3/2}-\frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750}+\frac{11\sqrt{\frac{11}{31}}(224510383+194487$$

```
[Out] -((1277 + 2240*x)*Sqrt[3 - x + 2*x^2])/7750 + (4*(4 - 5*x)*(3 - x + 2*x^2)^
(3/2))/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) - (4
799*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2500*Sqrt[2]) + (11*Sqrt[(11*(224510383 +
 194487500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*Sqrt[2]
))]*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/38750 - (11*Sqrt[(11*(-224510383 + 194487500*Sqrt[2]))/31]*ArcTanh[(Sqr
t[11/(62*(-224510383 + 194487500*Sqrt[2]))]*(21136 - 33287*Sqrt[2] + (87710
 - 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/38750
```

---

**Rubi [A]** time = 0.659844, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {971, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{31(5x^2+3x+2)}+\frac{4}{155}(4-5x)(2x^2-x+3)^{3/2}-\frac{(2240x+1277)\sqrt{2x^2-x+3}}{7750}+\frac{11\sqrt{\frac{11}{31}}(224510383+194487$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```


```
[Out] -((1277 + 2240*x)*Sqrt[3 - x + 2*x^2])/7750 + (4*(4 - 5*x)*(3 - x + 2*x^2)^
(3/2))/155 + ((3 + 10*x)*(3 - x + 2*x^2)^(5/2))/(31*(2 + 3*x + 5*x^2)) - (4
799*ArcSinh[(1 - 4*x)/Sqrt[23]])/(2500*Sqrt[2]) + (11*Sqrt[(11*(224510383 +
 194487500*Sqrt[2]))/31]*ArcTan[(Sqrt[11/(62*(224510383 + 194487500*Sqrt[2]
))]*(21136 + 33287*Sqrt[2] + (87710 + 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/38750 - (11*Sqrt[(11*(-224510383 + 194487500*Sqrt[2]))/31]*ArcTanh[(Sqr
t[11/(62*(-224510383 + 194487500*Sqrt[2]))]*(21136 - 33287*Sqrt[2] + (87710
 - 54423*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/38750
```


Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
```

```
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
```

```
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^2}\,dx = \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \frac{1}{31}\int \frac{(3-x+2x^2)^{3/2}\left(-\frac{75}{2}+15x+80x^2\right)}{2+3x+5x^2}\,dx$$

$$= \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} + \frac{\int \frac{(87660-54300x-53760x^2)\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx}{18600}$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \int$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - \int$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} + \int$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - 47$$

$$= -\frac{(1277+2240x)\sqrt{3-x+2x^2}}{7750} + \frac{4}{155}(4-5x)(3-x+2x^2)^{3/2} + \frac{(3+10x)(3-x+2x^2)^{5/2}}{31(2+3x+5x^2)} - 47$$

**Mathematica [C]** time = 1.69943, size = 685, normalized size = 2.69

$$-\frac{-1922000\sqrt{2x^2-x+3}x^3+7784100\sqrt{2x^2-x+3}x^2-5759180\sqrt{2x^2-x+3}x-5577520\sqrt{2x^2-x+3}-4611839\sqrt{2}(5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^2,x]
```

```
[Out] -(-5577520*Sqrt[3 - x + 2*x^2] - 5759180*x*Sqrt[3 - x + 2*x^2] + 7784100*x^
2*Sqrt[3 - x + 2*x^2] - 1922000*x^3*Sqrt[3 - x + 2*x^2] - 4611839*Sqrt[2]*(
2 + 3*x + 5*x^2)*ArcSinh[(-1 + 4*x)/Sqrt[23]] + (11*I)*Sqrt[286 + (22*I)*Sq
rt[31]]*(5177*I + 8771*Sqrt[31])*(2 + 3*x + 5*x^2)*ArcTanh[(63 + I*Sqrt[31]
 - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x
^2])] + (192962*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*Sqrt[31] +
22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2]
```

```
)] + 113894*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[31] + 22*x -
(4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (2
89443*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (
4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 170
841*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*
Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + (482405*
I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)
*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] + 284735*
Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31] + 22*x - (4*I)*Sq
rt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(4805000*(2
 + 3*x + 5*x^2))
```

---

**Maple [B]** time = 0.414, size = 40028, normalized size = 157.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)

[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2\,x^{2} - x + 3\right)^{\frac{5}{2}}}{\left(5\,x^{2} + 3\,x + 2\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")

[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^2, x)
```

---

**Fricas [B]** time = 5.25698, size = 9335, normalized size = 36.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")

[Out] 1/13227599224357079000000*(38925001324*1464599010050^(1/4)*sqrt(155590)*sqrt
(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) + 388975000)*arctan(1
/29697544706386636381999587 5*(110935670*sqrt(155590)*(4*1464599010050^(3/4)
*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5 - 298338397*x^4 + 15
6222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62947113*x^6 + 1354639
29*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 - 122404608*x + 714528
00) - 142905600*x + 122404608) + 2411645*1464599010050^(1/4)*sqrt(62)*(3035
566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4 + 807721920*x^3 - 57
9888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204950152*x^5 - 4510041
40*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x + 268655616) - 5373112
```

32*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000)
+ 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 70
4270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(87
46*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 +
396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(77795/920561)*
(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374*x^7 - 85793239*x^6
+ 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 601344*x^2 - 4*sqrt(2)*(1
7889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^4 + 15593832*x^3 + 5
8564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599010050^(1/4)*sqrt(62
)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984960*x^4 + 949582080*
x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036*x^6 + 505509740*x^5
 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 725677056*x) - 183223296
*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 388975000) + 759924265600
1778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x
^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^
7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 103
6800*x) + 3276288*x) + 345420120727353550*sqrt(31)*(254591*x^8 - 4815126*x^
7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*
x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*
x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(1464599010050^(1/4)*sqrt(15
5590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 387
58*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 6744561519183110*x^2 -
6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 2752882252
7278000)/x^2) + 9579853131092463349032125*sqrt(31)*(2828123*x^8 - 9696916*x
^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3798144
0*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^
4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(258519
1*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x
^3 - 34615296*x^2 - 24772608*x + 18579456)) + 38925001324*1464599010050^(1/
4)*sqrt(155590)*sqrt(62)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(224510383*sqrt(2) +
 388975000)*arctan(1/296975447063866363819995875*(110935670*sqrt(155590)*(4
*1464599010050^(3/4)*sqrt(62)*(18997882*x^7 - 82713851*x^6 + 169131062*x^5
- 298338397*x^4 + 156222120*x^3 - 89116200*x^2 - sqrt(2)*(18111018*x^7 - 62
947113*x^6 + 135463929*x^5 - 197908246*x^4 + 94500248*x^3 - 34095024*x^2 -
122404608*x + 71452800) - 142905600*x + 122404608) + 2411645*1464599010050^
(1/4)*sqrt(62)*(3035566*x^7 - 47612316*x^6 + 259553720*x^5 - 615321136*x^4
+ 807721920*x^3 - 579888000*x^2 - sqrt(2)*(2643323*x^7 - 39854517*x^6 + 204
950152*x^5 - 451004140*x^4 + 573424416*x^3 - 311722272*x^2 - 434377728*x +
268655616) - 537311232*x + 434377728))*sqrt(2*x^2 - x + 3)*sqrt(224510383*s
qrt(2) + 388975000) - 843027075536136774714827000*sqrt(31)*sqrt(2)*(28180*x
^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 984
96*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710
*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) -
 sqrt(77795/920561)*(sqrt(155590)*(4*1464599010050^(3/4)*sqrt(62)*(58767374
*x^7 - 85793239*x^6 + 285539949*x^5 - 168939120*x^4 + 253241640*x^3 + 60134
4*x^2 - 4*sqrt(2)*(17889302*x^7 - 25424283*x^6 + 80174553*x^5 - 21241168*x^
4 + 15593832*x^3 + 58564512*x^2 - 58564512*x) - 601344*x) + 2411645*1464599
010050^(1/4)*sqrt(62)*(9891184*x^7 - 128099264*x^6 + 496592960*x^5 - 666984
960*x^4 + 949582080*x^3 + 183223296*x^2 - sqrt(2)*(10181049*x^7 - 131588036
*x^6 + 505509740*x^5 - 637596864*x^4 + 754818336*x^3 + 725677056*x^2 - 7256
77056*x) - 183223296*x))*sqrt(2*x^2 - x + 3)*sqrt(224510383*sqrt(2) + 38897
5000) - 7599242656001778100*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 157
8888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(1
5550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
 + 1209600*x^2 - 1036800*x) + 3276288*x) - 345420120727353550*sqrt(31)*(254
591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219
328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^
5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((14645990
10050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(97

```
33*x + 29025) - 38758*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) + 6744
561519183110*x^2 + 6056340956001160*sqrt(2)*(2*x^2 - x + 3) - 2078426100809
4890*x + 27528822527278000)/x^2) - 957985313109246334903212 5*sqrt(31)*(2828
123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493
00096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1
0070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x -
 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 135629
44*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 11*1464599
010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*(1944875000*x^2 - 224510383*sqr
t(2)*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*sqrt(224510383*sqrt(2) +
 388975000)*log(14708117187500/920561*(1464599010050^(1/4)*sqrt(155590)*sqr
t(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 29025) - 38758*x + 19
292)*sqrt(224510383*sqrt(2) + 388975000) + 6744561519183110*x^2 + 605634095
6001160*sqrt(2)*(2*x^2 - x + 3) - 20784261008094890*x + 27528822527278000)/
x^2) - 11*1464599010050^(1/4)*sqrt(155590)*sqrt(62)*sqrt(31)*(1944875000*x^
2 - 224510383*sqrt(2)*(5*x^2 + 3*x + 2) + 1166925000*x + 777950000)*sqrt(22
4510383*sqrt(2) + 388975000)*log(-14708117187500/920561*(1464599010050^(1/4
)*sqrt(155590)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(9733*x + 290
25) - 38758*x + 19292)*sqrt(224510383*sqrt(2) + 388975000) - 67445615191831
10*x^2 - 6056340956001160*sqrt(2)*(2*x^2 - x + 3) + 20784261008094890*x - 2
7528822527278000)/x^2) + 6347924867768962 21210*sqrt(2)*(5*x^2 + 3*x + 2)*lo
g(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 17067869
9669123600*(3100*x^3 - 12555*x^2 + 9289*x + 8996)*sqrt(2*x^2 - x + 3))/(5*x
^2 + 3*x + 2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{5}{2}}}{\left(5x^2 + 3x + 2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**2, x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.78 $\int \frac{(3-x+2x^2)^{5/2}}{(2+3x+5x^2)^3} dx$

**Optimal**. Leaf size=281

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{62(5x^2+3x+2)^2}+\frac{(2336x+769)(2x^2-x+3)^{3/2}}{3844(5x^2+3x+2)}+\frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050}+\frac{\sqrt{11(1+4\sqrt{2})}(293734}{}$$

```
[Out] ((11359 - 12920*x)*Sqrt[3 - x + 2*x^2])/48050 + ((3 + 10*x)*(3 - x + 2*x^2)
^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + ((769 + 2336*x)*(3 - x + 2*x^2)^(3/2))/(
3844*(2 + 3*x + 5*x^2)) - (4*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/125 + (Sq
rt[11*(1 + 4*Sqrt[2])]*(2937349 + 1978861*Sqrt[2])*ArcTan[(Sqrt[11/(62*(353
1015707557 + 2498852071250*Sqrt[2]))]*(3957722 + 2937349*Sqrt[2] + (9832420
 + 6895071*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000 - ((2937349 - 197886
1*Sqrt[2])*Sqrt[11*(-1 + 4*Sqrt[2])]*ArcTanh[(Sqrt[11/(62*(-3531015707557 +
 2498852071250*Sqrt[2]))]*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*S
qrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000
```

---

**Rubi [A]** time = 0.654547, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.37, Rules used = {971, 1054, 1066, 1076, 619, 215, 1035, 1029, 206, 204}

$$\frac{(10x+3)(2x^2-x+3)^{5/2}}{62(5x^2+3x+2)^2}+\frac{(2336x+769)(2x^2-x+3)^{3/2}}{3844(5x^2+3x+2)}+\frac{(11359-12920x)\sqrt{2x^2-x+3}}{48050}+\frac{\sqrt{11(1+4\sqrt{2})}(293734}{}$$

Antiderivative was successfully verified.

```
[In] Int[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```


```
[Out] ((11359 - 12920*x)*Sqrt[3 - x + 2*x^2])/48050 + ((3 + 10*x)*(3 - x + 2*x^2)
^(5/2))/(62*(2 + 3*x + 5*x^2)^2) + ((769 + 2336*x)*(3 - x + 2*x^2)^(3/2))/(
3844*(2 + 3*x + 5*x^2)) - (4*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/125 + (Sq
rt[11*(1 + 4*Sqrt[2])]*(2937349 + 1978861*Sqrt[2])*ArcTan[(Sqrt[11/(62*(353
1015707557 + 2498852071250*Sqrt[2]))]*(3957722 + 2937349*Sqrt[2] + (9832420
 + 6895071*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000 - ((2937349 - 197886
1*Sqrt[2])*Sqrt[11*(-1 + 4*Sqrt[2])]*ArcTanh[(Sqrt[11/(62*(-3531015707557 +
 2498852071250*Sqrt[2]))]*(3957722 - 2937349*Sqrt[2] + (9832420 - 6895071*S
qrt[2])*x))/Sqrt[3 - x + 2*x^2]])/29791000
```


Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1054

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((A*b*c - 2
*a*B*c + a*b*C - (c*(b*B - 2*A*c) - C*(b^2 - 2*a*c))*x)*(a + b*x + c*x^2)^(
p + 1)*(d + e*x + f*x^2)^q)/(c*(b^2 - 4*a*c)*(p + 1)), x] - Dist[1/(c*(b^2
- 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*
Simp[e*q*(A*b*c - 2*a*B*c + a*b*C) - d*(c*(b*B - 2*A*c)*(2*p + 3) + C*(2*a*
c - b^2*(p + 2))) + (2*f*q*(A*b*c - 2*a*B*c + a*b*C) - e*(c*(b*B - 2*A*c)*(
2*p + q + 3) + C*(2*a*c*(q + 1) - b^2*(p + q + 2))))*x - f*(c*(b*B - 2*A*c)
*(2*p + 2*q + 3) + C*(2*a*c*(2*q + 1) - b^2*(p + 2*q + 2)))*x^2, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(3-x+2x^2\right)^{5/2}}{\left(2+3x+5x^2\right)^3}\,dx = \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} - \frac{1}{62}\int \frac{\left(3-x+2x^2\right)^{3/2}\left(-\frac{195}{2}+35x+40x^2\right)}{\left(2+3x+5x^2\right)^2}\,dx$$

$$= \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)} + \frac{\int \frac{\left(\frac{66735}{4}-7375x-25840x^2\right)\sqrt{3-x+2x^2}}{2+3x+5x^2}\,dx}{9610}$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)} -$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)} -$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)} +$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)} -$$

$$= \frac{(11359-12920x)\sqrt{3-x+2x^2}}{48050} + \frac{(3+10x)\left(3-x+2x^2\right)^{5/2}}{62\left(2+3x+5x^2\right)^2} + \frac{(769+2336x)\left(3-x+2x^2\right)^{3/2}}{3844\left(2+3x+5x^2\right)} -$$

**Mathematica [C]** time = 2.11623, size = 1009, normalized size = 3.59

$$12599950\sqrt{286-22i\sqrt{31}}\tanh^{-1}\left(\frac{-4i\sqrt{31}x+22x+i\sqrt{31}-63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)x^4-12290525i\sqrt{682\left(13-i\sqrt{31}\right)}\tanh^{-1}\left(\frac{-4i\sqrt{31}x+22x+i\sqrt{31}-}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 - x + 2*x^2)^(5/2)/(2 + 3*x + 5*x^2)^3,x]
```

```
[Out] (153804640*Sqrt[3 - x + 2*x^2] + 474815220*x*Sqrt[3 - x + 2*x^2] + 64020704
0*x^2*Sqrt[3 - x + 2*x^2] + 662597100*x^3*Sqrt[3 - x + 2*x^2] + 1906624*Sqr
t[2]*(2 + 3*x + 5*x^2)^2*ArcSinh[(-1 + 4*x)/Sqrt[23]] - I*Sqrt[286 + (22*I)
*Sqrt[31]]*(-503998*I + 491621*Sqrt[31])*(2 + 3*x + 5*x^2)^2*ArcTanh[(63 +
I*Sqrt[31] - 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
 - x + 2*x^2])] - (1966484*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(-63 + I*
Sqrt[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
 x + 2*x^2])] + 2015992*Sqrt[286 - (22*I)*Sqrt[31]]*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] - (5899452*I)*Sqrt[682*(13 - I*Sqrt[31])]*x*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] + 6047976*Sqrt[286 - (22*I)*Sqrt[31]]*x*ArcTanh[(-63 + I*Sqrt[31] +
 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2
])] - (14257009*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^2*ArcTanh[(-63 + I*Sqrt[31
] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])] + 14615942*Sqrt[286 - (22*I)*Sqrt[31]]*x^2*ArcTanh[(-63 + I*Sqrt[31]
 + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x
^2])] - (14748630*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^3*ArcTanh[(-63 + I*Sqrt[
31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
2*x^2])] + 15119940*Sqrt[286 - (22*I)*Sqrt[31]]*x^3*ArcTanh[(-63 + I*Sqrt[3
1] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])] - (12290525*I)*Sqrt[682*(13 - I*Sqrt[31])]*x^4*ArcTanh[(-63 + I*Sqr
t[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x
+ 2*x^2])] + 12599950*Sqrt[286 - (22*I)*Sqrt[31]]*x^4*ArcTanh[(-63 + I*Sqrt
[31] + 22*x - (4*I)*Sqrt[31]*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
 2*x^2])])/(59582000*(2 + 3*x + 5*x^2)^2)
```

---

**Maple [B]** time = 0.69, size = 119458, normalized size = 425.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2-x+3\right)^{\frac{5}{2}}}{\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")

[Out] integrate((2*x^2 - x + 3)^(5/2)/(5*x^2 + 3*x + 2)^3, x)
```

---

**Fricas [B]** time = 5.49537, size = 10927, normalized size = 38.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")

[Out] 1/758714159921174808909075728000*(3184949732636*3868444992270541948232^(1/4
)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*s
qrt(3531015707557*sqrt(2) + 4997704142500)*arctan(1/45354848806294031039917
89624695893204150231*(2850690442882*sqrt(1999081657)*(2*3868444992270541948
232^(3/4)*sqrt(62)*(2627559914*x^7 - 10187615527*x^6 + 21362956024*x^5 - 34
451465819*x^4 + 17321103240*x^3 - 8320757400*x^2 - sqrt(2)*(1893366636*x^7
- 7237484076*x^6 + 15226003533*x^5 - 24262105817*x^4 + 12127036096*x^3 - 56
64787848*x^2 - 13367586816*x + 9338025600) - 18676051200*x + 13367586816) +
 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(400116332*x^7 - 6149336
082*x^6 + 32552996440*x^5 - 74427496472*x^4 + 96235107840*x^3 - 61219656000
*x^2 - sqrt(2)*(286685371*x^7 - 4395067059*x^6 + 23180544704*x^5 - 52748573
780*x^4 + 68065744032*x^3 - 42544702944*x^2 - 48625837056*x + 34092306432)
- 68184612864*x + 48625837056))*sqrt(2*x^2 - x + 3)*sqrt(3531015707557*sqrt
(2) + 4997704142500) + 1287492482243185397262185441849156780532968 8*sqrt(31
)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4
- 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 78
3113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 115
4304*x - 456192) - sqrt(1999081657/828550919)*(sqrt(1999081657)*(2*38684449
92270541948232^(3/4)*sqrt(62)*(9351066298*x^7 - 13433496653*x^6 + 433103458
23*x^5 - 17374572240*x^4 + 20927636280*x^3 + 18483199488*x^2 - sqrt(2)*(683
9273266*x^7 - 9809465289*x^6 + 31524099699*x^5 - 12024617744*x^4 + 13914887
256*x^3 + 14839341696*x^2 - 14839341696*x) - 18483199488*x) + 61971531367*3
868444992270541948232^(1/4)*sqrt(62)*(1427210918*x^7 - 18462714328*x^6 + 71
210222920*x^5 - 92387041920*x^4 + 119489780160*x^3 + 68726817792*x^2 - sqrt
(2)*(1033310523*x^7 - 13365477772*x^6 + 51521534980*x^5 - 66583614528*x^4 +
 85122955872*x^3 + 53108877312*x^2 - 53108877312*x) - 68726817792*x))*sqrt(
2*x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 45164233298567
21284677540671884*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 -
 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 -
 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600
*x^2 - 1036800*x) + 3276288*x) + 2052919695389418765762518487 22*sqrt(31)*(2
54591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 742
19328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*
x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(38684
44992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(2141441*x + 1076175) - 3217616*x - 1065266)*sqrt(353101570755
7*sqrt(2) + 4997704142500) - 155990877430002205517374*x^2 - 140073440957553
000872744*sqrt(2)*(2*x^2 - x + 3) + 480706581467965980267826*x - 6366974588
97968185785200)/x^2) + 1463059638912710678707028911192223614242 01*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 31849
49732636*3868444992270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(2)*(25
```

*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(3531015707557*sqrt(2) + 49977041425
00)*arctan(1/4535484880629403103991789624695893204150231*(2850690442882*sqr
t(1999081657)*(2*3868444992270541948232^(3/4)*sqrt(62)*(2627559914*x^7 - 10
187615527*x^6 + 21362956024*x^5 - 34451465819*x^4 + 17321103240*x^3 - 83207
57400*x^2 - sqrt(2)*(1893366636*x^7 - 7237484076*x^6 + 15226003533*x^5 - 24
262105817*x^4 + 12127036096*x^3 - 5664787848*x^2 - 13367586816*x + 93380256
00) - 18676051200*x + 13367586816) + 61971531367*3868444992270541948232^(1/
4)*sqrt(62)*(400116332*x^7 - 6149336082*x^6 + 32552996440*x^5 - 74427496472
*x^4 + 96235107840*x^3 - 61219656000*x^2 - sqrt(2)*(286685371*x^7 - 4395067
059*x^6 + 23180544704*x^5 - 52748573780*x^4 + 68065744032*x^3 - 42544702944
*x^2 - 48625837056*x + 34092306432) - 68184612864*x + 48625837056))*sqrt(2*
x^2 - x + 3)*sqrt(3531015707557*sqrt(2) + 4997704142500) - 1287492482243185
39726218544184915678053296888*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704
270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(874
6*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 3
96144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - sqrt(1999081657/8285
50919)*(sqrt(1999081657)*(2*3868444992270541948232^(3/4)*sqrt(62)*(93510662
98*x^7 - 13433496653*x^6 + 43310345823*x^5 - 17374572240*x^4 + 20927636280*
x^3 + 18483199488*x^2 - sqrt(2)*(6839273266*x^7 - 9809465289*x^6 + 31524099
699*x^5 - 12024617744*x^4 + 13914887256*x^3 + 14839341696*x^2 - 14839341696
*x) - 18483199488*x) + 61971531367*3868444992270541948232^(1/4)*sqrt(62)*(1
427210918*x^7 - 18462714328*x^6 + 71210222920*x^5 - 92387041920*x^4 + 11948
9780160*x^3 + 68726817792*x^2 - sqrt(2)*(1033310523*x^7 - 13365477772*x^6 +
 51521534980*x^5 - 66583614528*x^4 + 85122955872*x^3 + 53108877312*x^2 - 53
108877312*x) - 68726817792*x))*sqrt(2*x^2 - x + 3)*sqrt(3531015707557*sqrt(
2) + 4997704142500) - 45164233298567212846775406718884*sqrt(31)*sqrt(2)*(123
408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 +
 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 205291
969538941876576251848722*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6
 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(
2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 -
1944*x) + 144820224*x))*sqrt((3868444992270541948232^(1/4)*sqrt(1999081657)
*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2141441*x + 1076175) - 321
7616*x - 1065266)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 15599087743
0002205517374*x^2 + 140073440957553000872744*sqrt(2)*(2*x^2 - x + 3) - 4807
06581467965980267826*x + 636697458897968185785200)/x^2) - 14630596389127106
787070289111922236142420l*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^
6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqr
t(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1
080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*
x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^
2 - 24772608*x + 18579456)) + 3868444992270541948232^(1/4)*sqrt(1999081657)
*sqrt(62)*sqrt(31)*(124942603562500*x^4 + 149931124275000*x^3 + 14493342013
2500*x^2 - 3531015707557*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 59
972449710000*x + 19990816570000)*sqrt(3531015707557*sqrt(2) + 4997704142500
)*log(3123565089062500/828550919*(3868444992270541948232^(1/4)*sqrt(1999081
657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2141441*x + 1076175) -
 3217616*x - 1065266)*sqrt(3531015707557*sqrt(2) + 4997704142500) + 1559908
77430002205517374*x^2 + 140073440957553000872744*sqrt(2)*(2*x^2 - x + 3) -
480706581467965980267826*x + 636697458897968185785200)/x^2) - 3868444992270
541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*(124942603562500*x^4 + 1
49931124275000*x^3 + 144933420132500*x^2 - 3531015707557*sqrt(2)*(25*x^4 +
30*x^3 + 29*x^2 + 12*x + 4) + 59972449710000*x + 19990816570000)*sqrt(35310
15707557*sqrt(2) + 4997704142500)*log(-3123565089062500/828550919*(38684449
92270541948232^(1/4)*sqrt(1999081657)*sqrt(62)*sqrt(31)*sqrt(2*x^2 - x + 3)
*(sqrt(2)*(2141441*x + 1076175) - 3217616*x - 1065266)*sqrt(3531015707557*s
qrt(2) + 4997704142500) - 155990877430002205517374*x^2 - 140073440957553000
872744*sqrt(2)*(2*x^2 - x + 3) + 480706581467965980267826*x - 6366974588979

```
68185785200)/x^2) + 1213942655873879694254521648*sqrt(2)*(25*x^4 + 30*x^3
+ 29*x^2 + 12*x + 4)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2
+ 16*x - 25) + 8684553339368286054110128O*(97155*x^3 + 93872*x^2 + 69621*x
+ 22552)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(2x^2 - x + 3\right)^{\frac{5}{2}}}{\left(5x^2 + 3x + 2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral((2*x**2 - x + 3)**(5/2)/(5*x**2 + 3*x + 2)**3, x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.79 $\int \frac{(2+3x+5x^2)^4}{\sqrt{3-x+2x^2}} dx$

**Optimal**. Leaf size=185

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7+\frac{57375}{448}\sqrt{2x^2-x+3}x^6+\frac{2116475\sqrt{2x^2-x+3}x^5}{10752}+\frac{686531\sqrt{2x^2-x+3}x^4}{6144}-\frac{19750457\sqrt{2x^2-}}{229376}$$

```
[Out] (16493087661*Sqrt[3 - x + 2*x^2])/29360128 + (1572007407*x*Sqrt[3 - x + 2*x
^2])/7340032 - (15428243*x^2*Sqrt[3 - x + 2*x^2])/131072 - (19750457*x^3*Sq
rt[3 - x + 2*x^2])/229376 + (686531*x^4*Sqrt[3 - x + 2*x^2])/6144 + (211647
5*x^5*Sqrt[3 - x + 2*x^2])/10752 + (57375*x^6*Sqrt[3 - x + 2*x^2])/448 + (6
25*x^7*Sqrt[3 - x + 2*x^2])/16 + (2899366573*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
8388608*Sqrt[2])
```

---

**Rubi [A]** time = 0.312208, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 619, 215}

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7+\frac{57375}{448}\sqrt{2x^2-x+3}x^6+\frac{2116475\sqrt{2x^2-x+3}x^5}{10752}+\frac{686531\sqrt{2x^2-x+3}x^4}{6144}-\frac{19750457\sqrt{2x^2-}}{229376}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```


```
[Out] (16493087661*Sqrt[3 - x + 2*x^2])/29360128 + (1572007407*x*Sqrt[3 - x + 2*x
^2])/7340032 - (15428243*x^2*Sqrt[3 - x + 2*x^2])/131072 - (19750457*x^3*Sq
rt[3 - x + 2*x^2])/229376 + (686531*x^4*Sqrt[3 - x + 2*x^2])/6144 + (211647
5*x^5*Sqrt[3 - x + 2*x^2])/10752 + (57375*x^6*Sqrt[3 - x + 2*x^2])/448 + (6
25*x^7*Sqrt[3 - x + 2*x^2])/16 + (2899366573*ArcSinh[(1 - 4*x)/Sqrt[23]])/(
8388608*Sqrt[2])
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\sqrt{3-x+2x^2}}\,dx = \frac{625}{16}x^7\sqrt{3-x+2x^2}+\frac{1}{16}\int \frac{256+1536x+6016x^2+14976x^3+28176x^4+37440x^5+24475x^6}{\sqrt{3-x+2x^2}}$$

$$= \frac{57375}{448}x^6\sqrt{3-x+2x^2}+\frac{625}{16}x^7\sqrt{3-x+2x^2}+\frac{1}{224}\int \frac{3584+21504x+84224x^2+209664x^3}{\sqrt{3-x+2}}$$

$$= \frac{2116475x^5\sqrt{3-x+2x^2}}{10752}+\frac{57375}{448}x^6\sqrt{3-x+2x^2}+\frac{625}{16}x^7\sqrt{3-x+2x^2}+\frac{\int \frac{43008+258048x+10106}{}}{}$$

$$= \frac{686531x^4\sqrt{3-x+2x^2}}{6144}+\frac{2116475x^5\sqrt{3-x+2x^2}}{10752}+\frac{57375}{448}x^6\sqrt{3-x+2x^2}+\frac{625}{16}x^7\sqrt{3-x+2}$$

$$= -\frac{19750457x^3\sqrt{3-x+2x^2}}{229376}+\frac{686531x^4\sqrt{3-x+2x^2}}{6144}+\frac{2116475x^5\sqrt{3-x+2x^2}}{10752}+\frac{57375}{448}x^6\sqrt{3}$$

$$= -\frac{15428243x^2\sqrt{3-x+2x^2}}{131072}-\frac{19750457x^3\sqrt{3-x+2x^2}}{229376}+\frac{686531x^4\sqrt{3-x+2x^2}}{6144}+\frac{2116475x^5}{10}$$

$$= \frac{1572007407x\sqrt{3-x+2x^2}}{7340032}-\frac{15428243x^2\sqrt{3-x+2x^2}}{131072}-\frac{19750457x^3\sqrt{3-x+2x^2}}{229376}+\frac{686531x}{}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128}+\frac{1572007407x\sqrt{3-x+2x^2}}{7340032}-\frac{15428243x^2\sqrt{3-x+2x^2}}{131072}-\frac{197504}{}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128}+\frac{1572007407x\sqrt{3-x+2x^2}}{7340032}-\frac{15428243x^2\sqrt{3-x+2x^2}}{131072}-\frac{197504}{}$$

$$= \frac{16493087661\sqrt{3-x+2x^2}}{29360128}+\frac{1572007407x\sqrt{3-x+2x^2}}{7340032}-\frac{15428243x^2\sqrt{3-x+2x^2}}{131072}-\frac{197504}{}$$

**Mathematica [A]** time = 0.246325, size = 75, normalized size = 0.41

$$\frac{4\sqrt{2x^2-x+3}\left(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x^2\right.}{352321536}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(49479262983 + 18864088884*x - 10367779296*x^2 - 758
4175488*x^3 + 9842108416*x^4 + 17338163200*x^5 + 11280384000*x^6 + 34406400
00*x^7) + 60886698033*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/352321536
```

---

**Maple [A]** time = 0.065, size = 147, normalized size = 0.8

$$\frac{625\,x^7}{16}\sqrt{2\,x^2-x+3}+\frac{2116475\,x^5}{10752}\sqrt{2\,x^2-x+3}+\frac{686531\,x^4}{6144}\sqrt{2\,x^2-x+3}-\frac{2899366573\sqrt{2}}{16777216}\mathrm{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 625/16*x^7*(2*x^2-x+3)^(1/2)+2116475/10752*x^5*(2*x^2-x+3)^(1/2)+686531/614
4*x^4*(2*x^2-x+3)^(1/2)-2899366573/16777216*2^(1/2)*arcsinh(4/23*23^(1/2)*(
x-1/4))-19750457/229376*x^3*(2*x^2-x+3)^(1/2)-15428243/131072*x^2*(2*x^2-x+
3)^(1/2)+57375/448*x^6*(2*x^2-x+3)^(1/2)+1572007407/7340032*x*(2*x^2-x+3)^(
1/2)+16493087661/29360128*(2*x^2-x+3)^(1/2)
```

---

**Maxima [A]** time = 1.53155, size = 200, normalized size = 1.08

$$\frac{625}{16}\sqrt{2x^2-x+3}x^7+\frac{57375}{448}\sqrt{2x^2-x+3}x^6+\frac{2116475}{10752}\sqrt{2x^2-x+3}x^5+\frac{686531}{6144}\sqrt{2x^2-x+3}x^4-\frac{19750457}{229376}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 625/16*sqrt(2*x^2 - x + 3)*x^7 + 57375/448*sqrt(2*x^2 - x + 3)*x^6 + 211647
5/10752*sqrt(2*x^2 - x + 3)*x^5 + 686531/6144*sqrt(2*x^2 - x + 3)*x^4 - 197
50457/229376*sqrt(2*x^2 - x + 3)*x^3 - 15428243/131072*sqrt(2*x^2 - x + 3)*
x^2 + 1572007407/7340032*sqrt(2*x^2 - x + 3)*x - 2899366573/16777216*sqrt(2
)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16493087661/29360128*sqrt(2*x^2 - x +
3)
```

---

**Fricas [A]** time = 1.36022, size = 355, normalized size = 1.92

$$\frac{1}{88080384}\left(3440640000x^7+11280384000x^6+17338163200x^5+9842108416x^4-7584175488x^3-10367779296x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/88080384*(3440640000*x^7 + 11280384000*x^6 + 17338163200*x^5 + 9842108416
*x^4 - 7584175488*x^3 - 10367779296*x^2 + 18864088884*x + 49479262983)*sqrt
(2*x^2 - x + 3) + 2899366573/33554432*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x
+ 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+3x+2\right)^4}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/sqrt(2*x**2 - x + 3), x)
```

**Giac [A]** time = 1.17971, size = 112, normalized size = 0.61

$$\frac{1}{88080384}\left(4\left(8\left(4\left(16\left(100\left(120\left(140\,x+459\right)x+84659\right)x+4805717\right)x-59251371\right)x-323993103\right)x+4716022221\right)x+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/88080384*(4*(8*(4*(16*(100*(120*(140*x + 459)*x + 84659)*x + 4805717)*x -
 59251371)*x - 323993103)*x + 4716022221)*x + 49479262983)*sqrt(2*x^2 - x +
 3) + 2899366573/16777216*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 -
x + 3)) + 1)
```

## 3.80 $\int \frac{(2+3x+5x^2)^3}{\sqrt{3-x+2x^2}} \, dx$

**Optimal**. Leaf size=143

$$\frac{125}{12}\sqrt{2x^2-x+3}x^5+\frac{1355}{48}\sqrt{2x^2-x+3}x^4+\frac{8185}{256}\sqrt{2x^2-x+3}x^3-\frac{3387\sqrt{2x^2-x+3}x^2}{1024}-\frac{372783\sqrt{2x^2-x+3}x}{8192}-$$

```
[Out] (-203373*Sqrt[3 - x + 2*x^2])/32768 - (372783*x*Sqrt[3 - x + 2*x^2])/8192 -
 (3387*x^2*Sqrt[3 - x + 2*x^2])/1024 + (8185*x^3*Sqrt[3 - x + 2*x^2])/256 +
 (1355*x^4*Sqrt[3 - x + 2*x^2])/48 + (125*x^5*Sqrt[3 - x + 2*x^2])/12 - (92
67707*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

---

**Rubi [A]** time = 0.168162, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 619, 215}

$$\frac{125}{12}\sqrt{2x^2-x+3}x^5+\frac{1355}{48}\sqrt{2x^2-x+3}x^4+\frac{8185}{256}\sqrt{2x^2-x+3}x^3-\frac{3387\sqrt{2x^2-x+3}x^2}{1024}-\frac{372783\sqrt{2x^2-x+3}x}{8192}-$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-203373*Sqrt[3 - x + 2*x^2])/32768 - (372783*x*Sqrt[3 - x + 2*x^2])/8192 -
 (3387*x^2*Sqrt[3 - x + 2*x^2])/1024 + (8185*x^3*Sqrt[3 - x + 2*x^2])/256 +
 (1355*x^4*Sqrt[3 - x + 2*x^2])/48 + (125*x^5*Sqrt[3 - x + 2*x^2])/12 - (92
67707*ArcSinh[(1 - 4*x)/Sqrt[23]])/(65536*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\sqrt{3-x+2x^2}}\,dx = \frac{125}{12}x^5\sqrt{3-x+2x^2}+\frac{1}{12}\int \frac{96+432x+1368x^2+2484x^3+1545x^4+\frac{6775x^5}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{1355}{48}x^4\sqrt{3-x+2x^2}+\frac{125}{12}x^5\sqrt{3-x+2x^2}+\frac{1}{120}\int \frac{960+4320x+13680x^2-15810x^3+\frac{12277}{4}}{\sqrt{3-x+2x^2}}$$

$$= \frac{8185}{256}x^3\sqrt{3-x+2x^2}+\frac{1355}{48}x^4\sqrt{3-x+2x^2}+\frac{125}{12}x^5\sqrt{3-x+2x^2}+\frac{1}{960}\int \frac{7680+34560x-}{\sqrt{3-x}}$$

$$= -\frac{3387x^2\sqrt{3-x+2x^2}}{1024}+\frac{8185}{256}x^3\sqrt{3-x+2x^2}+\frac{1355}{48}x^4\sqrt{3-x+2x^2}+\frac{125}{12}x^5\sqrt{3-x+2x^2}+$$

$$= -\frac{372783x\sqrt{3-x+2x^2}}{8192}-\frac{3387x^2\sqrt{3-x+2x^2}}{1024}+\frac{8185}{256}x^3\sqrt{3-x+2x^2}+\frac{1355}{48}x^4\sqrt{3-x+2x^2}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768}-\frac{372783x\sqrt{3-x+2x^2}}{8192}-\frac{3387x^2\sqrt{3-x+2x^2}}{1024}+\frac{8185}{256}x^3\sqrt{3-x+2x^2}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768}-\frac{372783x\sqrt{3-x+2x^2}}{8192}-\frac{3387x^2\sqrt{3-x+2x^2}}{1024}+\frac{8185}{256}x^3\sqrt{3-x+2x^2}$$

$$= -\frac{203373\sqrt{3-x+2x^2}}{32768}-\frac{372783x\sqrt{3-x+2x^2}}{8192}-\frac{3387x^2\sqrt{3-x+2x^2}}{1024}+\frac{8185}{256}x^3\sqrt{3-x+2x^2}$$

**Mathematica [A]** time = 0.13242, size = 65, normalized size = 0.45

$$\frac{4\sqrt{2x^2-x+3}\left(1024000x^5+2775040x^4+3143040x^3-325152x^2-4473396x-610119\right)-27803121\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{393216}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-610119 - 4473396*x - 325152*x^2 + 3143040*x^3 + 27
75040*x^4 + 1024000*x^5) - 27803121*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/39
3216
```

---

**Maple [A]** time = 0.054, size = 113, normalized size = 0.8

$$\frac{125\,x^5}{12}\sqrt{2\,x^2-x+3}+\frac{1355\,x^4}{48}\sqrt{2\,x^2-x+3}+\frac{9267707\,\sqrt{2}}{131072}\mathrm{Arcsinh}\left(\frac{4\,\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)+\frac{8185\,x^3}{256}\sqrt{2\,x^2-x+3}-\frac{3387\,x}{1024}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 125/12*x^5*(2*x^2-x+3)^(1/2)+1355/48*x^4*(2*x^2-x+3)^(1/2)+9267707/131072*2
^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+8185/256*x^3*(2*x^2-x+3)^(1/2)-3387/1
024*x^2*(2*x^2-x+3)^(1/2)-372783/8192*x*(2*x^2-x+3)^(1/2)-203373/32768*(2*x
```

```
^2-x+3)^(1/2)
```

---

**Maxima [A]** time = 1.44931, size = 154, normalized size = 1.08

$$\frac{125}{12}\sqrt{2x^2-x+3}x^5+\frac{1355}{48}\sqrt{2x^2-x+3}x^4+\frac{8185}{256}\sqrt{2x^2-x+3}x^3-\frac{3387}{1024}\sqrt{2x^2-x+3}x^2-\frac{372783}{8192}\sqrt{2x^2-x+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 125/12*sqrt(2*x^2 - x + 3)*x^5 + 1355/48*sqrt(2*x^2 - x + 3)*x^4 + 8185/256
*sqrt(2*x^2 - x + 3)*x^3 - 3387/1024*sqrt(2*x^2 - x + 3)*x^2 - 372783/8192*
sqrt(2*x^2 - x + 3)*x + 9267707/131072*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x -
 1)) - 203373/32768*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.34021, size = 266, normalized size = 1.86

$$\frac{1}{98304}\left(1024000\,x^5+2775040\,x^4+3143040\,x^3-325152\,x^2-4473396\,x-610119\right)\sqrt{2\,x^2-x+3}+\frac{9267707}{262144}\sqrt{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(1024000*x^5 + 2775040*x^4 + 3143040*x^3 - 325152*x^2 - 4473396*x -
 610119)*sqrt(2*x^2 - x + 3) + 9267707/262144*sqrt(2)*log(-4*sqrt(2)*sqrt(2
*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+3x+2\right)^3}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/sqrt(2*x**2 - x + 3), x)
```

---

**Giac [A]** time = 1.19468, size = 99, normalized size = 0.69

$$\frac{1}{98304}\left(4\left(8\left(20\left(16\left(100\,x+271\right)x+4911\right)x-10161\right)x-1118349\right)x-610119\right)\sqrt{2\,x^2-x+3}-\frac{9267707}{131072}\sqrt{2}\log\left(-2\sqrt{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/98304*(4*(8*(20*(16*(100*x + 271)*x + 4911)*x - 10161)*x - 1118349)*x - 6
10119)*sqrt(2*x^2 - x + 3) - 9267707/131072*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)
*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.81 $\int \frac{(2+3x+5x^2)^2}{\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=101

$$\frac{25}{8}\sqrt{2x^2-x+3}x^3+\frac{655}{96}\sqrt{2x^2-x+3}x^2+\frac{3443}{768}\sqrt{2x^2-x+3}x-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{30725\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

```
[Out] (-11373*Sqrt[3 - x + 2*x^2])/1024 + (3443*x*Sqrt[3 - x + 2*x^2])/768 + (655
*x^2*Sqrt[3 - x + 2*x^2])/96 + (25*x^3*Sqrt[3 - x + 2*x^2])/8 + (30725*ArcS
inh[(1 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

---

**Rubi [A]** time = 0.0883339, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 619, 215}

$$\frac{25}{8}\sqrt{2x^2-x+3}x^3+\frac{655}{96}\sqrt{2x^2-x+3}x^2+\frac{3443}{768}\sqrt{2x^2-x+3}x-\frac{11373\sqrt{2x^2-x+3}}{1024}+\frac{30725\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2048\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (-11373*Sqrt[3 - x + 2*x^2])/1024 + (3443*x*Sqrt[3 - x + 2*x^2])/768 + (655
*x^2*Sqrt[3 - x + 2*x^2])/96 + (25*x^3*Sqrt[3 - x + 2*x^2])/8 + (30725*ArcS
inh[(1 - 4*x)/Sqrt[23]])/(2048*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^2}{\sqrt{3-x+2x^2}}\,dx = \frac{25}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{8}\int \frac{32+96x+7x^2+\frac{655x^3}{2}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{655}{96}x^2\sqrt{3-x+2x^2}+\frac{25}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{48}\int \frac{192-1389x+\frac{3443x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{3443}{768}x\sqrt{3-x+2x^2}+\frac{655}{96}x^2\sqrt{3-x+2x^2}+\frac{25}{8}x^3\sqrt{3-x+2x^2}+\frac{1}{192}\int \frac{-\frac{7257}{4}-\frac{34119x}{8}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024}+\frac{3443}{768}x\sqrt{3-x+2x^2}+\frac{655}{96}x^2\sqrt{3-x+2x^2}+\frac{25}{8}x^3\sqrt{3-x+2x^2}-\ldots$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024}+\frac{3443}{768}x\sqrt{3-x+2x^2}+\frac{655}{96}x^2\sqrt{3-x+2x^2}+\frac{25}{8}x^3\sqrt{3-x+2x^2}-\ldots$$

$$= -\frac{11373\sqrt{3-x+2x^2}}{1024}+\frac{3443}{768}x\sqrt{3-x+2x^2}+\frac{655}{96}x^2\sqrt{3-x+2x^2}+\frac{25}{8}x^3\sqrt{3-x+2x^2}+\ldots$$

**Mathematica [A]** time = 0.0755216, size = 55, normalized size = 0.54

$$\frac{4\sqrt{2x^2-x+3}\left(9600x^3+20960x^2+13772x-34119\right)+92175\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{12288}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*Sqrt[3 - x + 2*x^2]*(-34119 + 13772*x + 20960*x^2 + 9600*x^3) + 92175*Sq
rt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]])/12288
```

---

**Maple [A]** time = 0.051, size = 79, normalized size = 0.8

$$\frac{25x^3}{8}\sqrt{2x^2-x+3}+\frac{655x^2}{96}\sqrt{2x^2-x+3}+\frac{3443x}{768}\sqrt{2x^2-x+3}-\frac{11373}{1024}\sqrt{2x^2-x+3}-\frac{30725\sqrt{2}}{4096}\mathrm{Arcsinh}\left(\frac{4\sqrt{2\ldots}}{23}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 25/8*x^3*(2*x^2-x+3)^(1/2)+655/96*x^2*(2*x^2-x+3)^(1/2)+3443/768*x*(2*x^2-x
+3)^(1/2)-11373/1024*(2*x^2-x+3)^(1/2)-30725/4096*2^(1/2)*arcsinh(4/23*23^(
1/2)*(x-1/4))
```

---

**Maxima [A]** time = 1.48942, size = 108, normalized size = 1.07

$$\frac{25}{8}\sqrt{2x^2-x+3}x^3+\frac{655}{96}\sqrt{2x^2-x+3}x^2+\frac{3443}{768}\sqrt{2x^2-x+3}x-\frac{30725}{4096}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{11373}{1024}\sqrt{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 25/8*sqrt(2*x^2 - x + 3)*x^3 + 655/96*sqrt(2*x^2 - x + 3)*x^2 + 3443/768*sq
rt(2*x^2 - x + 3)*x - 30725/4096*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) -
 11373/1024*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.37724, size = 211, normalized size = 2.09

$$\frac{1}{3072}\left(9600\,x^3+20960\,x^2+13772\,x-34119\right)\sqrt{2\,x^2-x+3}+\frac{30725}{8192}\,\sqrt{2}\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3072*(9600*x^3 + 20960*x^2 + 13772*x - 34119)*sqrt(2*x^2 - x + 3) + 30725
/8192*sqrt(2)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x -
 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+3x+2\right)^2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/sqrt(2*x**2 - x + 3), x)
```

---

**Giac [A]** time = 1.16484, size = 85, normalized size = 0.84

$$\frac{1}{3072}\,(4\,(40\,(60\,x+131)x+3443)x-34119)\sqrt{2\,x^2-x+3}+\frac{30725}{4096}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3072*(4*(40*(60*x + 131)*x + 3443)*x - 34119)*sqrt(2*x^2 - x + 3) + 30725
/4096*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.82 $\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx$

**Optimal**. Leaf size=59

$$\frac{5}{4}\sqrt{2x^2-x+3}x+\frac{39}{16}\sqrt{2x^2-x+3}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

```
[Out] (39*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*x^2])/4 + (17*ArcSinh[(1
- 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

---

**Rubi [A]** time = 0.032932, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1661, 640, 619, 215}

$$\frac{5}{4}\sqrt{2x^2-x+3}x+\frac{39}{16}\sqrt{2x^2-x+3}+\frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (39*Sqrt[3 - x + 2*x^2])/16 + (5*x*Sqrt[3 - x + 2*x^2])/4 + (17*ArcSinh[(1
- 4*x)/Sqrt[23]])/(32*Sqrt[2])
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{2+3x+5x^2}{\sqrt{3-x+2x^2}}\,dx = \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{1}{4}\int \frac{-7+\frac{39x}{2}}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{17}{32}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} - \frac{17\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{32\sqrt{46}}$$
$$= \frac{39}{16}\sqrt{3-x+2x^2} + \frac{5}{4}x\sqrt{3-x+2x^2} + \frac{17\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{32\sqrt{2}}$$

**Mathematica [A]** time = 0.0382121, size = 45, normalized size = 0.76

$$\frac{1}{64}\left(4\sqrt{2x^2-x+3}(20x+39)+17\sqrt{2}\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/Sqrt[3 - x + 2*x^2],x]
```

```
[Out] (4*(39 + 20*x)*Sqrt[3 - x + 2*x^2] + 17*Sqrt[2]*ArcSinh[(1 - 4*x)/Sqrt[23]]
)/64
```

---

**Maple [A]** time = 0.051, size = 45, normalized size = 0.8

$$\frac{5x}{4}\sqrt{2x^2-x+3}+\frac{39}{16}\sqrt{2x^2-x+3}-\frac{17\sqrt{2}}{64}\mathrm{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 5/4*x*(2*x^2-x+3)^(1/2)+39/16*(2*x^2-x+3)^(1/2)-17/64*2^(1/2)*arcsinh(4/23*
23^(1/2)*(x-1/4))
```

---

**Maxima [A]** time = 1.53221, size = 62, normalized size = 1.05

$$\frac{5}{4}\sqrt{2x^2-x+3}x-\frac{17}{64}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)+\frac{39}{16}\sqrt{2x^2-x+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2*x^2 - x + 3)*x - 17/64*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1))
+ 39/16*sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.32083, size = 163, normalized size = 2.76

$$\frac{1}{16}\sqrt{2x^2-x+3}(20x+39)+\frac{17}{128}\sqrt{2}\log\left(4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/128*sqrt(2)*log(4*sqrt(2)*sqrt(2*
x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{5x^2+3x+2}{\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/sqrt(2*x**2 - x + 3), x)
```

---

**Giac [A]** time = 1.15619, size = 72, normalized size = 1.22

$$\frac{1}{16}\sqrt{2x^2-x+3}(20x+39)+\frac{17}{64}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2*x^2 - x + 3)*(20*x + 39) + 17/64*sqrt(2)*log(-2*sqrt(2)*(sqrt(2
)*x - sqrt(2*x^2 - x + 3)) + 1)
```

## 3.83 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} dx$

**Optimal**. Leaf size=148

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)-\sqrt{\frac{1}{682}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(10\sqrt{2}-13\right)}}\left(\left(13\right.\right.}{\sqrt{2x^{}}}\right.$$

```
[Out] Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))]*(7 + 3*
Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] - Sqrt[(-13 + 10*Sqrt[
2])/682]*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))]*(7 - 3*Sqrt[2] + (13 - 1
0*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
```

---

**Rubi [A]** time = 0.314359, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {986, 1029, 204, 206}

$$\sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(\left(13+10\sqrt{2}\right)x+3\sqrt{2}+7\right)}{\sqrt{2x^2-x+3}}\right)-\sqrt{\frac{1}{682}\left(10\sqrt{2}-13\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{11}{31\left(10\sqrt{2}-13\right)}}\left(\left(13\right.\right.}{\sqrt{2x^{}}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] Sqrt[(13 + 10*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13 + 10*Sqrt[2]))]*(7 + 3*
Sqrt[2] + (13 + 10*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]] - Sqrt[(-13 + 10*Sqrt[
2])/682]*ArcTanh[(Sqrt[11/(31*(-13 + 10*Sqrt[2]))]*(7 - 3*Sqrt[2] + (13 - 1
0*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]]
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx = -\frac{\int \frac{11-11\sqrt{2}-11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{22\sqrt{2}} + \frac{\int \frac{11+11\sqrt{2}-11x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{22\sqrt{2}}$$

$$= -\left(\frac{1}{2}\left(11\left(20-13\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-3751\left(13-10\sqrt{2}\right)-11x^2}\,dx, x, \frac{11\left(7-3\sqrt{2}\right)}{\sqrt{3}}\right.\right.$$

$$= \sqrt{\frac{1}{682}\left(13+10\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(13+10\sqrt{2}\right)}}\left(7+3\sqrt{2}+\left(13+10\sqrt{2}\right)x\right)}{\sqrt{3-x+2x^2}}\right) - \sqrt{\frac{1}{682}}$$

**Mathematica [C]** time = 0.34685, size = 176, normalized size = 1.19

$$-\frac{\sqrt{13+i\sqrt{31}}\left(\sqrt{31}+13i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)+\sqrt{13-i\sqrt{31}}\left(\sqrt{31}-13i\right)\tanh^{-1}\left(\frac{\left(-22+4i\sqrt{31}\right)x-i\sqrt{31}+63}{2\sqrt{286-22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{20\sqrt{682}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)),x]
```

```
[Out] -(Sqrt[13 + I*Sqrt[31]]*(13*I + Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 -
 (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] +
Sqrt[13 - I*Sqrt[31]]*(-13*I + Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 +
(4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/(2
0*Sqrt[682])
```

---

**Maple [B]** time = 0.108, size = 684, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 1/21142*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^
2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(369*(-775687+549362*2^(1/2))^(1/2)*2^
(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))
^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-4
1))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)
^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4
+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2
```

```
)+1-x))+520*(-775687+549362*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arcta
n(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016
)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23
)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+465124*arctanh(31/2*(8*(2^(1/2
)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1
/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-866822*arctanh(31/2*(8*(2^(1
/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^
(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)
^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-1+x)/
(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/(-886
6+6820*2^(1/2))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(5x^2+3x+2)\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

---

**Fricas [B]** time = 4.6852, size = 7101, normalized size = 47.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/845680*sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(13*sqrt(2) + 20)*(13*sq
rt(2) - 20)*log(1240*(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) + 7595*x^2 + 6820*s
qrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) + 1/845680*sqrt(341)*200^(1/
4)*sqrt(31)*sqrt(5)*sqrt(13*sqrt(2) + 20)*(13*sqrt(2) - 20)*log(-1240*(sqrt
(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3
*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(2)*(2*x^2 - x + 3) + 2
3405*x - 31000)/x^2) - 1/6820*sqrt(341)*200^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*s
qrt(2) + 20)*arctan(1/2762875*(14260*sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*
(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838*x^5 - 93342*x^4 + 45376*x^3 - 1
8288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 + 40352*x^5 - 68777*x^4 + 35480*x^
3 - 19080*x^2 - 34560*x + 27648) - 55296*x + 34560) + 5*200^(1/4)*(18463*x^
7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4 + 4140576*x^3 - 2378592*x^2 - sq
rt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5 - 2237548*x^4 + 2920320*x^3 - 20
05920*x^2 - 1990656*x + 1534464) - 3068928*x + 1990656))*sqrt(13*sqrt(2) +
20) + 7843000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 13852
56*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*
x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546
048*x - 539136) + 1154304*x - 456192) - 2*sqrt(310)*(sqrt(341)*sqrt(5)*sqrt
(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 - 44014*x^6 + 139674*x^5 - 42464*x
^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454*x^7 - 22399*x^6 + 73509*x^5 - 3
```

```
7360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x) - 89856*x) + 5*200^(1/4)*(69479
*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x^4 + 5347296*x^3 + 4478976*x^2 -
 sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*x^5 - 2560320*x^4 + 3506400*x^3
+ 1202688*x^2 - 1202688*x) - 4478976*x))*sqrt(13*sqrt(2) + 20) + 550*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) + 25*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^
5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 7
6*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144
820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(5)*sqrt(2*x^2 - x + 3)*
(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2) + 20) - 7595*x^2 - 6820*sqrt(
2)*(2*x^2 - x + 3) + 23405*x - 31000)/x^2) + 89125*sqrt(31)*(2828123*x^8 -
9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3
+ 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 +
 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936)
)/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4
4249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 1/6820*sqrt(341)*200
^(1/4)*sqrt(5)*sqrt(2)*sqrt(13*sqrt(2) + 20)*arctan(1/2762875*(14260*sqrt(3
41)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(8056*x^7 - 28976*x^6 + 61838
*x^5 - 93342*x^4 + 45376*x^3 - 18288*x^2 - sqrt(2)*(4702*x^7 - 19541*x^6 +
40352*x^5 - 68777*x^4 + 35480*x^3 - 19080*x^2 - 34560*x + 27648) - 55296*x
+ 34560) + 5*200^(1/4)*(18463*x^7 - 280047*x^6 + 1453472*x^5 - 3238500*x^4
+ 4140576*x^3 - 2378592*x^2 - sqrt(2)*(11418*x^7 - 177633*x^6 + 957180*x^5
- 2237548*x^4 + 2920320*x^3 - 2005920*x^2 - 1990656*x + 1534464) - 3068928*
x + 1990656))*sqrt(13*sqrt(2) + 20) - 7843000*sqrt(31)*sqrt(2)*(28180*x^8 -
 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x
^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4
 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*s
qrt(310)*(sqrt(341)*sqrt(5)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(30876*x^7 -
44014*x^6 + 139674*x^5 - 42464*x^4 + 38736*x^3 + 89856*x^2 - sqrt(2)*(15454
*x^7 - 22399*x^6 + 73509*x^5 - 37360*x^4 + 52200*x^3 + 13824*x^2 - 13824*x)
 - 89856*x) + 5*200^(1/4)*(69479*x^7 - 898236*x^6 + 3454740*x^5 - 4394304*x
^4 + 5347296*x^3 + 4478976*x^2 - sqrt(2)*(38627*x^7 - 500012*x^6 + 1934180*
x^5 - 2560320*x^4 + 3506400*x^3 + 1202688*x^2 - 1202688*x) - 4478976*x))*sq
rt(13*sqrt(2) + 20) - 550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 15788
88*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(155
50*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 +
 1209600*x^2 - 1036800*x) + 3276288*x) - 25*sqrt(31)*(254591*x^8 - 4815126*
x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 16895692
8*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 361
8*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((sqrt(341)*200^(1/4)*sqrt(3
1)*sqrt(5)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(4*x - 1) - 3*x - 5)*sqrt(13*sqrt(2
) + 20) + 7595*x^2 + 6820*sqrt(2)*(2*x^2 - x + 3) - 23405*x + 31000)/x^2) -
 89125*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 26
92*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x -
5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6
+ 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18
579456))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2 - x + 3}\left(5x^2 + 3x + 2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.84 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx$

**Optimal**. Leaf size=188

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)}{1364}-$$

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727 +
 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))]*(2
119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1364 -
 (Sqrt[(-2343727 + 1678700*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-2343727 + 1
678700*Sqrt[2]))]*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/1364
```

---

**Rubi [A]** time = 0.428841, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{682\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{2}\right)}}\left(\left(5751+3935\sqrt{2}\right)x+1816\sqrt{2}+2119\right)}{\sqrt{2x^2-x+3}}\right)}{1364}-$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(682*(2 + 3*x + 5*x^2)) + (Sqrt[(2343727 +
 1678700*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(2343727 + 1678700*Sqrt[2]))]*(2
119 + 1816*Sqrt[2] + (5751 + 3935*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/1364 -
 (Sqrt[(-2343727 + 1678700*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-2343727 + 1
678700*Sqrt[2]))]*(2119 - 1816*Sqrt[2] + (5751 - 3935*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/1364
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{\int \frac{-1826+\frac{2255x}{2}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{121}{2}\left(537-332\sqrt{2}\right)-\frac{121}{2}\left(127-205\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{165044\sqrt{2}} + \frac{\int \frac{\frac{121}{2}\left(537+332\sqrt{2}\right)-\frac{12}{2}}{\sqrt{3-x+2x^2}\left(2+\right.}}{165044}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} - \frac{1}{496}\left(11\left(3357400-2343727\sqrt{2}\right)\right)\operatorname{Subst}\left(\int \frac{}{-\frac{453871}{4}\left(2\right.}\right.$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{682\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{682}\left(2343727+1678700\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(2343727+1678700\sqrt{\right.}}}}{}\right.}{1364}$$

**Mathematica [C]** time = 1.0074, size = 287, normalized size = 1.53

$$\frac{25\left(\frac{i\sqrt{286+22i\sqrt{31}}\left(224\sqrt{31}+1023i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{\left(\sqrt{31}-13i\right)^2}+\frac{10i\left(1364\left(\sqrt{31}+13i\right)(65x+4)\sqrt{2x^2-x+3}-5\sqrt{286-22i\sqrt{31}}\left(787\sqrt{31}-1271i\right)\left(5x^2+3x\right.\right.}{\left(\sqrt{31}+13i\right)^2\left(10ix+\sqrt{31}+3i\right)\left(5\left(\sqrt{31}-13i\right)x+8\sqrt{3}\right.}\right.}{116281}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (25*((I*Sqrt[286 + (22*I)*Sqrt[31]]*(1023*I + 224*Sqrt[31])*ArcTanh[(63 + I
*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3
 - x + 2*x^2])])/(-13*I + Sqrt[31])^2 + ((10*I)*(1364*(13*I + Sqrt[31])*(4
+ 65*x)*Sqrt[3 - x + 2*x^2] - 5*Sqrt[286 - (22*I)*Sqrt[31]]*(-1271*I + 787*
Sqrt[31])*(2 + 3*x + 5*x^2)*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqrt[31
])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]))/((13*I + Sqrt[
31])^2*(3*I + Sqrt[31] + (10*I)*x)*(-4*I + 8*Sqrt[31] + 5*(-13*I + Sqrt[31]
)*x))))/116281
```

---

**Maple [B]** time = 0.153, size = 5225, normalized size = 27.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*sqrt(2*x^2 - x + 3)), x)
```

---

**Fricas [B]** time = 5.14088, size = 8625, normalized size = 45.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/263043507934399808*(8422204*563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)
*(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2) + 3357400)*arctan(1/710190022151725
```

4683789*(47876524*sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(2950932*x^7 -
11691762*x^6 + 24397746*x^5 - 40053004*x^4 + 20309552*x^3 - 10145376*x^2 -
sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17801494*x^5 - 27869789*x^4 + 13808040
*x^3 - 6172200*x^2 - 15724800*x + 10596096) - 21192192*x + 15724800) + 5203
97*563606738^(1/4)*sqrt(341)*(226651*x^7 - 3496629*x^6 + 18614024*x^5 - 428
60780*x^4 + 55586592*x^3 - 36274464*x^2 - sqrt(2)*(168871*x^7 - 2579646*x^6
 + 13533020*x^5 - 30582616*x^4 + 39345120*x^3 - 23947200*x^2 - 28449792*x +
 19450368) - 38900736*x + 28449792))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(
2) + 3357400) + 2016023288688769071527 2*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(33
574/2191)*(sqrt(33574)*(22*563606738^(3/4)*sqrt(341)*(10257392*x^7 - 147733
68*x^6 + 47877288*x^5 - 20710528*x^4 + 26321472*x^3 + 17079552*x^2 - sqrt(2
)*(8292238*x^7 - 11867543*x^6 + 37968813*x^5 - 13449840*x^4 + 14570280*x^3
+ 20176128*x^2 - 20176128*x) - 17079552*x) + 520397*563606738^(1/4)*sqrt(34
1)*(795513*x^7 - 10292932*x^6 + 39734380*x^5 - 51864768*x^4 + 68281632*x^3
+ 34255872*x^2 - 8*sqrt(2)*(77213*x^7 - 998548*x^6 + 3846220*x^5 - 4943520*
x^4 + 6215760*x^3 + 4318272*x^2 - 4318272*x) - 34255872*x))*sqrt(2*x^2 - x
+ 3)*sqrt(2343727*sqrt(2) + 3357400) + 421088065768678*sqrt(31)*sqrt(2)*(12
3408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3
 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5
+ 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 19140
366625849*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5
+ 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*
x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 14482
0224*x))*sqrt(-(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
 x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357
400) - 1731948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 53372285
80187*x - 7069176927400)/x^2) + 229093555532814667219*sqrt(31)*(2828123*x^8
 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x
^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^
5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 948879
36))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4
+ 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 8422204*563606738
^(1/4)*sqrt(33574)*sqrt(341)*sqrt(2)*(5*x^2 + 3*x + 2)*sqrt(2343727*sqrt(2)
 + 3357400)*arctan(1/7101900221517254683789*(47876524*sqrt(33574)*(22*56360
6738^(3/4)*sqrt(341)*(2950932*x^7 - 11691762*x^6 + 24397746*x^5 - 40053004*
x^4 + 20309552*x^3 - 10145376*x^2 - sqrt(2)*(2248634*x^7 - 8421787*x^6 + 17
801494*x^5 - 27869789*x^4 + 13808040*x^3 - 6172200*x^2 - 15724800*x + 10596
096) - 21192192*x + 15724800) + 520397*563606738^(1/4)*sqrt(341)*(226651*x^
7 - 3496629*x^6 + 18614024*x^5 - 42860780*x^4 + 55586592*x^3 - 36274464*x^2
 - sqrt(2)*(168871*x^7 - 2579646*x^6 + 13533020*x^5 - 30582616*x^4 + 393451
20*x^3 - 23947200*x^2 - 28449792*x + 19450368) - 38900736*x + 28449792))*sq
rt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) - 20160232886887690715272
*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549
144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*
x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 53913
6) + 1154304*x - 456192) - 2*sqrt(33574/2191)*(sqrt(33574)*(22*563606738^(3
/4)*sqrt(341)*(10257392*x^7 - 14773368*x^6 + 47877288*x^5 - 20710528*x^4 +
26321472*x^3 + 17079552*x^2 - sqrt(2)*(8292238*x^7 - 11867543*x^6 + 3796881
3*x^5 - 13449840*x^4 + 14570280*x^3 + 20176128*x^2 - 20176128*x) - 17079552
*x) + 520397*563606738^(1/4)*sqrt(341)*(795513*x^7 - 10292932*x^6 + 3973438
0*x^5 - 51864768*x^4 + 68281632*x^3 + 34255872*x^2 - 8*sqrt(2)*(77213*x^7 -
 998548*x^6 + 3846220*x^5 - 4943520*x^4 + 6215760*x^3 + 4318272*x^2 - 43182
72*x) - 34255872*x))*sqrt(2*x^2 - x + 3)*sqrt(2343727*sqrt(2) + 3357400) -
421088065768678*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3
293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 1
18051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x

```
^2 - 1036800*x) + 3276288*x) - 19140366625849*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((563606738^(1/4)*sqrt(335
74)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x
 - 225)*sqrt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028
*sqrt(2)*(2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 22909355
5532814667219*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14283534
4*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x
^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 43
20*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 141919
20*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608
*x + 18579456)) + 563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*(16787000*
x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x + 2) + 10072200*x + 6714800)*sqrt(234372
7*sqrt(2) + 3357400)*log(335740000/2191*(563606738^(1/4)*sqrt(33574)*sqrt(3
41)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(1123*x + 898) - 2021*x - 225)*sq
rt(2343727*sqrt(2) + 3357400) + 1731948347213*x^2 + 1555218924028*sqrt(2)*(
2*x^2 - x + 3) - 5337228580187*x + 7069176927400)/x^2) - 563606738^(1/4)*sq
rt(33574)*sqrt(341)*sqrt(31)*(16787000*x^2 - 2343727*sqrt(2)*(5*x^2 + 3*x +
 2) + 10072200*x + 6714800)*sqrt(2343727*sqrt(2) + 3357400)*log(-335740000/
2191*(563606738^(1/4)*sqrt(33574)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(s
qrt(2)*(1123*x + 898) - 2021*x - 225)*sqrt(2343727*sqrt(2) + 3357400) - 173
1948347213*x^2 - 1555218924028*sqrt(2)*(2*x^2 - x + 3) + 5337228580187*x -
7069176927400)/x^2) + 385694293158944*sqrt(2*x^2 - x + 3)*(65*x + 4))/(5*x^
2 + 3*x + 2)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2 - x + 3}\,(5x^2 + 3x + 2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**2/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^2/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.85 $\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx$

**Optimal**. Leaf size=223

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}+\frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496\left(5x^2+3x+2\right)}+\frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\tan^{-1}\left(\sqrt{31(641486784\cdots}\right.}{3720992}$$

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + ((26794 + 862
65*x)*Sqrt[3 - x + 2*x^2])/(1860496*(2 + 3*x + 5*x^2)) + (25*Sqrt[(64148678
47 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600*
Sqrt[2]))]*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/3720992 - (25*Sqrt[(-6414867847 + 4536374600*Sqrt[2])/682]*Ar
cTanh[(Sqrt[11/(31*(-6414867847 + 4536374600*Sqrt[2]))]*(123161 - 85754*Sqr
t[2] + (294669 - 208915*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/3720992
```

---

**Rubi [A]** time = 0.469453, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{\sqrt{2x^2-x+3}(65x+4)}{1364\left(5x^2+3x+2\right)^2}+\frac{(86265x+26794)\sqrt{2x^2-x+3}}{1860496\left(5x^2+3x+2\right)}+\frac{25\sqrt{\frac{1}{682}\left(6414867847+4536374600\sqrt{2}\right)}\tan^{-1}\left(\sqrt{31(641486784\cdots}\right.}{3720992}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((4 + 65*x)*Sqrt[3 - x + 2*x^2])/(1364*(2 + 3*x + 5*x^2)^2) + ((26794 + 862
65*x)*Sqrt[3 - x + 2*x^2])/(1860496*(2 + 3*x + 5*x^2)) + (25*Sqrt[(64148678
47 + 4536374600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(6414867847 + 4536374600*
Sqrt[2]))]*(123161 + 85754*Sqrt[2] + (294669 + 208915*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/3720992 - (25*Sqrt[(-6414867847 + 4536374600*Sqrt[2])/682]*Ar
cTanh[(Sqrt[11/(31*(-6414867847 + 4536374600*Sqrt[2]))]*(123161 - 85754*Sqr
t[2] + (294669 - 208915*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/3720992
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
```

0]

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^3}\,dx = \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5775+\frac{6479x}{2}-2860x^2}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\int \frac{-\frac{28220225}{2}+\frac{22521125x}{4}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} \cdots}{112560008}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\int \frac{\frac{33275}{4}\left(26103-18658\sqrt{2}\right) \cdots}{\sqrt{3-x+2x^2}\left(\cdots\right.}}{247632\cdots}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} - \frac{\left(6875\left(9072749200 \cdots\right.\right.}{\cdots}$$

$$= \frac{(4+65x)\sqrt{3-x+2x^2}}{1364\left(2+3x+5x^2\right)^2} + \frac{(26794+86265x)\sqrt{3-x+2x^2}}{1860496\left(2+3x+5x^2\right)} + \frac{25\sqrt{\frac{1}{682}\left(6414867847 \cdots\right.}}{\cdots}$$

**Mathematica [C]** time = 6.23773, size = 1279, normalized size = 5.74

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] (2500*Sqrt[3 - x + 2*x^2])/(341*Sqrt[31]*(13*I + Sqrt[31])*(3 - I*Sqrt[31]
+ 10*x)^2) + (7500*Sqrt[3 - x + 2*x^2])/(10571*(13 - I*Sqrt[31])*(3 - I*Sqr
t[31] + 10*x)) - (2500*Sqrt[3 - x + 2*x^2])/(341*Sqrt[31]*(13*I - Sqrt[31])
*(3 + I*Sqrt[31] + 10*x)^2) + (7500*Sqrt[3 - x + 2*x^2])/(10571*(13 + I*Sqr
t[31])*(3 + I*Sqrt[31] + 10*x)) - (375*Sqrt[(2*(13 - I*Sqrt[31]))/11]*(11 -
 (2*I)*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] - 2*(11 - (2*I)*Sqrt[31])*x)/(2*S
qrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(10571*(13*I + Sqrt[31])^2
) - (750*Sqrt[(2*(13 - I*Sqrt[31]))/341]*ArcTanh[(63 - I*Sqrt[31] - 2*(11 -
 (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(9
61*(13*I + Sqrt[31])) + (750*Sqrt[(2*(13 + I*Sqrt[31]))/341]*ArcTanh[(63 +
I*Sqrt[31] - 2*(11 + (2*I)*Sqrt[31])*x)/(2*Sqrt[22*(13 + I*Sqrt[31])]*Sqrt[
3 - x + 2*x^2])])/(961*(13*I - Sqrt[31])) - (375*Sqrt[(2*(13 + I*Sqrt[31]))
/11]*(11 + (2*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] - 2*(11 + (2*I)*Sqrt[31
])*x)/(2*Sqrt[22*(13 + I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/(10571*(13*I - S
qrt[31])^2) + (((500*I)/31)*(-(((20*(-3 + I*Sqrt[31]) - 10*(27 - (4*I)*Sqrt
[31]))*Sqrt[3 - x + 2*x^2])/((300 - 10*(-3 + I*Sqrt[31]) + 2*(-3 + I*Sqrt[3
1])^2)*(-3 + I*Sqrt[31] - 10*x))) + (2*Sqrt[22*(13 - I*Sqrt[31])]*(20*(-3 +
 I*Sqrt[31]) + 10*(27 - (4*I)*Sqrt[31]) - 2*(600 + 2*(-3 + I*Sqrt[31])*(27
- (4*I)*Sqrt[31])))*ArcTanh[(-63 + I*Sqrt[31] - (-10 + 4*(-3 + I*Sqrt[31]))
*x)/(2*Sqrt[22*(13 - I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/((300 - 10*(-3 + I
*Sqrt[31]) + 2*(-3 + I*Sqrt[31])^2)*(1200 - 40*(-3 + I*Sqrt[31]) + 8*(-3 +
I*Sqrt[31])^2))))/(Sqrt[31]*(300 - 10*(-3 + I*Sqrt[31]) + 2*(-3 + I*Sqrt[31
])^2)) + (((500*I)/31)*(-(((-20*(3 + I*Sqrt[31]) + 10*(-27 - (4*I)*Sqrt[31]
))*Sqrt[3 - x + 2*x^2])/((300 + 10*(3 + I*Sqrt[31]) + 2*(3 + I*Sqrt[31])^2)
```

```
*(3 + I*Sqrt[31] + 10*x))) + (2*Sqrt[22*(13 + I*Sqrt[31])]*(-20*(3 + I*Sqrt
[31]) - 10*(-27 - (4*I)*Sqrt[31]) - 2*(600 + 2*(3 + I*Sqrt[31])*(-27 - (4*I
)*Sqrt[31])))*ArcTanh[(63 + I*Sqrt[31] - (10 + 4*(3 + I*Sqrt[31]))*x)/(2*Sq
rt[22*(13 + I*Sqrt[31])]*Sqrt[3 - x + 2*x^2])])/((300 + 10*(3 + I*Sqrt[31])
 + 2*(3 + I*Sqrt[31])^2)*(1200 + 40*(3 + I*Sqrt[31]) + 8*(3 + I*Sqrt[31])^2
))))/(Sqrt[31]*(300 + 10*(3 + I*Sqrt[31]) + 2*(3 + I*Sqrt[31])^2))
```

---

**Maple [B]** time = 0.191, size = 13040, normalized size = 58.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*sqrt(2*x^2 - x + 3)), x)
```

---

**Fricas [B]** time = 5.17811, size = 9927, normalized size = 44.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/21234400002747742634682214 4*(46113488900*4115738902305032^(1/4)*sqrt(226
81873)*sqrt(341)*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4)*sqrt(6414867
847*sqrt(2) + 9072749200)*arctan(1/38366683092940095300583223739487 69*(6468
8701796*sqrt(22681873)*(11*4115738902305032^(3/4)*sqrt(341)*(160344708*x^7
- 615873378*x^6 + 1294230774*x^5 - 2070733376*x^4 + 1037098288*x^3 - 489164
544*x^2 - sqrt(2)*(112700446*x^7 - 434839553*x^6 + 912850886*x^5 - 14661276
91*x^4 + 735661560*x^3 - 350098200*x^2 - 799200000*x + 567316224) - 1134632
448*x + 799200000) + 703138063*4115738902305032^(1/4)*sqrt(341)*(12162569*x
^7 - 186616851*x^6 + 985490056*x^5 - 2246141620*x^4 + 2900382048*x^3 - 1823
848416*x^2 - sqrt(2)*(8564099*x^7 - 131508024*x^6 + 695288980*x^5 - 1587105
104*x^4 + 2050714080*x^3 - 1296806400*x^2 - 1457077248*x + 1033108992) - 20
66217984*x + 1457077248))*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2) + 907
2749200) + 1089118745864105931113330222282 2312*sqrt(31)*sqrt(2)*(28180*x^8
- 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*
x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^
```

4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*
sqrt(45363746/479849)*(sqrt(22681873)*(11*4115738902305032^(3/4)*sqrt(341)*
(576322648*x^7 - 827050092*x^6 + 2660713572*x^5 - 1032439232*x^4 + 12116047
68*x^3 + 1213394688*x^2 - sqrt(2)*(403157522*x^7 - 578844217*x^6 + 18641293
47*x^5 - 735062160*x^4 + 873708120*x^3 + 823986432*x^2 - 823986432*x) - 121
3394688*x) + 703138063*4115738902305032^(1/4)*sqrt(341)*(43684647*x^7 - 565
067708*x^6 + 2178643220*x^5 - 2819241792*x^4 + 3618371808*x^3 + 2197767168*
x^2 - 2*sqrt(2)*(15328963*x^7 - 198290348*x^6 + 764653220*x^5 - 990717120*x
^4 + 1276256160*x^3 + 755350272*x^2 - 755350272*x) - 2197767168*x))*sqrt(2*
x^2 - x + 3)*sqrt(6414867847*sqrt(2) + 9072749200) + 168363055004367262339
322*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 2
44047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*
x) + 3276288*x) + 7652866136562148288151*sqrt(31)*(254591*x^8 - 4815126*x^7
 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x
^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x
^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(4115738902305032^(1/4)*sqrt(
22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012)
 - 93199*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187
753*x^2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*
x - 2091880597633419400)/x^2) + 123763493848193855808332979804799*sqrt(31)*
(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 -
 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^
6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 22306406
4*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 1
3562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 46113
488900*4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(2)*(25*x^4 + 30
*x^3 + 29*x^2 + 12*x + 4)*sqrt(6414867847*sqrt(2) + 9072749200)*arctan(1/38
36668309294009530058322373948769*(64688701796*sqrt(22681873)*(11*4115738902
305032^(3/4)*sqrt(341)*(160344708*x^7 - 615873378*x^6 + 1294230774*x^5 - 20
70733376*x^4 + 1037098288*x^3 - 489164544*x^2 - sqrt(2)*(112700446*x^7 - 43
4839553*x^6 + 912850886*x^5 - 1466127691*x^4 + 735661560*x^3 - 350098200*x^
2 - 799200000*x + 567316224) - 1134632448*x + 799200000) + 703138063*411573
8902305032^(1/4)*sqrt(341)*(12162569*x^7 - 186616851*x^6 + 985490056*x^5 -
2246141620*x^4 + 2900382048*x^3 - 1823848416*x^2 - sqrt(2)*(8564099*x^7 - 1
31508024*x^6 + 695288980*x^5 - 1587105104*x^4 + 2050714080*x^3 - 1296806400
*x^2 - 1457077248*x + 1033108992) - 2066217984*x + 1457077248))*sqrt(2*x^2
- x + 3)*sqrt(6414867847*sqrt(2) + 9072749200) - 10891187458641059311133302
222822312*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x
^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7
+ 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*
x - 539136) + 1154304*x - 456192) - 2*sqrt(45363746/479849)*(sqrt(22681873)
*(11*4115738902305032^(3/4)*sqrt(341)*(576322648*x^7 - 827050092*x^6 + 2660
713572*x^5 - 1032439232*x^4 + 1211604768*x^3 + 1213394688*x^2 - sqrt(2)*(40
3157522*x^7 - 578844217*x^6 + 1864129347*x^5 - 735062160*x^4 + 873708120*x^
3 + 823986432*x^2 - 823986432*x) - 1213394688*x) + 703138063*41157389023050
32^(1/4)*sqrt(341)*(43684647*x^7 - 565067708*x^6 + 2178643220*x^5 - 2819241
792*x^4 + 3618371808*x^3 + 2197767168*x^2 - 2*sqrt(2)*(15328963*x^7 - 19829
0348*x^6 + 764653220*x^5 - 990717120*x^4 + 1276256160*x^3 + 755350272*x^2 -
 755350272*x) - 2197767168*x))*sqrt(2*x^2 - x + 3)*sqrt(6414867847*sqrt(2)
+ 9072749200) - 168363055004367262339322*sqrt(31)*sqrt(2)*(123408*x^8 - 914
152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2
 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4
- 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 7652866136562148288
151*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 1087
81920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 +
517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x
))*sqrt((4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^
2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sq

```
rt(2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(
2*x^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) - 123763
4938481938558083329798047 99*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*
x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*s
qrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 +
 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 466120
0*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*
x^2 - 24772608*x + 18579456)) - 25*4115738902305032^(1/4)*sqrt(22681873)*sq
rt(341)*sqrt(31)*(226818730000*x^4 + 272182476000*x^3 + 263109726800*x^2 -
6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4) + 108872990400*x +
 36290996800)*sqrt(6414867847*sqrt(2) + 9072749200)*log(1134093650000000/47
9849*(4115738902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*sqrt(2*x^2 -
 x + 3)*(sqrt(2)*(67187*x + 26012) - 93199*x - 41175)*sqrt(6414867847*sqrt(
2) + 9072749200) + 512510746420187753*x^2 + 460213731479352268*sqrt(2)*(2*x
^2 - x + 3) - 1579369851213231647*x + 2091880597633419400)/x^2) + 25*411573
8902305032^(1/4)*sqrt(22681873)*sqrt(341)*sqrt(31)*(226818730000*x^4 + 2721
82476000*x^3 + 263109726800*x^2 - 6414867847*sqrt(2)*(25*x^4 + 30*x^3 + 29*
x^2 + 12*x + 4) + 108872990400*x + 36290996800)*sqrt(6414867847*sqrt(2) + 9
072749200)*log(-1134093650000000/479849*(4115738902305032^(1/4)*sqrt(226818
73)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(67187*x + 26012) - 931
99*x - 41175)*sqrt(6414867847*sqrt(2) + 9072749200) - 512510746420187753*x^
2 - 460213731479352268*sqrt(2)*(2*x^2 - x + 3) + 1579369851213231647*x - 20
91880597633419400)/x^2) - 114133005406879362464*(431325*x^3 + 392765*x^2 +
341572*x + 59044)*sqrt(2*x^2 - x + 3))/(25*x^4 + 30*x^3 + 29*x^2 + 12*x + 4
)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**3/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*(5*x**2 + 3*x + 2)**3), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^3/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.86 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=166

$$\frac{625}{24}\sqrt{2x^2-x+3}x^5+\frac{10075}{96}\sqrt{2x^2-x+3}x^4+\frac{79425}{512}\sqrt{2x^2-x+3}x^3-\frac{111315\sqrt{2x^2-x+3}x^2}{2048}-\frac{8992487\sqrt{2x^2-x+3}x}{16384}$$

```
[Out] (-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) - (31009685*Sqrt[3 - x + 2
*x^2])/65536 - (8992487*x*Sqrt[3 - x + 2*x^2])/16384 - (111315*x^2*Sqrt[3 -
 x + 2*x^2])/2048 + (79425*x^3*Sqrt[3 - x + 2*x^2])/512 + (10075*x^4*Sqrt[3
 - x + 2*x^2])/96 + (625*x^5*Sqrt[3 - x + 2*x^2])/24 - (310445587*ArcSinh[(
1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2])
```

---

**Rubi [A]** time = 0.203946, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{24}\sqrt{2x^2-x+3}x^5+\frac{10075}{96}\sqrt{2x^2-x+3}x^4+\frac{79425}{512}\sqrt{2x^2-x+3}x^3-\frac{111315\sqrt{2x^2-x+3}x^2}{2048}-\frac{8992487\sqrt{2x^2-x+3}x}{16384}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```



```
[Out] (-14641*(101 + 79*x))/(1472*Sqrt[3 - x + 2*x^2]) - (31009685*Sqrt[3 - x + 2
*x^2])/65536 - (8992487*x*Sqrt[3 - x + 2*x^2])/16384 - (111315*x^2*Sqrt[3 -
 x + 2*x^2])/2048 + (79425*x^3*Sqrt[3 - x + 2*x^2])/512 + (10075*x^4*Sqrt[3
 - x + 2*x^2])/96 + (625*x^5*Sqrt[3 - x + 2*x^2])/24 - (310445587*ArcSinh[(
1 - 4*x)/Sqrt[23]])/(131072*Sqrt[2])
```



Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{\frac{2821893}{256}-\frac{661181x}{128}-\frac{488267x^2}{64}+\frac{143635x^3}{32}+\frac{213325x^4}{16}+\frac{83375x^5}{8}+\frac{143\cdots}{}}{\sqrt{3-x+2x^2}}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{625}{24}x^5\sqrt{3-x+2x^2} + \frac{1}{138}\int \frac{\frac{8465679}{64}-\frac{1983543x}{32}-\frac{1464801x^2}{16}+\frac{430905x^3}{8}\cdots}{\sqrt{3-x+2x^2}}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x^5\sqrt{3-x+2x^2} + \frac{\int \frac{\frac{42328395}{32}-\frac{9917715x}{16}-\frac{73240\cdots}{8}\cdots}{\sqrt{3-\cdots}}}{1\cdots}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{10075}{96}x^4\sqrt{3-x+2x^2} + \frac{625}{24}x^5\sqrt{3-x+2x^2}\cdots$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3-x+2x^2} + \frac{10075}{96}x^4\sqrt{3-x+\cdots}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{3-x+2x^2}}{2048} + \frac{79425}{512}x^3\sqrt{3-x+\cdots}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{3-x\cdots}}{2048}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{3-x\cdots}}{2048}$$

$$= -\frac{14641(101+79x)}{1472\sqrt{3-x+2x^2}} - \frac{31009685\sqrt{3-x+2x^2}}{65536} - \frac{8992487x\sqrt{3-x+2x^2}}{16384} - \frac{111315x^2\sqrt{3-x\cdots}}{2048}$$

**Mathematica [A]** time = 0.376634, size = 95, normalized size = 0.57

$$\sqrt{2x^2-x+3}\left(\frac{625x^5}{24}+\frac{10075x^4}{96}+\frac{79425x^3}{512}-\frac{111315x^2}{2048}-\frac{14641(79x+101)}{1472\left(2x^2-x+3\right)}-\frac{8992487x}{16384}-\frac{31009685}{65536}\right)+\frac{3104455\cdots}{1\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] Sqrt[3 - x + 2*x^2]*(-31009685/65536 - (8992487*x)/16384 - (111315*x^2)/204
8 + (79425*x^3)/512 + (10075*x^4)/96 + (625*x^5)/24 - (14641*(101 + 79*x))/
(1472*(3 - x + 2*x^2))) + (310445587*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(131072*
Sqrt[2])
```

---

**Maple [A]** time = 0.066, size = 166, normalized size = 1.

$$\frac{8825 x^6}{48}\frac{1}{\sqrt{2 x^2-x+3}}+\frac{217675 x^5}{768}\frac{1}{\sqrt{2 x^2-x+3}}+\frac{52235 x^4}{1024}\frac{1}{\sqrt{2 x^2-x+3}}+\frac{310445587 \sqrt{2}}{262144}\text{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 8825/48*x^6/(2*x^2-x+3)^(1/2)+217675/768*x^5/(2*x^2-x+3)^(1/2)+52235/1024*x
^4/(2*x^2-x+3)^(1/2)+310445587/262144*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4)
)+1234044515/12058624*(-1+4*x)/(2*x^2-x+3)^(1/2)+625/12*x^7/(2*x^2-x+3)^(1/
2)-310445587/131072*x/(2*x^2-x+3)^(1/2)-4734827/8192*x^3/(2*x^2-x+3)^(1/2)-
18367831/32768*x^2/(2*x^2-x+3)^(1/2)-1217267299/524288/(2*x^2-x+3)^(1/2)
```

---

**Maxima [A]** time = 1.49527, size = 200, normalized size = 1.2

$$\frac{625 x^7}{12\sqrt{2 x^2-x+3}}+\frac{8825 x^6}{48\sqrt{2 x^2-x+3}}+\frac{217675 x^5}{768\sqrt{2 x^2-x+3}}+\frac{52235 x^4}{1024\sqrt{2 x^2-x+3}}-\frac{4734827 x^3}{8192\sqrt{2 x^2-x+3}}-\frac{18367831 x^2}{32768\sqrt{2 x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 625/12*x^7/sqrt(2*x^2 - x + 3) + 8825/48*x^6/sqrt(2*x^2 - x + 3) + 217675/7
68*x^5/sqrt(2*x^2 - x + 3) + 52235/1024*x^4/sqrt(2*x^2 - x + 3) - 4734827/8
192*x^3/sqrt(2*x^2 - x + 3) - 18367831/32768*x^2/sqrt(2*x^2 - x + 3) + 3104
45587/262144*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 2953101993/1507328*
x/sqrt(2*x^2 - x + 3) - 3653899049/1507328/sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.41592, size = 385, normalized size = 2.32

$$\frac{21420745503\sqrt{2}\left(2 x^2-x+3\right)\log\left(-4\sqrt{2}\sqrt{2 x^2-x+3}(4 x-1)-32 x^2+16 x-25\right)+8\left(235520000 x^7+831385600 x^6\right.}{36175872\left(2 x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/36175872*(21420745503*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 -
 x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(235520000*x^7 + 831385600*x^6
+ 1281670400*x^5 + 230669760*x^4 - 2613624504*x^3 - 2534760678*x^2 - 885930
5979*x - 10961697147)*sqrt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+3x+2\right)^4}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(3/2), x)
```

---

**Giac [A]** time = 1.1978, size = 111, normalized size = 0.67

$$-\frac{310445587}{262144}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(4(40(20(16(100x+353)x+8707)x+31341)x-142}{4521984\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -310445587/262144*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3))
+ 1) + 1/4521984*((46*(4*(40*(20*(16*(100*x + 353)*x + 8707)*x + 31341)*x -
 14204481)*x - 55103493)*x - 8859305979)*x - 10961697147)/sqrt(2*x^2 - x +
3)
```

# 3.87 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=124

$$\frac{125}{16}\sqrt{2x^2-x+3}x^3+\frac{1825}{64}\sqrt{2x^2-x+3}x^2+\frac{15565}{512}\sqrt{2x^2-x+3}x-\frac{181561\sqrt{2x^2-x+3}}{2048}-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}+\frac{116888}{}$$

```
[Out] (-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) - (181561*Sqrt[3 - x + 2*x^2]
)/2048 + (15565*x*Sqrt[3 - x + 2*x^2])/512 + (1825*x^2*Sqrt[3 - x + 2*x^2])
/64 + (125*x^3*Sqrt[3 - x + 2*x^2])/16 + (1168881*ArcSinh[(1 - 4*x)/Sqrt[23
]])/(4096*Sqrt[2])
```

---

**Rubi [A]** time = 0.127095, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{125}{16}\sqrt{2x^2-x+3}x^3+\frac{1825}{64}\sqrt{2x^2-x+3}x^2+\frac{15565}{512}\sqrt{2x^2-x+3}x-\frac{181561\sqrt{2x^2-x+3}}{2048}-\frac{1331(17-45x)}{368\sqrt{2x^2-x+3}}+\frac{116888}{}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-1331*(17 - 45*x))/(368*Sqrt[3 - x + 2*x^2]) - (181561*Sqrt[3 - x + 2*x^2]
)/2048 + (15565*x*Sqrt[3 - x + 2*x^2])/512 + (1825*x^2*Sqrt[3 - x + 2*x^2])
/64 + (125*x^3*Sqrt[3 - x + 2*x^2])/16 + (1168881*ArcSinh[(1 - 4*x)/Sqrt[23
]])/(4096*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
```

```
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^{3/2}}\,dx = -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{2}{23}\int \frac{-\frac{110285}{64}-\frac{19067x}{32}+\frac{22195x^2}{16}+\frac{13225x^3}{8}+\frac{2875x^4}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{125}{16}x^3\sqrt{3-x+2x^2} + \frac{1}{92}\int \frac{-\frac{110285}{8}-\frac{19067x}{4}+\frac{18515x^2}{4}+\frac{125925x^3}{8}}{\sqrt{3-x+2x^2}}\,dx$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \frac{125}{16}x^3\sqrt{3-x+2x^2} + \frac{1}{552}\int \frac{-\frac{330855}{4}-\frac{492177x}{4}\ldots}{\sqrt{3-x+\ldots}}$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \frac{125}{16}x^3\sqrt{3-x+2x^2} + \ldots$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \ldots$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \ldots$$

$$= -\frac{1331(17-45x)}{368\sqrt{3-x+2x^2}} - \frac{181561\sqrt{3-x+2x^2}}{2048} + \frac{15565}{512}x\sqrt{3-x+2x^2} + \frac{1825}{64}x^2\sqrt{3-x+2x^2} + \ldots$$

**Mathematica [A]** time = 0.228601, size = 65, normalized size = 0.52

$$\frac{\frac{4\left(736000x^5+2318400x^4+2624760x^3-5754186x^2+16138403x-15423965\right)}{\sqrt{2x^2-x+3}} - 26884263\sqrt{2}\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{188416}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] ((4*(-15423965 + 16138403*x - 5754186*x^2 + 2624760*x^3 + 2318400*x^4 + 736
000*x^5))/Sqrt[3 - x + 2*x^2] - 26884263*Sqrt[2]*ArcSinh[(-1 + 4*x)/Sqrt[23
]])/188416
```

---

**Maple [A]** time = 0.059, size = 132, normalized size = 1.1

$$\frac{125\,x^5}{8}\frac{1}{\sqrt{2\,x^2-x+3}} + \frac{1575\,x^4}{32}\frac{1}{\sqrt{2\,x^2-x+3}} - \frac{1168881\,\sqrt{2}}{8192}\mathrm{Arcsinh}\left(\frac{4\,\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right) + \frac{-5392543+21570172\,x}{376832}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 125/8*x^5/(2*x^2-x+3)^(1/2)+1575/32*x^4/(2*x^2-x+3)^(1/2)-1168881/8192*2^(1
/2)*arcsinh(4/23*23^(1/2)*(x-1/4))+5392543/376832*(-1+4*x)/(2*x^2-x+3)^(1/2
)+1168881/4096*x/(2*x^2-x+3)^(1/2)+14265/256*x^3/(2*x^2-x+3)^(1/2)-125091/1
024*x^2/(2*x^2-x+3)^(1/2)-5130399/16384/(2*x^2-x+3)^(1/2)
```

---

**Maxima [A]** time = 1.52628, size = 154, normalized size = 1.24

$$\frac{125\,x^5}{8\,\sqrt{2\,x^2-x+3}}+\frac{1575\,x^4}{32\,\sqrt{2\,x^2-x+3}}+\frac{14265\,x^3}{256\,\sqrt{2\,x^2-x+3}}-\frac{125091\,x^2}{1024\,\sqrt{2\,x^2-x+3}}-\frac{1168881}{8192}\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\sqrt{23}(4\,x-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 125/8*x^5/sqrt(2*x^2 - x + 3) + 1575/32*x^4/sqrt(2*x^2 - x + 3) + 14265/256
*x^3/sqrt(2*x^2 - x + 3) - 125091/1024*x^2/sqrt(2*x^2 - x + 3) - 1168881/81
92*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) + 16138403/47104*x/sqrt(2*x^2 -
 x + 3) - 15423965/47104/sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.32687, size = 311, normalized size = 2.51

$$\frac{26884263\,\sqrt{2}\left(2\,x^2-x+3\right)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+8\left(736000\,x^5+2318400\,x^4+262476\right.}{376832\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/376832*(26884263*sqrt(2)*(2*x^2 - x + 3)*log(4*sqrt(2)*sqrt(2*x^2 - x + 3
)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(736000*x^5 + 2318400*x^4 + 2624760*x
^3 - 5754186*x^2 + 16138403*x - 15423965)*sqrt(2*x^2 - x + 3))/(2*x^2 - x +
 3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+3x+2\right)^3}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(3/2), x)
```

---

**Giac [A]** time = 1.20565, size = 97, normalized size = 0.78

$$\frac{1168881}{8192}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{(46(20(40(20x+63)x+2853)x-125091)x+16138403)x-1}{47104\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1168881/8192*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1)
+ 1/47104*((46*(20*(40*(20*x + 63)*x + 2853)*x - 125091)*x + 16138403)*x -
15423965)/sqrt(2*x^2 - x + 3)
```

# 3.88 $\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=82

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}+\frac{25}{8}x\sqrt{2x^2-x+3}+\frac{415}{32}\sqrt{2x^2-x+3}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

```
[Out] (121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (415*Sqrt[3 - x + 2*x^2])/32 +
(25*x*Sqrt[3 - x + 2*x^2])/8 - (223*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2
])
```

---

**Rubi [A]** time = 0.0708539, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(19-7x)}{92\sqrt{2x^2-x+3}}+\frac{25}{8}x\sqrt{2x^2-x+3}+\frac{415}{32}\sqrt{2x^2-x+3}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (121*(19 - 7*x))/(92*Sqrt[3 - x + 2*x^2]) + (415*Sqrt[3 - x + 2*x^2])/32 +
(25*x*Sqrt[3 - x + 2*x^2])/8 - (223*ArcSinh[(1 - 4*x)/Sqrt[23]])/(64*Sqrt[2
])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^{3/2}}\,dx = \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{\frac{1173}{16}+\frac{1955x}{8}+\frac{575x^2}{4}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{25}{8}x\sqrt{3-x+2x^2}+\frac{1}{46}\int \frac{-138+\frac{9545x}{8}}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{415}{32}\sqrt{3-x+2x^2}+\frac{25}{8}x\sqrt{3-x+2x^2}+\frac{223}{64}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{415}{32}\sqrt{3-x+2x^2}+\frac{25}{8}x\sqrt{3-x+2x^2}+\frac{223\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+\cdots\right.}{64\sqrt{46}}$$

$$= \frac{121(19-7x)}{92\sqrt{3-x+2x^2}}+\frac{415}{32}\sqrt{3-x+2x^2}+\frac{25}{8}x\sqrt{3-x+2x^2}-\frac{223\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{64\sqrt{2}}$$

**Mathematica [A]** time = 0.140896, size = 55, normalized size = 0.67

$$\frac{4600x^3+16790x^2-9421x+47027}{736\sqrt{2x^2-x+3}}+\frac{223\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (47027 - 9421*x + 16790*x^2 + 4600*x^3)/(736*Sqrt[3 - x + 2*x^2]) + (223*Ar
cSinh[(-1 + 4*x)/Sqrt[23]])/(64*Sqrt[2])
```

---

**Maple [A]** time = 0.052, size = 98, normalized size = 1.2

$$\frac{25\,x^3}{4}\frac{1}{\sqrt{2\,x^2-x+3}}+\frac{365\,x^2}{16}\frac{1}{\sqrt{2\,x^2-x+3}}-\frac{223\,x}{64}\frac{1}{\sqrt{2\,x^2-x+3}}+\frac{15761}{256}\frac{1}{\sqrt{2\,x^2-x+3}}-\frac{-13713+54852\,x}{5888}\frac{\cdots}{\sqrt{2\,x^2\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x)
```

```
[Out] 25/4*x^3/(2*x^2-x+3)^(1/2)+365/16*x^2/(2*x^2-x+3)^(1/2)-223/64*x/(2*x^2-x+3
)^(1/2)+15761/256/(2*x^2-x+3)^(1/2)-13713/5888*(-1+4*x)/(2*x^2-x+3)^(1/2)+2
```

```
23/128*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

---

**Maxima [A]** time = 1.48402, size = 108, normalized size = 1.32

$$\frac{25\,x^3}{4\,\sqrt{2\,x^2-x+3}}+\frac{365\,x^2}{16\,\sqrt{2\,x^2-x+3}}+\frac{223}{128}\,\sqrt{2}\,\mathrm{arsinh}\left(\frac{1}{23}\,\sqrt{23}(4\,x-1)\right)-\frac{9421\,x}{736\,\sqrt{2\,x^2-x+3}}+\frac{47027}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 25/4*x^3/sqrt(2*x^2 - x + 3) + 365/16*x^2/sqrt(2*x^2 - x + 3) + 223/128*sqr
t(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 9421/736*x/sqrt(2*x^2 - x + 3) + 47
027/736/sqrt(2*x^2 - x + 3)
```

---

**Fricas [A]** time = 1.3002, size = 251, normalized size = 3.06

$$\frac{5129\,\sqrt{2}\left(2\,x^2-x+3\right)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+8\left(4600\,x^3+16790\,x^2-9421\,x+47027\right)\cdots}{5888\left(2\,x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5888*(5129*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*
x - 1) - 32*x^2 + 16*x - 25) + 8*(4600*x^3 + 16790*x^2 - 9421*x + 47027)*sq
rt(2*x^2 - x + 3))/(2*x^2 - x + 3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+3x+2\right)^2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(3/2), x)
```

---

**Giac [A]** time = 1.18148, size = 84, normalized size = 1.02

$$-\frac{223}{128}\,\sqrt{2}\log\left(-2\,\sqrt{2}\left(\sqrt{2}x-\sqrt{2\,x^2-x+3}\right)+1\right)+\frac{(230\,(20\,x+73)x-9421)x+47027}{736\,\sqrt{2\,x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -223/128*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1/
736*((230*(20*x + 73)*x - 9421)*x + 47027)/sqrt(2*x^2 - x + 3)
```

## 3.89 $\int \frac{2+3x+5x^2}{(3-x+2x^2)^{3/2}}\,dx$

**Optimal**. Leaf size=45

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(2*Sqrt[2])
```

---

**Rubi [A]** time = 0.0291616, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1660, 12, 619, 215}

$$-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) - (5*ArcSinh[(1 - 4*x)/Sqrt[23]])/
(2*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{3/2}}\,dx &= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{2}{23}\int \frac{115}{4\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{5}{2}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}+\frac{5\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx,x,-1+4x\right)}{2\sqrt{46}} \\
&= -\frac{11(5+3x)}{23\sqrt{3-x+2x^2}}-\frac{5\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{2\sqrt{2}}
\end{aligned}
$$

**Mathematica [A]** time = 0.0770898, size = 45, normalized size = 1.

$$
\frac{5\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{2\sqrt{2}}-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(3/2),x]
```

```
[Out] (-11*(5 + 3*x))/(23*Sqrt[3 - x + 2*x^2]) + (5*ArcSinh[(-1 + 4*x)/Sqrt[23]])
/(2*Sqrt[2])
```

---

**Maple [A]** time = 0.049, size = 64, normalized size = 1.4

$$
-\frac{5x}{2}\frac{1}{\sqrt{2x^2-x+3}}-\frac{17}{8}\frac{1}{\sqrt{2x^2-x+3}}+\frac{-49+196x}{184}\frac{1}{\sqrt{2x^2-x+3}}+\frac{5\sqrt{2}}{4}\text{Arcsinh}\left(\frac{4\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x)
```

```
[Out] -5/2*x/(2*x^2-x+3)^(1/2)-17/8/(2*x^2-x+3)^(1/2)+49/184*(-1+4*x)/(2*x^2-x+3)
^(1/2)+5/4*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

---

**Maxima [A]** time = 1.50947, size = 62, normalized size = 1.38

$$
\frac{5}{4}\sqrt{2}\,\text{arsinh}\left(\frac{1}{23}\sqrt{23}(4x-1)\right)-\frac{33x}{23\sqrt{2x^2-x+3}}-\frac{55}{23\sqrt{2x^2-x+3}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/4*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 33/23*x/sqrt(2*x^2 - x + 3)
- 55/23/sqrt(2*x^2 - x + 3)
```

**Fricas [B]** time = 1.30075, size = 209, normalized size = 4.64

$$\frac{115\sqrt{2}\left(2x^2-x+3\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)-88\sqrt{2x^2-x+3}(3x+5)}{184\left(2x^2-x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/184*(115*sqrt(2)*(2*x^2 - x + 3)*log(-4*sqrt(2)*sqrt(2*x^2 - x + 3)*(4*x
- 1) - 32*x^2 + 16*x - 25) - 88*sqrt(2*x^2 - x + 3)*(3*x + 5))/(2*x^2 - x +
 3)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2+3x+2}{\left(2x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(3/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(3/2), x)
```

**Giac [A]** time = 1.22882, size = 72, normalized size = 1.6

$$-\frac{5}{4}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{11(3x+5)}{23\sqrt{2x^2-x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(3/2),x, algorithm="giac")
```

```
[Out] -5/4*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/23*
(3*x + 5)/sqrt(2*x^2 - x + 3)
```

## 3.90 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx$

**Optimal**. Leaf size=176

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}+\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{22}\sqrt{\frac{1}{682}}\left(500\sqrt{2}\right.$$

```
[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcTa
n[(Sqrt[11/(31*(247 + 500*Sqrt[2]))]*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*x)
)/Sqrt[3 - x + 2*x^2]])/22 - (Sqrt[(-247 + 500*Sqrt[2])/682]*ArcTanh[(Sqrt[
11/(31*(-247 + 500*Sqrt[2]))]*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x))/Sqrt[
3 - x + 2*x^2]])/22
```

---

**Rubi [A]** time = 0.407876, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {974, 1035, 1029, 206, 204}

$$\frac{13-6x}{253\sqrt{2x^2-x+3}}+\frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(\left(69+65\sqrt{2}\right)x+4\sqrt{2}+61\right)}{\sqrt{2x^2-x+3}}\right)-\frac{1}{22}\sqrt{\frac{1}{682}}\left(500\sqrt{2}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(253*Sqrt[3 - x + 2*x^2]) + (Sqrt[(247 + 500*Sqrt[2])/682]*ArcTa
n[(Sqrt[11/(31*(247 + 500*Sqrt[2]))]*(61 + 4*Sqrt[2] + (69 + 65*Sqrt[2])*x)
)/Sqrt[3 - x + 2*x^2]])/22 - (Sqrt[(-247 + 500*Sqrt[2])/682]*ArcTanh[(Sqrt[
11/(31*(-247 + 500*Sqrt[2]))]*(61 - 4*Sqrt[2] + (69 - 65*Sqrt[2])*x))/Sqrt[
3 - x + 2*x^2]])/22
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] && !( !IntegerQ[p] && ILtQ[q, -1]) && !IGtQ[q,
 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
```

```
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx = \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{\int \frac{-1012-\frac{1265x}{2}}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{2783}$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{\int \frac{\frac{2783}{2}\left(3+8\sqrt{2}\right)-\frac{2783}{2}\left(13-5\sqrt{2}\right)x}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}\,dx}{61226\sqrt{2}} - \frac{\int \frac{\frac{2783}{2}\left(3-8\sqrt{2}\right)-\frac{2783}{2}\left(13+5\sqrt{2}\right.}{\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}}{61226\sqrt{2}}$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} - \frac{1}{8}\left(253\left(1000-247\sqrt{2}\right)\right)\text{Subst}\left(\int \frac{1}{-\frac{240097759}{4}\left(247-50\right.}\right.$$

$$= \frac{13-6x}{253\sqrt{3-x+2x^2}} + \frac{1}{22}\sqrt{\frac{1}{682}\left(247+500\sqrt{2}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(247+500\sqrt{2}\right)}}\left(61+4\right.}{\sqrt{3-x}}\right.$$

**Mathematica [C]** time = 1.36352, size = 202, normalized size = 1.15

$$\frac{-\frac{27280(6x-13)}{\sqrt{2x^2-x+3}} - 23\sqrt{682\left(13+i\sqrt{31}\right)}\left(13\sqrt{31}+69i\right)\tanh^{-1}\left(\frac{\left(-22-4i\sqrt{31}\right)x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right) - 23\sqrt{682\left(13-i\sqrt{31}\right)}\left(13\sqrt{31}-6\right.}{6901840}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] ((-27280*(-13 + 6*x))/Sqrt[3 - x + 2*x^2] - 23*Sqrt[682*(13 + I*Sqrt[31])]*
(69*I + 13*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(
2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - 23*Sqrt[682*(13 - I*S
qrt[31])]*(-69*I + 13*Sqrt[31])*ArcTanh[(63 - I*Sqrt[31] + (-22 + (4*I)*Sqr
t[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/6901840
```

---

**Maple [B]** time = 0.124, size = 718, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/465124*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)
^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(2197*(-775687+549362*2^(1/2))^(1/2)*
2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2
))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)
-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+
x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)
^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1
/2)+1-x))+3070*(-775687+549362*2^(1/2))^(1/2)*(-8866+6820*2^(1/2))^(1/2)*ar
ctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(
1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^
(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32
016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2
+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+1712502*arctanh(31/2*(8*(2^
(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*
2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-6617446*arctanh(31/2*(8*
(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8
-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(2^(1/2)-1+x)^2/(2^(1/2)
+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))/(1+(2^(1/2)-
1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))/(8+3*2^(1/2))/
(-8866+6820*2^(1/2))^(1/2)+1/22/(2*x^2-x+3)^(1/2)-3/506*(-1+4*x)/(2*x^2-x+3
)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(3/2)), x)
```

---

**Fricas [B]** time = 4.79084, size = 7618, normalized size = 43.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] -1/50921775520*(339388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*
sqrt(247*sqrt(2) + 1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(
2*x^2 - x + 3)*(22*50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x
^4 + 235088*x^3 - 46944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5
- 427391*x^4 + 234360*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 17
2800) + 5*50^(1/4)*(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 +
 24624288*x^3 - 10862496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^
5 - 12895624*x^4 + 17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19
989504*x + 10533888))*sqrt(247*sqrt(2) + 1000) + 933317000*sqrt(31)*sqrt(2)
*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*
x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5
+ 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x -
456192) - 2*sqrt(310/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3
/4)*(246848*x^7 - 348192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 10298
88*x^2 - sqrt(2)*(46522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*
x^3 - 269568*x^2 + 269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 667694
8*x^6 + 25569820*x^5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(
2)*(38689*x^7 - 502244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689
984*x^2 + 1689984*x) - 46199808*x))*sqrt(247*sqrt(2) + 1000) + 65450*sqrt(3
1)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^
4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276
288*x) + 2975*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*
x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 -
 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1
44820224*x))*sqrt(-(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3
)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 81
1580*sqrt(2)*(2*x^2 - x + 3) + 2785195*x - 3689000)/x^2) + 10605875*sqrt(31
)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4
 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*
x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064
064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 -
 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 339
388*sqrt(341)*50^(1/4)*sqrt(10)*sqrt(2)*(2*x^2 - x + 3)*sqrt(247*sqrt(2) +
1000)*arctan(1/328782125*(14260*sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*
50^(3/4)*(57708*x^7 - 181278*x^6 + 400374*x^5 - 525676*x^4 + 235088*x^3 - 4
6944*x^2 - sqrt(2)*(20846*x^7 - 109153*x^6 + 215386*x^5 - 427391*x^4 + 2343
60*x^3 - 156600*x^2 - 172800*x + 186624) - 373248*x + 172800) + 5*50^(1/4)*
(125839*x^7 - 1864281*x^6 + 9323336*x^5 - 19725020*x^4 + 24624288*x^3 - 108
62496*x^2 - sqrt(2)*(56119*x^7 - 908994*x^6 + 5175980*x^5 - 12895624*x^4 +
17261280*x^3 - 14184000*x^2 - 10533888*x + 9994752) - 19989504*x + 10533888
))*sqrt(247*sqrt(2) + 1000) - 933317000*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(31
0/119)*(sqrt(341)*sqrt(10)*sqrt(2*x^2 - x + 3)*(22*50^(3/4)*(246848*x^7 - 3
48192*x^6 + 1080672*x^5 - 178432*x^4 - 18432*x^3 + 1029888*x^2 - sqrt(2)*(4
6522*x^7 - 71117*x^6 + 257247*x^5 - 273360*x^4 + 484920*x^3 - 269568*x^2 +
269568*x) - 1029888*x) + 5*50^(1/4)*(516957*x^7 - 6676948*x^6 + 25569820*x^
5 - 31522752*x^4 + 34450848*x^3 + 46199808*x^2 - 4*sqrt(2)*(38689*x^7 - 502
244*x^6 + 1967660*x^5 - 2828160*x^4 + 4711680*x^3 - 1689984*x^2 + 1689984*x
) - 46199808*x))*sqrt(247*sqrt(2) + 1000) - 65450*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 2975*sqrt(
31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4
 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 -
```

```
 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((
sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 3
8) + x - 75)*sqrt(247*sqrt(2) + 1000) + 903805*x^2 + 811580*sqrt(2)*(2*x^2
 - x + 3) - 2785195*x + 3689000)/x^2) - 10605875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) - 23*sqrt(341)*50^(1/4)*s
qrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2 - x + 3) - 1000*x + 3000)*s
qrt(247*sqrt(2) + 1000)*log(3100000/119*(sqrt(341)*50^(1/4)*sqrt(31)*sqrt(1
0)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) + x - 75)*sqrt(247*sqrt(2) + 10
00) + 903805*x^2 + 811580*sqrt(2)*(2*x^2 - x + 3) - 2785195*x + 3689000)/x^
2) + 23*sqrt(341)*50^(1/4)*sqrt(31)*sqrt(10)*(2000*x^2 - 247*sqrt(2)*(2*x^2
 - x + 3) - 1000*x + 3000)*sqrt(247*sqrt(2) + 1000)*log(-3100000/119*(sqrt(
341)*50^(1/4)*sqrt(31)*sqrt(10)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(37*x - 38) +
x - 75)*sqrt(247*sqrt(2) + 1000) - 903805*x^2 - 811580*sqrt(2)*(2*x^2 - x +
 3) + 2785195*x - 3689000)/x^2) + 201271840*sqrt(2*x^2 - x + 3)*(6*x - 13))
/(2*x^2 - x + 3)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(2x^2 - x + 3)^{\frac{3}{2}} (5x^2 + 3x + 2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.91 $\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx$

**Optimal**. Leaf size=211

$$-\frac{6315-2306x}{345092\sqrt{2x^2-x+3}}+\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{11}{31(129694447+1}}\right.}{30008}$$

```
[Out] -(6315 - 2306*x)/(345092*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*Sqrt[3 - x
+ 2*x^2]*(2 + 3*x + 5*x^2)) + (Sqrt[(129694447 + 103775000*Sqrt[2])/682]*Ar
cTan[(Sqrt[11/(31*(129694447 + 103775000*Sqrt[2]))]*(12611 + 16454*Sqrt[2]
+ (45519 + 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/30008 - (Sqrt[(-1296944
47 + 103775000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-129694447 + 103775000*S
qrt[2]))]*(12611 - 16454*Sqrt[2] + (45519 - 29065*Sqrt[2])*x))/Sqrt[3 - x +
 2*x^2]])/30008
```

---

**Rubi [A]** time = 0.473226, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{6315-2306x}{345092\sqrt{2x^2-x+3}}+\frac{65x+4}{682\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{\sqrt{\frac{1}{682}\left(129694447+103775000\sqrt{2}\right)}\tan^{-1}\left(\sqrt{\frac{11}{31(129694447+1}}\right.}{30008}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(6315 - 2306*x)/(345092*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*Sqrt[3 - x
+ 2*x^2]*(2 + 3*x + 5*x^2)) + (Sqrt[(129694447 + 103775000*Sqrt[2])/682]*Ar
cTan[(Sqrt[11/(31*(129694447 + 103775000*Sqrt[2]))]*(12611 + 16454*Sqrt[2]
+ (45519 + 29065*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/30008 - (Sqrt[(-1296944
47 + 103775000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(-129694447 + 103775000*S
qrt[2]))]*(12611 - 16454*Sqrt[2] + (45519 - 29065*Sqrt[2])*x))/Sqrt[3 - x +
 2*x^2]])/30008
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
```

```
0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx = \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} - \frac{\int \frac{-1782+\frac{3333x}{2}-2860x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} - \frac{\int \frac{-\frac{3002857}{2}+\frac{6943\cdots}{\cdots}}{\sqrt{3-x+2x^2}\left(2+3x\cdots\right.}}{2087806\cdots}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} + \frac{\int \frac{\frac{30613}{4}\left(4653+215\cdots\right.}{\sqrt{3-x\cdots}}}{45\cdots}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} - \frac{\left(253\left(2075500\cdots\right.\right.}{\cdots}$$

$$= -\frac{6315-2306x}{345092\sqrt{3-x+2x^2}} + \frac{4+65x}{682\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)} + \frac{\sqrt{\frac{1}{682}\left(1296944\cdots\right.}}{\cdots}$$

**Mathematica [C]** time = 1.48804, size = 740, normalized size = 3.51

$$\frac{100\left(\frac{682\left(\left(22-4i\sqrt{31}\right)x+i\sqrt{31}+52\right)}{\left(\sqrt{31}+13i\right)\left(10ix+\sqrt{31}+3i\right)\sqrt{2x^2-x+3}} + \frac{682\left(\left(22+4i\sqrt{31}\right)x-i\sqrt{31}+52\right)}{\left(\sqrt{31}-13i\right)\left(-10ix+\sqrt{31}-3i\right)\sqrt{2x^2-x+3}} + \frac{22\left(2\left(11\sqrt{31}-62i\right)x+52\sqrt{31}+31i\right)}{\left(\sqrt{31}+13i\right)\sqrt{2x^2-x+3}} + \frac{22\left(2\left(11\sqrt{31}+62i\right)x+52\sqrt{31}\cdots\right.}{\left(\sqrt{31}-13i\right)\sqrt{2x^2-x+3}\cdots}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] (100*((682*(52 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x))/((13*I + Sqrt[31])*
(3*I + Sqrt[31] + (10*I)*x)*Sqrt[3 - x + 2*x^2]) + (682*(52 - I*Sqrt[31] +
(22 + (4*I)*Sqrt[31])*x))/((-13*I + Sqrt[31])*(-3*I + Sqrt[31] - (10*I)*x)*
Sqrt[3 - x + 2*x^2]) + (22*(31*I + 52*Sqrt[31] + 2*(-62*I + 11*Sqrt[31])*x)
)/((13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + (22*(-31*I + 52*Sqrt[31] + 2*(6
2*I + 11*Sqrt[31])*x))/((-13*I + Sqrt[31])*Sqrt[3 - x + 2*x^2]) + ((575*I)*
Sqrt[682*(13 + I*Sqrt[31])]*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31
])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]])/(-13*I + Sqrt[3
1])^2 + (155*(44*(16353 + (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 345*Sqrt[
286 + (22*I)*Sqrt[31]]*(-29 + (17*I)*Sqrt[31] + 10*(11 + (2*I)*Sqrt[31])*x)
*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*
Sqrt[31]]*Sqrt[3 - x + 2*x^2])]]))/(22*(-13*I + Sqrt[31])^3*(-3*I + Sqrt[31]
 - (10*I)*x)) + (155*(44*(16353 - (581*I)*Sqrt[31])*Sqrt[3 - x + 2*x^2] + 3
45*Sqrt[286 - (22*I)*Sqrt[31]]*(29 + (17*I)*Sqrt[31] + (-110 + (20*I)*Sqrt[
31])*x)*ArcTanh[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 -
(22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])]]))/(22*(13*I + Sqrt[31])^3*(3*I + Sqr
t[31] + (10*I)*x)) - ((575*I)*Sqrt[682*(13 - I*Sqrt[31])]*ArcTanh[(63 - I*S
qrt[31] + (-22 + (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 -
 x + 2*x^2])]])/(13*I + Sqrt[31])^2))/2674463
```

**Maple [B]** time = 0.179, size = 5942, normalized size = 28.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^2\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(3/2)), x)
```

**Fricas [B]** time = 5.23242, size = 9287, normalized size = 44.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/29889247038841109870720*(35183643812*3446160200^(1/4)*sqrt(20755)*sqrt(34
1)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sqrt(2) + 20755
0000)*arctan(1/2437871055247532640924125*(59193260*sqrt(20755)*(11*34461602
00^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^5 - 311740976
*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 - 65709803*x^6
+ 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2 - 126316800*
x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(1/4)*sqrt(341
)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4 + 425065248*x
^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106575580*x^5 -
236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x + 143534592) -
287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 2075
50000) + 6920408156831705561333000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7
 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2
)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x
^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(41510/3
97951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x^7 - 96938292
*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 38700288*x^2 - sqrt(
2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^4 + 79054920*
x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446160200^(1/4)*
sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358592*x^4 + 506
287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948*x^6 + 128512
220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 173705472*x) - 14
4903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 207550000) + 11691
```

```
2097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329
3072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118
051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2
 - 1036800*x) + 3276288*x) + 5314186228782025*sqrt(31)*(254591*x^8 - 481512
6*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956
928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3
618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(3446160200^(1/4)*sqrt(2
0755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 18
949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 - 3
49243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 1587470711561
000)/x^2) + 786410017821784722878 75*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 5
3385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2
 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 55
68*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8
 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 3
4615296*x^2 - 24772608*x + 18579456)) + 35183643812*3446160200^(1/4)*sqrt(2
0755)*sqrt(341)*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6)*sqrt(129694447*sq
rt(2) + 207550000)*arctan(1/2437871055247532640924125*(59193260*sqrt(20755)
*(11*3446160200^(3/4)*sqrt(341)*(20748108*x^7 - 87744678*x^6 + 180517074*x^
5 - 311740976*x^4 + 161753488*x^3 - 89046144*x^2 - sqrt(2)*(18515146*x^7 -
65709803*x^6 + 140687186*x^5 - 209710441*x^4 + 101256360*x^3 - 39198600*x^2
 - 126316800*x + 76909824) - 153819648*x + 126316800) + 643405*3446160200^(
1/4)*sqrt(341)*(1637219*x^7 - 25548801*x^6 + 138274456*x^5 - 324967420*x^4
+ 425065248*x^3 - 297030816*x^2 - sqrt(2)*(1361849*x^7 - 20608224*x^6 + 106
575580*x^5 - 236322704*x^4 + 301502880*x^3 - 169632000*x^2 - 225358848*x +
143534592) - 287069184*x + 225358848))*sqrt(2*x^2 - x + 3)*sqrt(129694447*s
qrt(2) + 207550000) - 692040815683170556133300 0*sqrt(31)*sqrt(2)*(28180*x^8
 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496
*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x
^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2
*sqrt(41510/397951)*(sqrt(20755)*(11*3446160200^(3/4)*sqrt(341)*(66710248*x
^7 - 96938292*x^6 + 319739772*x^5 - 172116032*x^4 + 247423968*x^3 + 3870028
8*x^2 - sqrt(2)*(71827622*x^7 - 102266467*x^6 + 323714097*x^5 - 93357360*x^
4 + 79054920*x^3 + 219532032*x^2 - 219532032*x) - 38700288*x) + 643405*3446
160200^(1/4)*sqrt(341)*(5462397*x^7 - 70721108*x^6 + 273784220*x^5 - 364358
592*x^4 + 506287008*x^3 + 144903168*x^2 - 2*sqrt(2)*(2586013*x^7 - 33428948
*x^6 + 128512220*x^5 - 162918720*x^4 + 196126560*x^3 + 173705472*x^2 - 1737
05472*x) - 144903168*x))*sqrt(2*x^2 - x + 3)*sqrt(129694447*sqrt(2) + 20755
0000) - 116912097033204550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578
888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15
550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3
+ 1209600*x^2 - 1036800*x) + 3276288*x) - 5314186228782025*sqrt(31)*(254591
*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328
*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 +
 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((3446160200^
(1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x +
 12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 38893032433
2445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 1198540387228555*x + 1
587470711561000)/x^2) - 78641001782178472287875*sqrt(31)*(2828123*x^8 - 969
6916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 3
7981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15
569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(
2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 4424
9088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 23*3446160200^(1/4)*sqr
t(20755)*sqrt(341)*sqrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^
2 - 129694447*sqrt(2)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 12
45300000)*sqrt(129694447*sqrt(2) + 207550000)*log(1037750000000/397951*(344
6160200^(1/4)*sqrt(20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(
6137*x + 12812) - 18949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) + 388
```

```
930324332445*x^2 + 349243556543420*sqrt(2)*(2*x^2 - x + 3) - 11985403872285
55*x + 1587470711561000)/x^2) - 23*3446160200^(1/4)*sqrt(20755)*sqrt(341)*s
qrt(31)*(2075500000*x^4 + 207550000*x^3 + 3320800000*x^2 - 129694447*sqrt(2
)*(10*x^4 + x^3 + 16*x^2 + 7*x + 6) + 1452850000*x + 1245300000)*sqrt(12969
4447*sqrt(2) + 207550000)*log(-1037750000000/397951*(3446160200^(1/4)*sqrt(
20755)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(6137*x + 12812) - 1
8949*x + 6675)*sqrt(129694447*sqrt(2) + 207550000) - 388930324332445*x^2 -
349243556543420*sqrt(2)*(2*x^2 - x + 3) + 1198540387228555*x - 158747071156
1000)/x^2) + 86612402022768160*(11530*x^3 - 24657*x^2 + 18557*x - 10606)*sq
rt(2*x^2 - x + 3))/(10*x^4 + x^3 + 16*x^2 + 7*x + 6)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^2 - x + 3\right)^{\frac{3}{2}} \left(5x^2 + 3x + 2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.92 $\int \frac{1}{(3-x+2x^2)^{3/2}(2+3x+5x^2)^3}\,dx$

**Optimal**. Leaf size=246

$$-\frac{4353943-6508666x}{941410976\sqrt{2x^2-x+3}}+\frac{5(17315x+7318)}{1860496\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}+\frac{3\sqrt{\frac{1}{682}}(13}{}$$

```
[Out] -(4353943 - 6508666*x)/(941410976*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*S
qrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + (5*(7318 + 17315*x))/(1860496*Sqr
t[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (3*Sqrt[(13874275807943 + 98197386500
00*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2
]))]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/81861824 - (3*Sqrt[(-13874275807943 + 9819738650000*Sqrt[2])/
682]*ArcTanh[(Sqrt[11/(31*(-13874275807943 + 9819738650000*Sqrt[2]))]*(5538
393 - 4123702*Sqrt[2] + (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/81861824
```

---

**Rubi [A]** time = 0.525246, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{4353943-6508666x}{941410976\sqrt{2x^2-x+3}}+\frac{5(17315x+7318)}{1860496\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)}+\frac{65x+4}{1364\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2}+\frac{3\sqrt{\frac{1}{682}}(13}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```


```
[Out] -(4353943 - 6508666*x)/(941410976*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*S
qrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + (5*(7318 + 17315*x))/(1860496*Sqr
t[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)) + (3*Sqrt[(13874275807943 + 98197386500
00*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(13874275807943 + 9819738650000*Sqrt[2
]))]*(5538393 + 4123702*Sqrt[2] + (13785797 + 9662095*Sqrt[2])*x))/Sqrt[3 -
 x + 2*x^2]])/81861824 - (3*Sqrt[(-13874275807943 + 9819738650000*Sqrt[2])/
682]*ArcTanh[(Sqrt[11/(31*(-13874275807943 + 9819738650000*Sqrt[2]))]*(5538
393 - 4123702*Sqrt[2] + (13785797 - 9662095*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2
]])/81861824
```


Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
```

```
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5731+\frac{7557x}{2}-5720x^2}{\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{5(7318+17315x)}{1860496\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{}{1860496}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{}{1860496}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{}{1860496}$$

$$= -\frac{4353943-6508666x}{941410976\sqrt{3-x+2x^2}} + \frac{4+65x}{1364\sqrt{3-x+2x^2}\left(2+3x+5x^2\right)^2} + \frac{}{1860496}$$

**Mathematica [C]** time = 2.24957, size = 231, normalized size = 0.94

$$\frac{\frac{27280\left(162716650x^5+86411405x^4+277167774x^3+175833195x^2+161806828x+22374044\right)}{\sqrt{2x^2-x+3}\left(5x^2+3x+2\right)^2} + 69\sqrt{286-22i\sqrt{31}}\left(13785797\sqrt{31}+14026539\right.}{256816914}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((27280*(22374044 + 161806828*x + 175833195*x^2 + 277167774*x^3 + 86411405*
x^4 + 162716650*x^5))/(Sqrt[3 - x + 2*x^2]*(2 + 3*x + 5*x^2)^2) + 69*Sqrt[2
86 - (22*I)*Sqrt[31]]*(14026539*I + 13785797*Sqrt[31])*ArcTan[(63*I + Sqrt[
31] - 2*(11*I + 2*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x +
2*x^2])] - (69*I)*Sqrt[286 + (22*I)*Sqrt[31]]*(-14026539*I + 13785797*Sqrt[
31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22
*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])])/25681691425280
```

---

**Maple [B]** time = 0.266, size = 18981, normalized size = 77.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3\left(2\,x^2-x+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(3/2)), x)
```

**Fricas [B]** time = 5.4732, size = 11105, normalized size = 45.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/33652296632397026886019646994897920*(920746859815884*1928545343086076450^
(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^
3 + 83*x^2 + 32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arcta
n(1/2252270155289097943751876925347228391692375*(2800589462980*sqrt(1963947
730)*(22*1928545343086076450^(3/4)*sqrt(341)*(7361410004*x^7 - 28555361914*
x^6 + 59872788262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2
 - sqrt(2)*(5311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 6800681868
3*x^4 + 33985514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) -
52334913024*x + 37489478400) + 30441189815*1928545343086076450^(1/4)*sqrt(3
41)*(560592897*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 +
134890825824*x^3 - 85859939808*x^2 - sqrt(2)*(402019087*x^7 - 6162703212*x^
6 + 32499503540*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 -
 68177562624*x + 47773380096) - 95546760192*x + 68177562624))*sqrt(2*x^2 -
x + 3)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 63935410859819554532
31134497759874144159000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x
^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8
 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144
*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(1963947730/3471424
919)*(sqrt(1963947730)*(22*1928545343086076450^(3/4)*sqrt(341)*(26184810824
*x^7 - 37618468196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x
^3 + 51583129344*x^2 - sqrt(2)*(19194187986*x^7 - 27528525721*x^6 + 8845761
3411*x^5 - 33685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 4176467481
6*x) - 51583129344*x) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(19
98926311*x^7 - 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 16744
6420704*x^3 + 96037622784*x^2 - 22*sqrt(2)*(65886479*x^7 - 852213084*x^6 +
3285070260*x^5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259
776*x) - 96037622784*x))*sqrt(2*x^2 - x + 3)*sqrt(13874275807943*sqrt(2) +
19639477300000) + 2282926923240949861309948624550*sqrt(31)*sqrt(2)*(123408*
x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38
22336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105
3960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 1037694056
```

01861357332270392025*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90
866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(
4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944
*x) + 144820224*x))*sqrt(-(1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(
341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*
x - 1471975)*sqrt(13874275807943*sqrt(2) + 19639477300000) - 16051926912456
8199977215*x^2 - 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) + 4946614
21179791799929785*x - 655180690304359999907000)/x^2) + 72653875977067675604
899255656362206183625*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 -
142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)
*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*
x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7
 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 -
24772608*x + 18579456)) + 920746859815884*1928545343086076450^(1/4)*sqrt(19
63947730)*sqrt(341)*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 +
32*x + 12)*sqrt(13874275807943*sqrt(2) + 19639477300000)*arctan(1/225227015
5289097943751876925347228391692375*(2800589462980*sqrt(1963947730)*(22*1928
545343086076450^(3/4)*sqrt(341)*(7361410004*x^7 - 28555361914*x^6 + 5987278
8262*x^5 - 96593638888*x^4 + 48573560944*x^3 - 23355012672*x^2 - sqrt(2)*(5
311119598*x^7 - 20292577289*x^6 + 42695479118*x^5 - 68006818683*x^4 + 33985
514680*x^3 - 15860251800*x^2 - 37489478400*x + 26167456512) - 52334913024*x
 + 37489478400) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(56059289
7*x^7 - 8616399363*x^6 + 45618625128*x^5 - 104316505460*x^4 + 134890825824*
x^3 - 85859939808*x^2 - sqrt(2)*(402019087*x^7 - 6162703212*x^6 + 324995035
40*x^5 - 73942829952*x^4 + 95407993440*x^3 - 59600016000*x^2 - 68177562624*
x + 47773380096) - 95546760192*x + 68177562624))*sqrt(2*x^2 - x + 3)*sqrt(1
3874275807943*sqrt(2) + 19639477300000) - 639354108598195545323113449775987
4144159000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*
x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7
 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048
*x - 539136) + 1154304*x - 456192) - 2*sqrt(1963947730/3471424919)*(sqrt(19
63947730)*(22*1928545343086076450^(3/4)*sqrt(341)*(26184810824*x^7 - 376184
68196*x^6 + 121297463436*x^5 - 48741866816*x^4 + 58784153184*x^3 + 51583129
344*x^2 - sqrt(2)*(19194187986*x^7 - 27528525721*x^6 + 88457613411*x^5 - 33
685377680*x^4 + 38926767960*x^3 + 41764674816*x^2 - 41764674816*x) - 515831
29344*x) + 30441189815*1928545343086076450^(1/4)*sqrt(341)*(1998926311*x^7
 - 25858659004*x^6 + 99738083860*x^5 - 129415692096*x^4 + 167446420704*x^3 +
 96037622784*x^2 - 22*sqrt(2)*(65886479*x^7 - 852213084*x^6 + 3285070260*x^
5 - 4244909760*x^4 + 5424792480*x^3 + 3393259776*x^2 - 3393259776*x) - 9603
7622784*x))*sqrt(2*x^2 - x + 3)*sqrt(13874275807943*sqrt(2) + 1963947730000
0) - 2282926923240949861309948624550*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*
x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - s
qrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 16
67952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 10376940560186135733227
0392025*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 +
108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^
7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 1448202
24*x))*sqrt((1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(31)*
sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 1471975)*s
qrt(13874275807943*sqrt(2) + 19639477300000) + 160519269124568199977215*x^2
 + 144139751866959199979540*sqrt(2)*(2*x^2 - x + 3) - 494661421179791799929
785*x + 655180690304359999907000)/x^2) - 7265387597706767560489925565636220
6183625*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5
+ 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2
692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x -
 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6
 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 1
8579456)) + 69*1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt(31
)*(981973865000000*x^6 + 687381705500000*x^5 + 2022866161900000*x^4 + 16693

```
55570500000*x^3 + 1630076615900000*x^2 - 13874275807943*sqrt(2)*(50*x^6 + 3
5*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12) + 628463273600000*x + 235673
727600000)*sqrt(13874275807943*sqrt(2) + 19639477300000)*log(17675529570000
00000/3471424919*(1928545343086076450^(1/4)*sqrt(1963947730)*sqrt(341)*sqrt
(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(2995431*x + 1523456) - 4518887*x - 14719
75)*sqrt(13874275807943*sqrt(2) + 19639477300000) + 16051926912456819997721
5*x^2 + 14413975186695919997954 0*sqrt(2)*(2*x^2 - x + 3) - 4946614211797917
99929785*x + 6551806903043599999907000)/x^2) - 69*1928545343086076450^(1/4)*
sqrt(1963947730)*sqrt(341)*sqrt(31)*(981973865000000*x^6 + 687381705500000*
x^5 + 2022866161900000*x^4 + 1669355570500000*x^3 + 1630076615900000*x^2 -
13874275807943*sqrt(2)*(50*x^6 + 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x
+ 12) + 628463273600000*x + 235673727600000)*sqrt(13874275807943*sqrt(2) +
19639477300000)*log(-1767552957000000000/3471424919*(1928545343086076450^(1
/4)*sqrt(1963947730)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(29954
31*x + 1523456) - 4518887*x - 1471975)*sqrt(13874275807943*sqrt(2) + 196394
77300000) - 160519269124568199977215*x^2 - 144139751866959199979540*sqrt(2)
*(2*x^2 - x + 3) + 494661421179791799929785*x - 6551806903043599999907000)/x
^2) + 357466584630058815949259 20*(162716650*x^5 + 86411405*x^4 + 277167774*
x^3 + 175833195*x^2 + 161806828*x + 22374044)*sqrt(2*x^2 - x + 3))/(50*x^6
+ 35*x^5 + 103*x^4 + 85*x^3 + 83*x^2 + 32*x + 12)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^2-x+3\right)^{\frac{3}{2}}\left(5x^2+3x+2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(3/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(3/2)*(5*x**2 + 3*x + 2)**3), x)
```

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(3/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.93 $\int \frac{(2+3x+5x^2)^4}{(3-x+2x^2)^{5/2}}\,dx$

**Optimal**. Leaf size=147

$$\frac{625}{32}\sqrt{2x^2-x+3}x^3+\frac{38375}{384}\sqrt{2x^2-x+3}x^2+\frac{526075\sqrt{2x^2-x+3}x}{3072}-\frac{1308645\sqrt{2x^2-x+3}}{4096}+\frac{1331(116368x+74}{101568\sqrt{2x^2-x+}}$$

```
[Out] (-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + (1331*(7409 + 116368*x
))/(101568*Sqrt[3 - x + 2*x^2]) - (1308645*Sqrt[3 - x + 2*x^2])/4096 + (526
075*x*Sqrt[3 - x + 2*x^2])/3072 + (38375*x^2*Sqrt[3 - x + 2*x^2])/384 + (62
5*x^3*Sqrt[3 - x + 2*x^2])/32 + (16955197*ArcSinh[(1 - 4*x)/Sqrt[23]])/(819
2*Sqrt[2])
```

---

**Rubi [A]** time = 0.167403, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{625}{32}\sqrt{2x^2-x+3}x^3+\frac{38375}{384}\sqrt{2x^2-x+3}x^2+\frac{526075\sqrt{2x^2-x+3}x}{3072}-\frac{1308645\sqrt{2x^2-x+3}}{4096}+\frac{1331(116368x+74}{101568\sqrt{2x^2-x+}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```


```
[Out] (-14641*(101 + 79*x))/(4416*(3 - x + 2*x^2)^(3/2)) + (1331*(7409 + 116368*x
))/(101568*Sqrt[3 - x + 2*x^2]) - (1308645*Sqrt[3 - x + 2*x^2])/4096 + (526
075*x*Sqrt[3 - x + 2*x^2])/3072 + (38375*x^2*Sqrt[3 - x + 2*x^2])/384 + (62
5*x^3*Sqrt[3 - x + 2*x^2])/32 + (16955197*ArcSinh[(1 - 4*x)/Sqrt[23]])/(819
2*Sqrt[2])
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^4}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{\frac{3839123}{256} - \frac{1983543x}{128} - \frac{1464801x^2}{64} + \frac{430905x^3}{32} + \frac{639975x^4}{16} + \frac{250125x^5}{8} +}{\left(3-x+2x^2\right)^{3/2}}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{4\int \frac{-\frac{141812733}{256} - \frac{1880595x}{16} + \frac{15512925x^2}{64} + \frac{3372375x^3}{16} + \frac{991875}{16}}{\sqrt{3-x+2x^2}}}{1587}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{625}{32}x^3\sqrt{3-x+2x^2} + \frac{\int \frac{-\frac{141812733}{32} - \frac{1880595x}{2} + 22}{\sqrt{3-x+}}}{317}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{38375}{384}x^2\sqrt{3-x+2x^2} + \frac{625}{32}x^3\sqrt{3-x+2x}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} + \frac{526075x\sqrt{3-x+2x^2}}{3072} + \frac{38375}{384}x^2\sqrt{3-x+}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{526075x\sqrt{3-x+}}{3072}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{526075x\sqrt{3-x+}}{3072}$$

$$= -\frac{14641(101+79x)}{4416\left(3-x+2x^2\right)^{3/2}} + \frac{1331(7409+116368x)}{101568\sqrt{3-x+2x^2}} - \frac{1308645\sqrt{3-x+2x^2}}{4096} + \frac{526075x\sqrt{3-x+}}{3072}$$

**Mathematica [A]** time = 0.517884, size = 75, normalized size = 0.51

$$\frac{507840000x^7 + 2090608000x^6 + 3504730800x^5 - 5076781260x^4 + 39848900984x^3 - 36481630395x^2 + 49883864262x}{6500352\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^4/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-18974698519 + 49883864262*x - 36481630395*x^2 + 39848900984*x^3 - 5076781
260*x^4 + 3504730800*x^5 + 2090608000*x^6 + 507840000*x^7)/(6500352*(3 - x
+ 2*x^2)^(3/2)) - (16955197*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(8192*Sqrt[2])
```

---

**Maple [A]** time = 0.071, size = 214, normalized size = 1.5

$$\frac{138025\,x^5}{256}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}+\frac{16955197\,x^3}{12288}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}-\frac{67488035\,x^2}{16384}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}-\frac{16955197\,\sqrt{2}}{16384}\text{Arcsinh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 138025/256*x^5/(2*x^2-x+3)^(3/2)+16955197/12288*x^3/(2*x^2-x+3)^(3/2)-67488
035/16384*x^2/(2*x^2-x+3)^(3/2)-16955197/16384*2^(1/2)*arcsinh(4/23*23^(1/2
)*(x-1/4))+992926033/13000704*(-1+4*x)/(2*x^2-x+3)^(1/2)+5141612725/3617587
2*(-1+4*x)/(2*x^2-x+3)^(3/2)+55167267/131072*x/(2*x^2-x+3)^(3/2)-799745/102
4*x^4/(2*x^2-x+3)^(3/2)+30875/96*x^6/(2*x^2-x+3)^(3/2)+16955197/8192*x/(2*x
^2-x+3)^(1/2)+16955197/32768/(2*x^2-x+3)^(1/2)-2149616639/524288/(2*x^2-x+3
)^(3/2)+625/8*x^7/(2*x^2-x+3)^(3/2)
```

---

**Maxima [B]** time = 1.91995, size = 342, normalized size = 2.33

$$\frac{625\,x^7}{8\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{30875\,x^6}{96\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{138025\,x^5}{256\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{799745\,x^4}{1024\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{16955197}{13000704}x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 625/8*x^7/(2*x^2 - x + 3)^(3/2) + 30875/96*x^6/(2*x^2 - x + 3)^(3/2) + 1380
25/256*x^5/(2*x^2 - x + 3)^(3/2) - 799745/1024*x^4/(2*x^2 - x + 3)^(3/2) -
16955197/13000704*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(
3/2) - 71/sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 -
 x + 3)^(3/2)) - 16955197/16384*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) +
1203818987/6500352*sqrt(2*x^2 - x + 3) + 3536205583/3250176*x/sqrt(2*x^2 -
x + 3) - 2638851/512*x^2/(2*x^2 - x + 3)^(3/2) + 257773037/1083392/sqrt(2*x
^2 - x + 3) + 29484067/23552*x/(2*x^2 - x + 3)^(3/2) - 374445479/70656/(2*x
^2 - x + 3)^(3/2)
```

---

**Fricas [A]** time = 1.37283, size = 441, normalized size = 3.

$$\frac{26907897639\,\sqrt{2}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\log\left(4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+8\left(507840000\,x\right.}{520028}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/52002816*(26907897639*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(4*sq
rt(2)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(507840000*x^
7 + 2090608000*x^6 + 3504730800*x^5 - 5076781260*x^4 + 39848900984*x^3 - 36
481630395*x^2 + 49883864262*x - 18974698519)*sqrt(2*x^2 - x + 3))/(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(5x^2+3x+2\right)^4}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**4/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**4/(2*x**2 - x + 3)**(5/2), x)
```

---

**Giac [A]** time = 1.15042, size = 109, normalized size = 0.74

$$\frac{16955197}{16384}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{((4(2645(20(40(60x+247)x+16563)x-479847)x+9962225}{6500352\left(2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^4/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] 16955197/16384*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1
) + 1/6500352*(((4*(2645*(20*(40*(60*x + 247)*x + 16563)*x - 479847)*x + 99
62225246)*x - 36481630395)*x + 49883864262)*x - 18974698519)/(2*x^2 - x + 3
)^(3/2)
```

## 3.94 $\int \frac{(2+3x+5x^2)^3}{(3-x+2x^2)^{5/2}} \, dx$

**Optimal**. Leaf size=105

$$\frac{121(10679-6744x)}{8464\sqrt{2x^2-x+3}}+\frac{125}{16}x\sqrt{2x^2-x+3}+\frac{3175}{64}\sqrt{2x^2-x+3}-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}-\frac{7495\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

```
[Out] (-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + (121*(10679 - 6744*x))/(
8464*Sqrt[3 - x + 2*x^2]) + (3175*Sqrt[3 - x + 2*x^2])/64 + (125*x*Sqrt[3 -
 x + 2*x^2])/16 - (7495*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

---

**Rubi [A]** time = 0.105366, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 619, 215}

$$\frac{121(10679-6744x)}{8464\sqrt{2x^2-x+3}}+\frac{125}{16}x\sqrt{2x^2-x+3}+\frac{3175}{64}\sqrt{2x^2-x+3}-\frac{1331(17-45x)}{1104\left(2x^2-x+3\right)^{3/2}}-\frac{7495\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-1331*(17 - 45*x))/(1104*(3 - x + 2*x^2)^(3/2)) + (121*(10679 - 6744*x))/(
8464*Sqrt[3 - x + 2*x^2]) + (3175*Sqrt[3 - x + 2*x^2])/64 + (125*x*Sqrt[3 -
 x + 2*x^2])/16 - (7495*ArcSinh[(1 - 4*x)/Sqrt[23]])/(128*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{\left(2+3x+5x^2\right)^3}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{2}{69}\int \frac{-\frac{91275}{64}-\frac{57201x}{32}+\frac{66585x^2}{16}+\frac{39675x^3}{8}+\frac{8625x^4}{4}}{\left(3-x+2x^2\right)^{3/2}}\,dx$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{4\int \frac{\frac{1452105}{64}+\frac{277725x}{8}+\frac{198375x^2}{16}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{125}{16}x\sqrt{3-x+2x^2}+\frac{\int \frac{\frac{214245}{4}+\frac{5038725x}{32}}{\sqrt{3-x+2x^2}}\,dx}{1587}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3-x+2x^2}+\frac{749}{12}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3-x+2x^2}+\frac{749}{}$$

$$= -\frac{1331(17-45x)}{1104\left(3-x+2x^2\right)^{3/2}}+\frac{121(10679-6744x)}{8464\sqrt{3-x+2x^2}}+\frac{3175}{64}\sqrt{3-x+2x^2}+\frac{125}{16}x\sqrt{3-x+2x^2}-\frac{749}{}$$

**Mathematica [A]** time = 0.335113, size = 65, normalized size = 0.62

$$\frac{3174000x^5+16980900x^4-29423976x^3+101546529x^2-62463282x+89784565}{101568\left(2x^2-x+3\right)^{3/2}}+\frac{7495\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^3/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (89784565 - 62463282*x + 101546529*x^2 - 29423976*x^3 + 16980900*x^4 + 3174
000*x^5)/(101568*(3 - x + 2*x^2)^(3/2)) + (7495*ArcSinh[(-1 + 4*x)/Sqrt[23]
])/(128*Sqrt[2])
```

---

**Maple [B]** time = 0.057, size = 180, normalized size = 1.7

$$\frac{125\,x^5}{4}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}-\frac{7495\,x^3}{192}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}+\frac{222809\,x^2}{256}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}+\frac{7495\,\sqrt{2}}{256}\text{Arcsinh}\left(\frac{4\,\sqrt{23}}{23}\left(x-\frac{1}{4}\right)\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 125/4*x^5/(2*x^2-x+3)^(3/2)-7495/192*x^3/(2*x^2-x+3)^(3/2)+222809/256*x^2/(
2*x^2-x+3)^(3/2)+7495/256*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))-3391139/20
3136*(-1+4*x)/(2*x^2-x+3)^(1/2)-14081711/565248*(-1+4*x)/(2*x^2-x+3)^(3/2)-
281177/2048*x/(2*x^2-x+3)^(3/2)+2675/16*x^4/(2*x^2-x+3)^(3/2)-7495/128*x/(2
*x^2-x+3)^(1/2)-7495/512/(2*x^2-x+3)^(1/2)+20961031/24576/(2*x^2-x+3)^(3/2)
```

---

**Maxima [B]** time = 1.50078, size = 296, normalized size = 2.82

$$\frac{125\,x^5}{4\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{2675\,x^4}{16\left(2\,x^2-x+3\right)^{\frac{3}{2}}}+\frac{7495}{203136}\,x\left(\frac{284\,x}{\sqrt{2\,x^2-x+3}}-\frac{3174\,x^2}{\left(2\,x^2-x+3\right)^{\frac{3}{2}}}-\frac{71}{\sqrt{2\,x^2-x+3}}+\frac{805\,x}{\left(2\,x^2-x+3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 125/4*x^5/(2*x^2 - x + 3)^(3/2) + 2675/16*x^4/(2*x^2 - x + 3)^(3/2) + 7495/
203136*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/s
qrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3/
2)) + 7495/256*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 532145/101568*sqr
t(2*x^2 - x + 3) - 4515389/50784*x/sqrt(2*x^2 - x + 3) + 7197/8*x^2/(2*x^2
- x + 3)^(3/2) + 396211/50784/sqrt(2*x^2 - x + 3) - 269783/1104*x/(2*x^2 -
x + 3)^(3/2) + 1002137/1104/(2*x^2 - x + 3)^(3/2)
```

---

**Fricas [A]** time = 1.37072, size = 370, normalized size = 3.52

$$\frac{11894565\,\sqrt{2}\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)\log\left(-4\,\sqrt{2}\sqrt{2\,x^2-x+3}(4\,x-1)-32\,x^2+16\,x-25\right)+8\left(3174000\,x^5+1\right.}{812544\left(4\,x^4-4\,x^3+13\,x^2-6\,x+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/812544*(11894565*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2
)*sqrt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) + 8*(3174000*x^5 + 16
980900*x^4 - 29423976*x^3 + 101546529*x^2 - 62463282*x + 89784565)*sqrt(2*x
^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+3x+2\right)^3}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**3/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**3/(2*x**2 - x + 3)**(5/2), x)
```

---

**Giac [A]** time = 1.22387, size = 97, normalized size = 0.92

$$-\frac{7495}{256}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)+\frac{3\left(\left(4\left(13225\left(20x+107\right)x-2451998\right)x+33848843\right)x-20821094\right)x}{101568\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^3/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -7495/256*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) + 1
/101568*(3*((4*(13225*(20*x + 107)*x - 2451998)*x + 33848843)*x - 20821094)
*x + 89784565)/(2*x^2 - x + 3)^(3/2)
```

## 3.95 $\int \frac{\left(2+3x+5x^2\right)^2}{\left(3-x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=68

$$\frac{121(19-7x)}{276\left(2x^2-x+3\right)^{3/2}}-\frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

```
[Out] (121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) - (11*(7351 + 2336*x))/(6348*S
qrt[3 - x + 2*x^2]) - (25*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

---

**Rubi [A]** time = 0.0612398, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1660, 12, 619, 215}

$$\frac{121(19-7x)}{276\left(2x^2-x+3\right)^{3/2}}-\frac{11(2336x+7351)}{6348\sqrt{2x^2-x+3}}-\frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (121*(19 - 7*x))/(276*(3 - x + 2*x^2)^(3/2)) - (11*(7351 + 2336*x))/(6348*S
qrt[3 - x + 2*x^2]) - (25*ArcSinh[(1 - 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rubi steps

$$\int \frac{(2+3x+5x^2)^2}{(3-x+2x^2)^{5/2}}\,dx = \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} + \frac{2}{69}\int \frac{\frac{131}{16}+\frac{5865x}{8}+\frac{1725x^2}{4}}{(3-x+2x^2)^{3/2}}\,dx$$
$$= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{4\int \frac{39675}{16\sqrt{3-x+2x^2}}\,dx}{1587}$$
$$= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{25}{4}\int \frac{1}{\sqrt{3-x+2x^2}}\,dx$$
$$= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} + \frac{25\,\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{23}}}\,dx, x, -1+4x\right)}{4\sqrt{46}}$$
$$= \frac{121(19-7x)}{276(3-x+2x^2)^{3/2}} - \frac{11(7351+2336x)}{6348\sqrt{3-x+2x^2}} - \frac{25\sinh^{-1}\left(\frac{1-4x}{\sqrt{23}}\right)}{4\sqrt{2}}$$

**Mathematica [A]** time = 0.240769, size = 55, normalized size = 0.81

$$\frac{25\sinh^{-1}\left(\frac{4x-1}{\sqrt{23}}\right)}{4\sqrt{2}} - \frac{11(2336x^3+6183x^2+714x+8623)}{3174(2x^2-x+3)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)^2/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(8623 + 714*x + 6183*x^2 + 2336*x^3))/(3174*(3 - x + 2*x^2)^(3/2)) + (
25*ArcSinh[(-1 + 4*x)/Sqrt[23]])/(4*Sqrt[2])
```

---

**Maple [B]** time = 0.052, size = 146, normalized size = 2.2

$$-\frac{25x^3}{6}(2x^2-x+3)^{-\frac{3}{2}} - \frac{145x^2}{8}(2x^2-x+3)^{-\frac{3}{2}} - \frac{319x}{64}(2x^2-x+3)^{-\frac{3}{2}} - \frac{15775}{768}(2x^2-x+3)^{-\frac{3}{2}} + \frac{-8493+33972x}{5888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x)
```

```
[Out] -25/6*x^3/(2*x^2-x+3)^(3/2)-145/8*x^2/(2*x^2-x+3)^(3/2)-319/64*x/(2*x^2-x+3
)^(3/2)-15775/768/(2*x^2-x+3)^(3/2)+8493/5888*(-1+4*x)/(2*x^2-x+3)^(3/2)+22
67/2116*(-1+4*x)/(2*x^2-x+3)^(1/2)-25/4*x/(2*x^2-x+3)^(1/2)-25/16/(2*x^2-x+
3)^(1/2)+25/8*2^(1/2)*arcsinh(4/23*23^(1/2)*(x-1/4))
```

---

**Maxima [B]** time = 1.77602, size = 250, normalized size = 3.68

$$\frac{25}{6348}x\left(\frac{284x}{\sqrt{2x^2-x+3}} - \frac{3174x^2}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{71}{\sqrt{2x^2-x+3}} + \frac{805x}{(2x^2-x+3)^{\frac{3}{2}}} - \frac{3243}{(2x^2-x+3)^{\frac{3}{2}}}\right) + \frac{25}{8}\sqrt{2}\,\text{arsinh}\left(\frac{1}{23}\sqrt{23}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 25/6348*x*(284*x/sqrt(2*x^2 - x + 3) - 3174*x^2/(2*x^2 - x + 3)^(3/2) - 71/
sqrt(2*x^2 - x + 3) + 805*x/(2*x^2 - x + 3)^(3/2) - 3243/(2*x^2 - x + 3)^(3
/2)) + 25/8*sqrt(2)*arcsinh(1/23*sqrt(23)*(4*x - 1)) - 1775/3174*sqrt(2*x^2
 - x + 3) + 1017/529*x/sqrt(2*x^2 - x + 3) - 15*x^2/(2*x^2 - x + 3)^(3/2) -
 6413/3174/sqrt(2*x^2 - x + 3) - 1/138*x/(2*x^2 - x + 3)^(3/2) - 2593/138/(
2*x^2 - x + 3)^(3/2)
```

---

**Fricas [B]** time = 1.37113, size = 302, normalized size = 4.44

$$\frac{39675\sqrt{2}\left(4x^4-4x^3+13x^2-6x+9\right)\log\left(-4\sqrt{2}\sqrt{2x^2-x+3}(4x-1)-32x^2+16x-25\right)-88\left(2336x^3+6183x^2\right.}{25392\left(4x^4-4x^3+13x^2-6x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/25392*(39675*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*log(-4*sqrt(2)*sq
rt(2*x^2 - x + 3)*(4*x - 1) - 32*x^2 + 16*x - 25) - 88*(2336*x^3 + 6183*x^2
 + 714*x + 8623)*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(5x^2+3x+2\right)^2}{\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)**2/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)**2/(2*x**2 - x + 3)**(5/2), x)
```

---

**Giac [A]** time = 1.26898, size = 82, normalized size = 1.21

$$-\frac{25}{8}\sqrt{2}\log\left(-2\sqrt{2}\left(\sqrt{2}x-\sqrt{2x^2-x+3}\right)+1\right)-\frac{11\,(((2336x+6183)x+714)x+8623)}{3174\left(2x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)^2/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] -25/8*sqrt(2)*log(-2*sqrt(2)*(sqrt(2)*x - sqrt(2*x^2 - x + 3)) + 1) - 11/31
74*(((2336*x + 6183)*x + 714)*x + 8623)/(2*x^2 - x + 3)^(3/2)
```

## 3.96 $\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=47

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69\left(2x^2-x+3\right)^{3/2}}$$

```
[Out] (-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 - x
 + 2*x^2])
```

---

**Rubi [A]** time = 0.0223468, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1660, 12, 613}

$$-\frac{71(1-4x)}{529\sqrt{2x^2-x+3}}-\frac{11(3x+5)}{69\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (-11*(5 + 3*x))/(69*(3 - x + 2*x^2)^(3/2)) - (71*(1 - 4*x))/(529*Sqrt[3 - x
 + 2*x^2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{2+3x+5x^2}{\left(3-x+2x^2\right)^{5/2}}\,dx = -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}} + \frac{2}{69}\int \frac{213}{4\left(3-x+2x^2\right)^{3/2}}\,dx$$
$$= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}} + \frac{71}{46}\int \frac{1}{\left(3-x+2x^2\right)^{3/2}}\,dx$$
$$= -\frac{11(5+3x)}{69\left(3-x+2x^2\right)^{3/2}} - \frac{71(1-4x)}{529\sqrt{3-x+2x^2}}$$

**Mathematica [A]** time = 0.104221, size = 33, normalized size = 0.7

$$\frac{2\left(852x^3-639x^2+1005x-952\right)}{1587\left(2x^2-x+3\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x + 5*x^2)/(3 - x + 2*x^2)^(5/2),x]
```

```
[Out] (2*(-952 + 1005*x - 639*x^2 + 852*x^3))/(1587*(3 - x + 2*x^2)^(3/2))
```

---

**Maple [A]** time = 0.045, size = 30, normalized size = 0.6

$$\frac{1704\,x^3-1278\,x^2+2010\,x-1904}{1587}\left(2\,x^2-x+3\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x)
```

```
[Out] 2/1587/(2*x^2-x+3)^(3/2)*(852*x^3-639*x^2+1005*x-952)
```

---

**Maxima [A]** time = 1.07712, size = 80, normalized size = 1.7

$$\frac{284\,x}{529\,\sqrt{2\,x^2-x+3}} - \frac{71}{529\,\sqrt{2\,x^2-x+3}} - \frac{11\,x}{23\left(2\,x^2-x+3\right)^{\frac{3}{2}}} - \frac{55}{69\left(2\,x^2-x+3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="maxima")
```

```
[Out] 284/529*x/sqrt(2*x^2 - x + 3) - 71/529/sqrt(2*x^2 - x + 3) - 11/23*x/(2*x^2
 - x + 3)^(3/2) - 55/69/(2*x^2 - x + 3)^(3/2)
```

---

**Fricas [A]** time = 1.34092, size = 132, normalized size = 2.81

$$\frac{2\left(852\,x^3-639\,x^2+1005\,x-952\right)\sqrt{2\,x^2-x+3}}{1587\left(4\,x^4-4\,x^3+13\,x^2-6\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/1587*(852*x^3 - 639*x^2 + 1005*x - 952)*sqrt(2*x^2 - x + 3)/(4*x^4 - 4*x^
3 + 13*x^2 - 6*x + 9)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{5x^2 + 3x + 2}{\left(2x^2 - x + 3\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**2+3*x+2)/(2*x**2-x+3)**(5/2),x)
```

```
[Out] Integral((5*x**2 + 3*x + 2)/(2*x**2 - x + 3)**(5/2), x)
```

---

**Giac [A]** time = 1.15732, size = 39, normalized size = 0.83

$$\frac{2\,(3\,(71\,(4\,x - 3)x + 335)x - 952)}{1587\left(2\,x^2 - x + 3\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^2+3*x+2)/(2*x^2-x+3)^(5/2),x, algorithm="giac")
```

```
[Out] 2/1587*(3*(71*(4*x - 3)*x + 335)*x - 952)/(2*x^2 - x + 3)^(3/2)
```

## 3.97 $\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx$

**Optimal**. Leaf size=199

$$\frac{3603-658x}{128018\sqrt{2x^2-x+3}}+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left(\left(247+345\right.\right.}{\sqrt{2x^2-x+}}\right.$$

```
[Out] (13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + (3603 - 658*x)/(128018*Sqrt[3 - x
+ 2*x^2]) + (Sqrt[(-15457 + 25000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(-15457
 + 25000*Sqrt[2]))]*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/484 - (Sqrt[(15457 + 25000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(1
5457 + 25000*Sqrt[2]))]*(443 + 98*Sqrt[2] + (247 - 345*Sqrt[2])*x))/Sqrt[3
- x + 2*x^2]])/484
```

---

**Rubi [A]** time = 0.455598, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$\frac{3603-658x}{128018\sqrt{2x^2-x+3}}+\frac{13-6x}{759\left(2x^2-x+3\right)^{3/2}}+\frac{1}{484}\sqrt{\frac{1}{682}\left(25000\sqrt{2}-15457\right)}\tan^{-1}\left(\frac{\sqrt{\frac{11}{31\left(25000\sqrt{2}-15457\right)}}\left(\left(247+345\right.\right.}{\sqrt{2x^2-x+}}\right.$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (13 - 6*x)/(759*(3 - x + 2*x^2)^(3/2)) + (3603 - 658*x)/(128018*Sqrt[3 - x
+ 2*x^2]) + (Sqrt[(-15457 + 25000*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31*(-15457
 + 25000*Sqrt[2]))]*(443 - 98*Sqrt[2] + (247 + 345*Sqrt[2])*x))/Sqrt[3 - x
+ 2*x^2]])/484 - (Sqrt[(15457 + 25000*Sqrt[2])/682]*ArcTanh[(Sqrt[11/(31*(1
5457 + 25000*Sqrt[2]))]*(443 + 98*Sqrt[2] + (247 - 345*Sqrt[2])*x))/Sqrt[3
- x + 2*x^2]])/484
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(3-x+2x^2)^{5/2}(2+3x+5x^2)}\,dx = \frac{13-6x}{759(3-x+2x^2)^{3/2}} - \frac{\int \frac{-2772-\frac{3003x}{2}+660x^2}{(3-x+2x^2)^{3/2}(2+3x+5x^2)}\,dx}{8349}$$

$$= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{\int \frac{-\frac{5184729}{2}-\frac{12481755x}{4}}{\sqrt{3-x+2x^2}(2+3x+5x^2)}\,dx}{23235267}$$

$$= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{\int \frac{-\frac{2112297}{4}(11-54\sqrt{2})-\frac{2112297}{4}(119-6}{\sqrt{3-x+2x^2}(2+3x+5x^2)}}{511175874\sqrt{2}}$$

$$= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} - \frac{1}{32}\left(17457\left(50000-15457\sqrt{2}\right.\right.$$

$$= \frac{13-6x}{759(3-x+2x^2)^{3/2}} + \frac{3603-658x}{128018\sqrt{3-x+2x^2}} + \frac{1}{484}\sqrt{\frac{1}{682}\left(-15457+25000\right.}$$

**Mathematica [C]** time = 0.909804, size = 218, normalized size = 1.1

$$\frac{-3948x^3+23592x^2-19767x+39005}{384054(2x^2-x+3)^{3/2}} - \frac{\sqrt{\frac{1}{682}(13+i\sqrt{31})}(119\sqrt{31}+247i)\tanh^{-1}\left(\frac{(-22-4i\sqrt{31})x+i\sqrt{31}+63}{2\sqrt{286+22i\sqrt{31}}\sqrt{2x^2-x+3}}\right)}{9680} + \frac{\sqrt{\frac{1}{682}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)),x]
```

```
[Out] (39005 - 19767*x + 23592*x^2 - 3948*x^3)/(384054*(3 - x + 2*x^2)^(3/2)) - (
Sqrt[(13 + I*Sqrt[31])/682]*(247*I + 119*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31]
 + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*
x^2])])/9680 + (Sqrt[(13 - I*Sqrt[31])/682]*(-247*I + 119*Sqrt[31])*ArcTanh
[(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]
]*Sqrt[3 - x + 2*x^2])])/9680
```

---

**Maple [B]** time = 0.121, size = 751, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x)
```

```
[Out] 1/66/(2*x^2-x+3)^(3/2)-1/506*(-1+4*x)/(2*x^2-x+3)^(3/2)-329/256036*(-1+4*x)
/(2*x^2-x+3)^(1/2)+1/10232728*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)
-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)*2^(1/2)*(10111*(-775687+
549362*2^(1/2))^(1/2)*2^(1/2)*(-8866+6820*2^(1/2))^(1/2)*arctan(1/11692487*
(-775687+549362*2^(1/2))^(1/2)*(-23*(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(
1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(
1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2
```

```
)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2
))*(2^(1/2)-1+x)/(2^(1/2)+1-x))+13910*(-775687+549362*2^(1/2))^(1/2)*(-8866
+6820*2^(1/2))^(1/2)*arctan(1/11692487*(-775687+549362*2^(1/2))^(1/2)*(-23*
(8+3*2^(1/2))*(-23*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+24*2^(1/2)-41))^(1/2)*(6
485*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+10368*(2^(1/2)-1+x)^2/(2^(1/2)+
1-x)^2+22379*2^(1/2)+32016)/(23*(2^(1/2)-1+x)^4/(2^(1/2)+1-x)^4+82*(2^(1/2)
-1+x)^2/(2^(1/2)+1-x)^2+23)*(8+3*2^(1/2))*(2^(1/2)-1+x)/(2^(1/2)+1-x))-9936
74*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/
2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2))*2^(1/2)-42
685698*arctanh(31/2*(8*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2
^(1/2)+1-x)^2*2^(1/2)+8-3*2^(1/2))^(1/2)/(-8866+6820*2^(1/2))^(1/2)))/((8*(
2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2+3*(2^(1/2)-1+x)^2/(2^(1/2)+1-x)^2*2^(1/2)+8-
3*2^(1/2))/(1+(2^(1/2)-1+x)/(2^(1/2)+1-x))^2)^(1/2)/(1+(2^(1/2)-1+x)/(2^(1/
2)+1-x))/(8+3*2^(1/2))/(-8866+6820*2^(1/2))^(1/2)+13/484/(2*x^2-x+3)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)\left(2\,x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)*(2*x^2 - x + 3)^(5/2)), x)
```

---

**Fricas [B]** time = 4.94658, size = 8280, normalized size = 41.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="fricas")
```

```
[Out] 1/370971467791584000*(1123856268*sqrt(341)*200^(1/4)*sqrt(2)*(4*x^4 - 4*x^3
 + 13*x^2 - 6*x + 9)*sqrt(-772850000*sqrt(2) + 2500000000)*arctan(-1/788938
9562500*(71300*sqrt(341)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(347404*x^7 - 90
7814*x^6 + 2112962*x^5 - 2166688*x^4 + 787344*x^3 + 304128*x^2 - sqrt(2)*(3
5898*x^7 - 441939*x^6 + 782418*x^5 - 2117233*x^4 + 1272680*x^3 - 1081800*x^
2 - 518400*x + 1043712) - 2087424*x + 518400) + 5*200^(1/4)*(712757*x^7 - 1
0233303*x^6 + 48529768*x^5 - 94500260*x^4 + 113086944*x^3 - 22282848*x^2 -
sqrt(2)*(158647*x^7 - 2935272*x^6 + 19428740*x^5 - 55765712*x^4 + 78380640*
x^3 - 84096000*x^2 - 37407744*x + 53208576) - 106417152*x + 37407744))*sqrt
(-772850000*sqrt(2) + 2500000000) + 22395686500000*sqrt(31)*sqrt(2)*(28180*
x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98
496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 132071
0*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192)
- sqrt(310/5711)*(sqrt(341)*sqrt(2*x^2 - x + 3)*(11*200^(3/4)*(1665224*x^7
- 2325796*x^6 + 7065036*x^5 - 196416*x^4 - 2176416*x^3 + 8895744*x^2 + sqrt
(2)*(167914*x^7 - 195429*x^6 + 331239*x^5 + 1685680*x^4 - 3693960*x^3 + 419
5584*x^2 - 4195584*x) - 8895744*x) + 5*200^(1/4)*(3246491*x^7 - 41888524*x^
6 + 159670660*x^5 - 190080576*x^4 + 180496224*x^3 + 376648704*x^2 - 2*sqrt(
2)*(40239*x^7 - 558044*x^6 + 2804660*x^5 - 9524160*x^4 + 34843680*x^3 - 740
06784*x^2 + 74006784*x) - 376648704*x))*sqrt(-772850000*sqrt(2) + 250000000
```

```
0) + 314105000*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 32
93072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 11
8051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^
2 - 1036800*x) + 3276288*x) + 14277500*sqrt(31)*(254591*x^8 - 4815126*x^7 +
 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2
 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3
 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(sqrt(341)*200^(1/4)*sqrt(31)*s
qrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sqrt(-772850000*sq
rt(2) + 2500000000) - 4337504500*x^2 - 3894902000*sqrt(2)*(2*x^2 - x + 3) +
 13366595500*x - 17704100000)/x^2) + 254496437500*sqrt(31)*(2828123*x^8 - 9
696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 +
 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 +
15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))
/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44
249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 1123856268*sqrt(341)*
200^(1/4)*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)*sqrt(-772850000*sqrt(2
) + 2500000000)*arctan(-1/7889389562500*(71300*sqrt(341)*sqrt(2*x^2 - x + 3
)*(11*200^(3/4)*(347404*x^7 - 907814*x^6 + 2112962*x^5 - 2166688*x^4 + 7873
44*x^3 + 304128*x^2 - sqrt(2)*(35898*x^7 - 441939*x^6 + 782418*x^5 - 211723
3*x^4 + 1272680*x^3 - 1081800*x^2 - 518400*x + 1043712) - 2087424*x + 51840
0) + 5*200^(1/4)*(712757*x^7 - 10233303*x^6 + 48529768*x^5 - 94500260*x^4 +
 113086944*x^3 - 22282848*x^2 - sqrt(2)*(158647*x^7 - 2935272*x^6 + 1942874
0*x^5 - 55765712*x^4 + 78380640*x^3 - 84096000*x^2 - 37407744*x + 53208576)
 - 106417152*x + 37407744))*sqrt(-772850000*sqrt(2) + 2500000000) - 2239568
6500000*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5
 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
- 539136) + 1154304*x - 456192) - sqrt(310/5711)*(sqrt(341)*sqrt(2*x^2 - x
+ 3)*(11*200^(3/4)*(1665224*x^7 - 2325796*x^6 + 7065036*x^5 - 196416*x^4 -
2176416*x^3 + 8895744*x^2 + sqrt(2)*(167914*x^7 - 195429*x^6 + 331239*x^5 +
 1685680*x^4 - 3693960*x^3 + 4195584*x^2 - 4195584*x) - 8895744*x) + 5*200^
(1/4)*(3246491*x^7 - 41888524*x^6 + 159670660*x^5 - 190080576*x^4 + 1804962
24*x^3 + 376648704*x^2 - 2*sqrt(2)*(40239*x^7 - 558044*x^6 + 2804660*x^5 -
9524160*x^4 + 34843680*x^3 - 74006784*x^2 + 74006784*x) - 376648704*x))*sqr
t(-772850000*sqrt(2) + 2500000000) - 314105000*sqrt(31)*sqrt(2)*(123408*x^8
 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 38223
36*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 105396
0*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 3276288*x) - 14277500*sqrt
(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^
4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6
- 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(
(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) +
163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 38949
02000*sqrt(2)*(2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 2544964
37500*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 +
254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 269
2*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5
184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 +
 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 185
79456)) + 1587*sqrt(341)*200^(1/4)*sqrt(31)*(200000*x^4 - 200000*x^3 + 6500
00*x^2 + 15457*sqrt(2)*(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 4500
00)*sqrt(-772850000*sqrt(2) + 2500000000)*log(77500000/5711*(sqrt(341)*200^
(1/4)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sq
rt(-772850000*sqrt(2) + 2500000000) + 4337504500*x^2 + 3894902000*sqrt(2)*(
2*x^2 - x + 3) - 13366595500*x + 17704100000)/x^2) - 1587*sqrt(341)*200^(1/
4)*sqrt(31)*(200000*x^4 - 200000*x^3 + 650000*x^2 + 15457*sqrt(2)*(4*x^4 -
4*x^3 + 13*x^2 - 6*x + 9) - 300000*x + 450000)*sqrt(-772850000*sqrt(2) + 25
00000000)*log(-77500000/5711*(sqrt(341)*200^(1/4)*sqrt(31)*sqrt(2*x^2 - x +
 3)*(sqrt(2)*(281*x - 444) + 163*x - 725)*sqrt(-772850000*sqrt(2) + 2500000
```

```
000) - 4337504500*x^2 - 3894902000*sqrt(2)*(2*x^2 - x + 3) + 13366595500*x
- 17704100000)/x^2) - 965935696000*(3948*x^3 - 23592*x^2 + 19767*x - 39005)
*sqrt(2*x^2 - x + 3))/(4*x^4 - 4*x^3 + 13*x^2 - 6*x + 9)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^2 - x + 3\right)^{\frac{5}{2}} \left(5x^2 + 3x + 2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2),x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.98 $\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx$

**Optimal**. Leaf size=234

$$-\frac{15101-8654x}{1035276\left(2x^2-x+3\right)^{3/2}}-\frac{1352542x+3133427}{523849656\sqrt{2x^2-x+3}}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}+\frac{625\sqrt{\frac{1}{682}\left(30463+23\right.}}{}$$

```
[Out] -(15101 - 8654*x)/(1035276*(3 - x + 2*x^2)^(3/2)) - (3133427 + 1352542*x)/(
523849656*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 +
 3*x + 5*x^2)) + (625*Sqrt[(30463 + 23600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31
*(30463 + 23600*Sqrt[2]))]*(203 + 242*Sqrt[2] + (687 + 445*Sqrt[2])*x))/Sqr
t[3 - x + 2*x^2]])/660176 - (625*Sqrt[(-30463 + 23600*Sqrt[2])/682]*ArcTanh
[(Sqrt[11/(31*(-30463 + 23600*Sqrt[2]))]*(203 - 242*Sqrt[2] + (687 - 445*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/660176
```

---

**Rubi [A]** time = 0.543007, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{15101-8654x}{1035276\left(2x^2-x+3\right)^{3/2}}-\frac{1352542x+3133427}{523849656\sqrt{2x^2-x+3}}+\frac{65x+4}{682\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}+\frac{625\sqrt{\frac{1}{682}\left(30463+23\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(15101 - 8654*x)/(1035276*(3 - x + 2*x^2)^(3/2)) - (3133427 + 1352542*x)/(
523849656*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(682*(3 - x + 2*x^2)^(3/2)*(2 +
 3*x + 5*x^2)) + (625*Sqrt[(30463 + 23600*Sqrt[2])/682]*ArcTan[(Sqrt[11/(31
*(30463 + 23600*Sqrt[2]))]*(203 + 242*Sqrt[2] + (687 + 445*Sqrt[2])*x))/Sqr
t[3 - x + 2*x^2]])/660176 - (625*Sqrt[(-30463 + 23600*Sqrt[2])/682]*ArcTanh
[(Sqrt[11/(31*(-30463 + 23600*Sqrt[2]))]*(203 - 242*Sqrt[2] + (687 - 445*Sq
rt[2])*x))/Sqrt[3 - x + 2*x^2]])/660176
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
```

0]

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx = \frac{4+65x}{682\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)} - \frac{\int \frac{-1738+\frac{4411x}{2}-5720x^2}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)}\,dx}{7502}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} + \frac{4+65x}{682\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)} - \frac{\int \frac{-\frac{940666\ldots}{2}}{(3-\ldots}}{\ldots}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+6\ldots}{682\left(3-x+2x^2\right)^{3/\ldots}}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+6\ldots}{682\left(3-x+2x^2\right)^{3/\ldots}}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+6\ldots}{682\left(3-x+2x^2\right)^{3/\ldots}}$$

$$= -\frac{15101-8654x}{1035276\left(3-x+2x^2\right)^{3/2}} - \frac{3133427+1352542x}{523849656\sqrt{3-x+2x^2}} + \frac{4+6\ldots}{682\left(3-x+2x^2\right)^{3/\ldots}}$$

**Mathematica [C]** time = 1.16162, size = 296, normalized size = 1.26

$$-\frac{5456\left(13525420x^5+32686812x^4+2879479x^3+84671384x^2-5712309x+31010342\right)+198375i\sqrt{286+22i\sqrt{31}}\,(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^2),x]
```

```
[Out] -(5456*(31010342 - 5712309*x + 84671384*x^2 + 2879479*x^3 + 32686812*x^4 +
13525420*x^5) + (198375*I)*Sqrt[286 + (22*I)*Sqrt[31]]*(31*I + 687*Sqrt[31]
)*Sqrt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*ArcTanh[(63 + I*Sqr
t[31] + (-22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x
 + 2*x^2])] + 198375*Sqrt[286 - (22*I)*Sqrt[31]]*(31 + (687*I)*Sqrt[31])*Sq
rt[3 - x + 2*x^2]*(6 + 7*x + 16*x^2 + x^3 + 10*x^4)*ArcTanh[(-63 + I*Sqrt[3
1] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]*Sqrt[3 - x + 2
*x^2])])/(2858123723136*(3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2))
```

---

**Maple [B]** time = 0.175, size = 5975, normalized size = 25.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5x^2+3x+2\right)^2\left(2x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^2*(2*x^2 - x + 3)^(5/2)), x)
```

---

**Fricas [B]** time = 4.83618, size = 8498, normalized size = 36.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="fricas")
```

```
[Out] 1/256043356025379141 12*(301208632500*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)
*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)*sqrt(30463*sqrt(2) +
47200)*arctan(1/11117215998613*(168268*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(
321084*x^7 - 1338894*x^6 + 2762802*x^5 - 4721048*x^4 + 2438224*x^3 - 131731
2*x^2 - sqrt(2)*(277258*x^7 - 994619*x^6 + 2123978*x^5 - 3198193*x^4 + 1552
680*x^3 - 621000*x^2 - 1900800*x + 1181952) - 2363904*x + 1900800) + 1829*6
962^(1/4)*sqrt(341)*(25187*x^7 - 392073*x^6 + 2114488*x^5 - 4948060*x^4 + 6
460704*x^3 - 4452768*x^2 - sqrt(2)*(20477*x^7 - 310452*x^6 + 1610140*x^5 -
3584192*x^4 + 4580640*x^3 - 2620800*x^2 - 3400704*x + 2198016) - 4396032*x
+ 3400704))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) + 3155854864122
4*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(118/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(3
41)*(1050904*x^7 - 1523916*x^6 + 5005956*x^5 - 2572736*x^4 + 3615264*x^3 +
877824*x^2 - sqrt(2)*(1065206*x^7 - 1518091*x^6 + 4815081*x^5 - 1448880*x^4
 + 1303560*x^3 + 3131136*x^2 - 3131136*x) - 877824*x) + 1829*6962^(1/4)*sqr
t(341)*(84981*x^7 - 1100084*x^6 + 4256060*x^5 - 5639616*x^4 + 7745184*x^3 +
 2571264*x^2 - 242*sqrt(2)*(319*x^7 - 4124*x^6 + 15860*x^5 - 20160*x^4 + 24
480*x^3 + 20736*x^2 - 20736*x) - 2571264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463
*sqrt(2) + 47200) + 187549318*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1
578888*x^6 - 3293072*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*
(15550*x^8 - 118051*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x
^3 + 1209600*x^2 - 1036800*x) + 3276288*x) + 8524969*sqrt(31)*(254591*x^8 -
 4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
 168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*
x^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(6962^(1/4)*sqrt(
341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x
+ 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)*(2*x^
2 - x + 3) + 676362371*x - 895844200)/x^2) + 358619870923*sqrt(31)*(2828123
*x^8 - 9696916*x^7 + 53385560*x^6 - 142835344*x^5 + 254146592*x^4 - 2493000
```

```
96*x^3 + 37981440*x^2 - 7744*sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 1007
0*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^2 + 4320*x - 5184) + 223064064*x - 94
887936))/(2585191*x^8 - 4661200*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*
x^4 + 44249088*x^3 - 34615296*x^2 - 24772608*x + 18579456)) + 301208632500*
6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*
x^2 + 15*x + 18)*sqrt(30463*sqrt(2) + 47200)*arctan(1/11117215998613*(16826
8*sqrt(118)*(22*6962^(3/4)*sqrt(341)*(321084*x^7 - 1338894*x^6 + 2762802*x^
5 - 4721048*x^4 + 2438224*x^3 - 1317312*x^2 - sqrt(2)*(277258*x^7 - 994619*
x^6 + 2123978*x^5 - 3198193*x^4 + 1552680*x^3 - 621000*x^2 - 1900800*x + 11
81952) - 2363904*x + 1900800) + 1829*6962^(1/4)*sqrt(341)*(25187*x^7 - 3920
73*x^6 + 2114488*x^5 - 4948060*x^4 + 6460704*x^3 - 4452768*x^2 - sqrt(2)*(2
0477*x^7 - 310452*x^6 + 1610140*x^5 - 3584192*x^4 + 4580640*x^3 - 2620800*x
^2 - 3400704*x + 2198016) - 4396032*x + 3400704))*sqrt(2*x^2 - x + 3)*sqrt(
30463*sqrt(2) + 47200) - 31558548641224*sqrt(31)*sqrt(2)*(28180*x^8 - 25466
6*x^7 + 704270*x^6 - 1385256*x^5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - s
qrt(2)*(8746*x^8 - 102335*x^7 + 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752
088*x^3 + 396144*x^2 + 546048*x - 539136) + 1154304*x - 456192) - 2*sqrt(11
8/79)*(sqrt(118)*(22*6962^(3/4)*sqrt(341)*(1050904*x^7 - 1523916*x^6 + 5005
956*x^5 - 2572736*x^4 + 3615264*x^3 + 877824*x^2 - sqrt(2)*(1065206*x^7 - 1
518091*x^6 + 4815081*x^5 - 1448880*x^4 + 1303560*x^3 + 3131136*x^2 - 313113
6*x) - 877824*x) + 1829*6962^(1/4)*sqrt(341)*(84981*x^7 - 1100084*x^6 + 425
6060*x^5 - 5639616*x^4 + 7745184*x^3 + 2571264*x^2 - 242*sqrt(2)*(319*x^7 -
 4124*x^6 + 15860*x^5 - 20160*x^4 + 24480*x^3 + 20736*x^2 - 20736*x) - 2571
264*x))*sqrt(2*x^2 - x + 3)*sqrt(30463*sqrt(2) + 47200) - 187549318*sqrt(31
)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 + 396480*x^4
 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 + 244047*x^6
- 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800*x) + 32762
88*x) - 8524969*sqrt(31)*(254591*x^8 - 4815126*x^7 + 32303580*x^6 - 9086680
8*x^5 + 108781920*x^4 - 74219328*x^3 - 168956928*x^2 - 15488*sqrt(2)*(4*x^8
 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 3618*x^3 + 2268*x^2 - 1944*x) +
 144820224*x))*sqrt((6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x
 + 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sqrt(30463*sqrt(2) + 47200) + 21
9481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x + 3) - 676362371*x + 895844200)/
x^2) - 358619870923*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 14
2835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*(
1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x^
2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 +
14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 24
772608*x + 18579456)) + 991875*6962^(1/4)*sqrt(341)*sqrt(118)*sqrt(31)*(944
000*x^6 - 377600*x^5 + 2879200*x^4 + 47200*x^3 + 2501600*x^2 - 30463*sqrt(2
)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) + 708000*x + 849600)
*sqrt(30463*sqrt(2) + 47200)*log(7375000000000/79*(6962^(1/4)*sqrt(341)*sqr
t(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) - 277*x + 75)*sq
rt(30463*sqrt(2) + 47200) + 219481829*x^2 + 197085724*sqrt(2)*(2*x^2 - x +
3) - 676362371*x + 895844200)/x^2) - 991875*6962^(1/4)*sqrt(341)*sqrt(118)*
sqrt(31)*(944000*x^6 - 377600*x^5 + 2879200*x^4 + 47200*x^3 + 2501600*x^2 -
 30463*sqrt(2)*(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18) + 70800
0*x + 849600)*sqrt(30463*sqrt(2) + 47200)*log(-7375000000000/79*(6962^(1/4)
*sqrt(341)*sqrt(118)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(101*x + 176) -
277*x + 75)*sqrt(30463*sqrt(2) + 47200) - 219481829*x^2 - 197085724*sqrt(2)
*(2*x^2 - x + 3) + 676362371*x - 895844200)/x^2) - 48877259552*(13525420*x^
5 + 32686812*x^4 + 2879479*x^3 + 84671384*x^2 - 5712309*x + 31010342)*sqrt(
2*x^2 - x + 3))/(20*x^6 - 8*x^5 + 61*x^4 + x^3 + 53*x^2 + 15*x + 18)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^2-x+3\right)^{\frac{5}{2}}\left(5x^2+3x+2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**2,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.99 $\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx$

**Optimal**. Leaf size=269

$$-\frac{1134826571-1504660754x}{476353953856\sqrt{2x^2-x+3}}+\frac{86885x+46386}{1860496\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}-\frac{12280939-19536786x}{2824232928\left(2x^2-x+3\right)^{3/2}}+\frac{\cdots}{1364\left(2x^2\cdots\right.}$$

```
[Out] -(12280939 - 19536786*x)/(2824232928*(3 - x + 2*x^2)^(3/2)) - (1134826571 -
 1504660754*x)/(476353953856*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*(3 - x
 + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + (46386 + 86885*x)/(1860496*(3 - x +
2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (35*Sqrt[(2243059557247 + 2011748500000*S
qrt[2])/682]*ArcTan[(Sqrt[11/(31*(2243059557247 + 2011748500000*Sqrt[2]))]*
(1432939 + 2428746*Sqrt[2] + (6290431 + 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/1800960128 - (35*Sqrt[(-2243059557247 + 2011748500000*Sqrt[2])/682]
*ArcTanh[(Sqrt[11/(31*(-2243059557247 + 2011748500000*Sqrt[2]))]*(1432939 -
 2428746*Sqrt[2] + (6290431 - 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/18
00960128
```

---

**Rubi [A]** time = 0.589435, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {974, 1060, 1035, 1029, 206, 204}

$$-\frac{1134826571-1504660754x}{476353953856\sqrt{2x^2-x+3}}+\frac{86885x+46386}{1860496\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)}-\frac{12280939-19536786x}{2824232928\left(2x^2-x+3\right)^{3/2}}+\frac{\cdots}{1364\left(2x^2\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] -(12280939 - 19536786*x)/(2824232928*(3 - x + 2*x^2)^(3/2)) - (1134826571 -
 1504660754*x)/(476353953856*Sqrt[3 - x + 2*x^2]) + (4 + 65*x)/(1364*(3 - x
 + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + (46386 + 86885*x)/(1860496*(3 - x +
2*x^2)^(3/2)*(2 + 3*x + 5*x^2)) + (35*Sqrt[(2243059557247 + 2011748500000*S
qrt[2])/682]*ArcTan[(Sqrt[11/(31*(2243059557247 + 2011748500000*Sqrt[2]))]*
(1432939 + 2428746*Sqrt[2] + (6290431 + 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2
*x^2]])/1800960128 - (35*Sqrt[(-2243059557247 + 2011748500000*Sqrt[2])/682]
*ArcTanh[(Sqrt[11/(31*(-2243059557247 + 2011748500000*Sqrt[2]))]*(1432939 -
 2428746*Sqrt[2] + (6290431 - 3861685*Sqrt[2])*x))/Sqrt[3 - x + 2*x^2]])/18
00960128
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
```

```
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1060

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((a + b*x +
 c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q + 1)*((A*c - a*C)*(2*a*c*e - b*(c*d +
a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*(A*(2*c^2*d + b
^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*e + a*b*f) + C*(b^2*d - a*b*e -
2*a*(c*d - a*f)))*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*
f))*(p + 1)), x] + Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b
*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^
2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*
(c*C*d - B*c*e - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^
2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d +
 A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*
(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*
c*e + a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*
p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] &&
 NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f
)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !
IGtQ[q, 0]
```

Rule 1035

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f), 2]}, Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d - a
*f - q) - (g*(c*e - b*f) - h*(c*d - a*f + q))*x, x]/((a + b*x + c*x^2)*Sqrt
[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[h*(b*d - a*e) - g*(c*d
- a*f + q) - (g*(c*e - b*f) - h*(c*d - a*f - q))*x, x]/((a + b*x + c*x^2)*S
qrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[b*d - a*e, 0] && NegQ[b^2 -
4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^3}\,dx = \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} - \frac{\int \frac{-5687+\frac{8635x}{2}-8580x^2}{\left(3-x+2x^2\right)^{5/2}\left(2+3x+5x^2\right)^2}\,dx}{15004}$$

$$= \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} + \frac{46386+86885x}{1860496\left(3-x+2x^2\right)^{3/2}\left(2+3x+\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} + \frac{4+65x}{1364\left(3-x+2x^2\right)^{3/2}\left(2+3x+5x^2\right)^2} + \frac{}{18}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{}{1364\left(3-x\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{}{1364\left(3-x\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{}{1364\left(3-x\right.}$$

$$= -\frac{12280939-19536786x}{2824232928\left(3-x+2x^2\right)^{3/2}} - \frac{1134826571-1504660754x}{476353953856\sqrt{3-x+2x^2}} + \frac{}{1364\left(3-x\right.}$$

**Mathematica [C]** time = 1.97068, size = 242, normalized size = 0.9

$$\frac{\frac{5456\left(225699113100x^7-12234606480x^6+592923725931x^5+174241614961x^4+519223213785x^3+178650961091x^2+218659985088x+9739335532\right)}{\left(2x^2-x+3\right)^{3/2}\left(5x^2+3x+2\right)^2}+1110}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 - x + 2*x^2)^(5/2)*(2 + 3*x + 5*x^2)^3),x]
```

```
[Out] ((5456*(9739335532 + 218659985088*x + 178650961091*x^2 + 519223213785*x^3 +
 174241614961*x^4 + 592923725931*x^5 - 12234606480*x^6 + 225699113100*x^7))
/((3 - x + 2*x^2)^(3/2)*(2 + 3*x + 5*x^2)^2) + 11109*Sqrt[286 + (22*I)*Sqrt
[31]]*(4541903 - (6290431*I)*Sqrt[31])*ArcTanh[(63 + I*Sqrt[31] + (-22 - (4
*I)*Sqrt[31])*x)/(2*Sqrt[286 + (22*I)*Sqrt[31]]*Sqrt[3 - x + 2*x^2])] - (11
109*I)*Sqrt[286 - (22*I)*Sqrt[31]]*(-4541903*I + 6290431*Sqrt[31])*ArcTanh[
(-63 + I*Sqrt[31] + (22 - (4*I)*Sqrt[31])*x)/(2*Sqrt[286 - (22*I)*Sqrt[31]]
```

```
*Sqrt[3 - x + 2*x^2])])/7796961516715008
```

**Maple [B]** time = 0.277, size = 19014, normalized size = 70.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x)
```

```
[Out] result too large to display
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5\,x^2+3\,x+2\right)^3\left(2\,x^2-x+3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 3*x + 2)^3*(2*x^2 - x + 3)^(5/2)), x)
```

**Fricas [B]** time = 5.31818, size = 11142, normalized size = 41.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="fricas")
```

```
[Out] 1/611377875290135815296770157063555072*(2164988593398757980*129508224872072
^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 4
023497000000)*arctan(1/4525340115746282619252370338578594 39*(11475013444*sq
rt(4023497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 117686842
22*x^6 + 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*
x^2 - sqrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626
609*x^4 + 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) -
 20220682752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(
214583731*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 576317
17152*x^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 146
88003420*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 311565
03552*x + 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*
sqrt(2243059557247*sqrt(2) + 4023497000000) + 128461267801829958223938254 77
25536472*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^
5 + 1549144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 +
 396104*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x
 - 539136) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(402349
7)*(11*129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 +
39777303828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt
```

(2)*(10387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4
+ 8224291080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 12472
8407*129508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 3480
3726780*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(
2)*(367903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 27
054001440*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x
^2 - x + 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 11194868609848920
9076292438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 329307
2*x^5 + 396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051
*x^7 + 244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 -
1036800*x) + 3276288*x) + 5088576640840418594376929*sqrt(31)*(254591*x^8 -
4815126*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 -
168956928*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x
^4 - 3618*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt(-(129508224872072^(
1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*
x + 2195288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000
000) - 19211019462513817817 83*x^2 - 17250711354094040489 48*sqrt(2)*(2*x^2 -
x + 3) + 5920130487427727531617*x - 7841232433679109313400)/x^2) + 1459787
134111704070726571076960 8369*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560
*x^6 - 142835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*
sqrt(2)*(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3
+ 1080*x^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 46612
00*x^7 + 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296
*x^2 - 24772608*x + 18579456)) + 2164988593398757980*129508224872072^(1/4)*
sqrt(4023497)*sqrt(341)*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390
*x^4 + 236*x^3 + 241*x^2 + 84*x + 36)*sqrt(2243059557247*sqrt(2) + 40234970
00000)*arctan(1/4525340115746282619252370338578594 39*(11475013444*sqrt(4023
497)*(11*129508224872072^(3/4)*sqrt(341)*(2673027292*x^7 - 11768684222*x^6
+ 24008796626*x^5 - 42687622824*x^4 + 22428040912*x^3 - 12956821056*x^2 - s
qrt(2)*(2612082154*x^7 - 9010050347*x^6 + 19426337114*x^5 - 28170626609*x^4
+ 13394761640*x^3 - 4698131400*x^2 - 17594323200*x + 10110341376) - 202206
82752*x + 17594323200) + 124728407*129508224872072^(1/4)*sqrt(341)*(2145837
31*x^7 - 3372306249*x^6 + 18434388344*x^5 - 43845503580*x^4 + 57631717152*x
^3 - 41786349984*x^2 - sqrt(2)*(190078101*x^7 - 2862100476*x^6 + 1468800342
0*x^5 - 32231022496*x^4 + 40927641120*x^3 - 21959568000*x^2 - 31156503552*x
+ 19060075008) - 38120150016*x + 31156503552))*sqrt(2*x^2 - x + 3)*sqrt(22
43059557247*sqrt(2) + 4023497000000) - 1284612678018299582239382547725536 47
2*sqrt(31)*sqrt(2)*(28180*x^8 - 254666*x^7 + 704270*x^6 - 1385256*x^5 + 154
9144*x^4 - 642048*x^3 - 98496*x^2 - sqrt(2)*(8746*x^8 - 102335*x^7 + 396104
*x^6 - 783113*x^5 + 1320710*x^4 - 752088*x^3 + 396144*x^2 + 546048*x - 5391
36) + 1154304*x - 456192) - 2*sqrt(8046994/10139750351)*(sqrt(4023497)*(11*
129508224872072^(3/4)*sqrt(341)*(8140972152*x^7 - 11907581308*x^6 + 3977730
3828*x^5 - 24395365568*x^4 + 37103094432*x^3 - 1836165888*x^2 - sqrt(2)*(10
387383478*x^7 - 14753211883*x^6 + 46462095753*x^5 - 11926110640*x^4 + 82242
91080*x^3 + 34793549568*x^2 - 34793549568*x) + 1836165888*x) + 124728407*12
9508224872072^(1/4)*sqrt(341)*(692762453*x^7 - 8972954292*x^6 + 34803726780
*x^5 - 46915651008*x^4 + 67421983392*x^3 + 10625375232*x^2 - 2*sqrt(2)*(367
903387*x^7 - 4754813452*x^6 + 18261523780*x^5 - 22991417280*x^4 + 270540014
40*x^3 + 26759248128*x^2 - 26759248128*x) - 10625375232*x))*sqrt(2*x^2 - x
+ 3)*sqrt(2243059557247*sqrt(2) + 4023497000000) - 1119486860984892090762 92
438*sqrt(31)*sqrt(2)*(123408*x^8 - 914152*x^7 + 1578888*x^6 - 3293072*x^5 +
396480*x^4 + 798336*x^3 - 3822336*x^2 - sqrt(2)*(15550*x^8 - 118051*x^7 +
244047*x^6 - 707374*x^5 + 1053960*x^4 - 1667952*x^3 + 1209600*x^2 - 1036800
*x) + 3276288*x) - 5088576640840418594376929*sqrt(31)*(254591*x^8 - 4815126
*x^7 + 32303580*x^6 - 90866808*x^5 + 108781920*x^4 - 74219328*x^3 - 1689569
28*x^2 - 15488*sqrt(2)*(4*x^8 - 76*x^7 + 517*x^6 - 1536*x^5 + 2385*x^4 - 36
18*x^3 + 2268*x^2 - 1944*x) + 144820224*x))*sqrt((129508224872072^(1/4)*sqr
t(4023497)*sqrt(341)*sqrt(31)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195
288) - 2838501*x + 1552075)*sqrt(2243059557247*sqrt(2) + 4023497000000) + 1

```
921101946251381781783*x^2 + 1725071135409404048948*sqrt(2)*(2*x^2 - x + 3)
- 5920130487427727531617*x + 7841232433679109313400)/x^2) - 145978713411170
40707265710769608369*sqrt(31)*(2828123*x^8 - 9696916*x^7 + 53385560*x^6 - 1
42835344*x^5 + 254146592*x^4 - 249300096*x^3 + 37981440*x^2 - 7744*sqrt(2)*
(1348*x^8 - 2692*x^7 + 9789*x^6 - 10070*x^5 + 15569*x^4 - 5568*x^3 + 1080*x
^2 + 4320*x - 5184) + 223064064*x - 94887936))/(2585191*x^8 - 4661200*x^7 +
 14191920*x^6 + 490880*x^5 - 13562944*x^4 + 44249088*x^3 - 34615296*x^2 - 2
4772608*x + 18579456)) + 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(341
)*sqrt(31)*(402349700000000*x^8 + 80469940000000*x^7 + 1291542537000000*x^6
 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96966
2777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172*x^
5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484589
2000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(246439191250000000
0000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31)*sq
rt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*sqrt
(2243059557247*sqrt(2) + 4023497000000) + 1921101946251381781783*x^2 + 1725
071135409404048948*sqrt(2)*(2*x^2 - x + 3) - 5920130487427727531617*x + 784
1232433679109313400)/x^2) - 55545*129508224872072^(1/4)*sqrt(4023497)*sqrt(
341)*sqrt(31)*(402349700000000*x^8 + 80469940000000*x^7 + 1291542537000000*
x^6 + 692041484000000*x^5 + 1569163830000000*x^4 + 949545292000000*x^3 + 96
9662777000000*x^2 - 2243059557247*sqrt(2)*(100*x^8 + 20*x^7 + 321*x^6 + 172
*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36) + 337973748000000*x + 14484
5892000000)*sqrt(2243059557247*sqrt(2) + 4023497000000)*log(-24643919125000
00000000/10139750351*(129508224872072^(1/4)*sqrt(4023497)*sqrt(341)*sqrt(31
)*sqrt(2*x^2 - x + 3)*(sqrt(2)*(643213*x + 2195288) - 2838501*x + 1552075)*
sqrt(2243059557247*sqrt(2) + 4023497000000) - 1921101946251381781783*x^2 -
1725071135409404048948*sqrt(2)*(2*x^2 - x + 3) + 5920130487427727531617*x -
 7841232433679109313400)/x^2) + 427817641581532204139104*(225699113100*x^7
- 12234606480*x^6 + 592923725931*x^5 + 174241614961*x^4 + 519223213785*x^3
+ 178650961091*x^2 + 218659985088*x + 9739335532)*sqrt(2*x^2 - x + 3))/(100
*x^8 + 20*x^7 + 321*x^6 + 172*x^5 + 390*x^4 + 236*x^3 + 241*x^2 + 84*x + 36
)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(2x^2 - x + 3\right)^{\frac{5}{2}} \left(5x^2 + 3x + 2\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2-x+3)**(5/2)/(5*x**2+3*x+2)**3,x)
```

```
[Out] Integral(1/((2*x**2 - x + 3)**(5/2)*(5*x**2 + 3*x + 2)**3), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2-x+3)^(5/2)/(5*x^2+3*x+2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.100 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2\,dx$

**Optimal**. Leaf size=436

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b}{512c^5}$$

```
[Out] ((128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(
e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(b +
2*c*x)*Sqrt[a + b*x + c*x^2])/(512*c^5) + ((640*c^3*d*e - 105*b^3*f^2 + 28*
b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(a + b*x +
c*x^2)^(3/2))/(960*c^4) + ((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(e
^2 + 2*d*f))*x*(a + b*x + c*x^2)^(3/2))/(160*c^3) + (f*(8*c*e - 3*b*f)*x^2*
(a + b*x + c*x^2)^(3/2))/(20*c^2) + (f^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c)
 - ((b^2 - 4*a*c)*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c
^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2
 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c
^(11/2))
```

---

**Rubi [A]** time = 0.789373, antiderivative size = 436, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(8c^2\left(2a^2f^2+12abef+5b^2\left(2df+e^2\right)\right)-56b^2cf(af+be)-32c^3\left(a\left(2df+e^2\right)+4bde\right)+21b}{512c^5}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```


```
[Out] ((128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c^3*(4*b*d*e + a*(
e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2 + 2*d*f)))*(b +
2*c*x)*Sqrt[a + b*x + c*x^2])/(512*c^5) + ((640*c^3*d*e - 105*b^3*f^2 + 28*
b*c*f*(10*b*e + 7*a*f) - 8*c^2*(32*a*e*f + 25*b*(e^2 + 2*d*f)))*(a + b*x +
c*x^2)^(3/2))/(960*c^4) + ((21*b^2*f^2 - 4*c*f*(14*b*e + 5*a*f) + 40*c^2*(e
^2 + 2*d*f))*x*(a + b*x + c*x^2)^(3/2))/(160*c^3) + (f*(8*c*e - 3*b*f)*x^2*
(a + b*x + c*x^2)^(3/2))/(20*c^2) + (f^2*x^3*(a + b*x + c*x^2)^(3/2))/(6*c)
 - ((b^2 - 4*a*c)*(128*c^4*d^2 + 21*b^4*f^2 - 56*b^2*c*f*(b*e + a*f) - 32*c
^3*(4*b*d*e + a*(e^2 + 2*d*f)) + 8*c^2*(12*a*b*e*f + 2*a^2*f^2 + 5*b^2*(e^2
 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c
^(11/2))
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2 dx = \frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2}\left(6cd^2+12cdex-3\left(af^2-2c\left(e^2+2df\right.\right.\right.}{6c}$$

$$= \frac{f(8ce-3bf)x^2\left(a+bx+cx^2\right)^{3/2}}{20c^2} + \frac{f^2x^3\left(a+bx+cx^2\right)^{3/2}}{6c} + \frac{\int \sqrt{a+bx+cx^2}\left(30\right.}{}$$

$$= \frac{\left(21b^2f^2-4cf(14be+5af)+40c^2\left(e^2+2df\right)\right)x\left(a+bx+cx^2\right)^{3/2}}{160c^3} + \frac{f(8ce-3bf)}{}$$

$$= \frac{\left(640c^3de-105b^3f^2+28bcf(10be+7af)-8c^2\left(32aef+25b\left(e^2+2df\right)\right)\right)\left(a+b\right.}{960c^4}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8c^2\left(12abef\right.\right.}{512c^5}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8c^2\left(12abef\right.\right.}{512c^5}$$

$$= \frac{\left(128c^4d^2+21b^4f^2-56b^2cf(be+af)-32c^3\left(4bde+a\left(e^2+2df\right)\right)+8c^2\left(12abef\right.\right.}{512c^5}$$

**Mathematica [A]** time = 0.964738, size = 657, normalized size = 1.51

$$-f^2\left(-15\left(16a^2c^2-56ab^2c+21b^4\right)\left(2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}-\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\right)+16c^{3/2}\left(-196abc\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2,x]
```

```
[Out] (3840*c^(9/2)*d^2*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + 10240*c^(9/2)*d*e*(a
+ x*(b + c*x))^(3/2) + 3840*c^(9/2)*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(3/2)
 + 6144*c^(9/2)*e*f*x^2*(a + x*(b + c*x))^(3/2) + 2560*c^(9/2)*f^2*x^3*(a +
```

```
x*(b + c*x))^(3/2) - 1920*c^4*(b^2 - 4*a*c)*d^2*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + x*(b + c*x)])] - 1920*b*c^3*d*e*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a
 + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]) + 8*c*e*f*(-16*c^(3/2)*(-35*b^2 + 32*a*c + 42*b*c*x)*(a + x*(b
 + c*x))^(3/2) - 15*b*(7*b^2 - 12*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b
 + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x
)])])) - 40*c^2*(e^2 + 2*d*f)*(80*b*c^(3/2)*(a + x*(b + c*x))^(3/2) - 3*(5*
b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])) - f^2*(2304*b*c^(7/
2)*x^2*(a + x*(b + c*x))^(3/2) + 16*c^(3/2)*(105*b^3 - 196*a*b*c - 126*b^2*
c*x + 120*a*c^2*x)*(a + x*(b + c*x))^(3/2) - 15*(21*b^4 - 56*a*b^2*c + 16*a
^2*c^2)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTan
h[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/(15360*c^(11/2))
```

---

**Maple [B]** time = 0.063, size = 1429, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x)
```

```
[Out] 3/8*e*f*b/c^2*a*x*(c*x^2+b*x+a)^(1/2)-7/32*e*f*b^3/c^3*x*(c*x^2+b*x+a)^(1/2
)-5/16*e*f*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/16*e
*f*b^2/c^3*a*(c*x^2+b*x+a)^(1/2)+3/8*e*f*b/c^(5/2)*a^2*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))+2/3*d*e*(c*x^2+b*x+a)^(3/2)/c-7/64*f^2*b^3/c^4*(c*x
^2+b*x+a)^(3/2)+21/512*f^2*b^5/c^5*(c*x^2+b*x+a)^(1/2)-21/1024*f^2*b^6/c^(1
1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/16*f^2*a^3/c^(5/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/128*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))*e^2-1/8*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))*e^2+1/4*d^2/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d^2/c^(1/2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))*b^2-7/32*f^2*b^2/c^3*a*x*(c*x^2+b*x+a)^(1/2)-7/20*e*
f*b/c^2*x*(c*x^2+b*x+a)^(3/2)-1/8*a/c^2*(c*x^2+b*x+a)^(1/2)*b*d*f+5/16*b^2/
c^2*x*(c*x^2+b*x+a)^(1/2)*d*f-1/4*a/c*x*(c*x^2+b*x+a)^(1/2)*d*f-1/2*d*e*b/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/8*b^2/c^(5/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*f-1/2*d*e*b/c*x*(c*x^2+b*x+a)^(1/
2)+7/24*e*f*b^2/c^3*(c*x^2+b*x+a)^(3/2)-7/64*e*f*b^4/c^4*(c*x^2+b*x+a)^(1/2
)-1/4*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f+1/2*d^2*x
*(c*x^2+b*x+a)^(1/2)-5/12*b/c^2*(c*x^2+b*x+a)^(3/2)*d*f-1/16*a/c^2*(c*x^2+b
*x+a)^(1/2)*b*e^2+1/8*d*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))+5/32*b^2/c^2*x*(c*x^2+b*x+a)^(1/2)*e^2+5/32*b^3/c^3*(c*x^2+b*x+a)^(1
/2)*d*f+3/16*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*e^2-
5/64*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-1/8*a/c*x*
(c*x^2+b*x+a)^(1/2)*e^2+2/5*e*f*x^2*(c*x^2+b*x+a)^(3/2)/c+1/2*x*(c*x^2+b*x+
a)^(3/2)/c*d*f-1/8*f^2*a/c^2*x*(c*x^2+b*x+a)^(3/2)-1/4*d*e*b^2/c^2*(c*x^2+b
*x+a)^(1/2)-3/20*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(3/2)+1/16*f^2*a^2/c^2*x*(c*x^
2+b*x+a)^(1/2)+1/32*f^2*a^2/c^3*(c*x^2+b*x+a)^(1/2)*b+21/160*f^2*b^2/c^3*x*
(c*x^2+b*x+a)^(3/2)+21/256*f^2*b^4/c^4*x*(c*x^2+b*x+a)^(1/2)+35/256*f^2*b^4
/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-7/64*f^2*b^3/c^4*a*(
c*x^2+b*x+a)^(1/2)-15/64*f^2*b^2/c^(7/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))+49/240*f^2*b/c^3*a*(c*x^2+b*x+a)^(3/2)+7/128*e*f*b^5/c^(9/2)*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/15*e*f*a/c^2*(c*x^2+b*x+a)^(3
/2)+1/4*x*(c*x^2+b*x+a)^(3/2)/c*e^2-5/24*b/c^2*(c*x^2+b*x+a)^(3/2)*e^2+5/64
*b^3/c^3*(c*x^2+b*x+a)^(1/2)*e^2+1/6*f^2*x^3*(c*x^2+b*x+a)^(3/2)/c
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 2.83322, size = 2866, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a*b*c^4)*d*e
+ 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140*a*b^4*c + 2
40*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*
c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*sqrt(c)*log(-8*c^2*x
^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) -
 4*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*x^4 +
16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*e)*f)*x
^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e^2 + (31
5*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e + 40*b*c^5*
e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4 - 16*a*c^5
)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 - 460*a*b^2*
c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*(5*b^2*c^4 -
 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2 - 8*(10*(5
*b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^6, 1/15360*(15*(128*(b^2*c^4 - 4*a*c^5)*d^2 - 128*(b^3*c^3 - 4*a
*b*c^4)*d*e + 8*(5*b^4*c^2 - 24*a*b^2*c^3 + 16*a^2*c^4)*e^2 + (21*b^6 - 140
*a*b^4*c + 240*a^2*b^2*c^2 - 64*a^3*c^3)*f^2 + 8*(2*(5*b^4*c^2 - 24*a*b^2*c
^3 + 16*a^2*c^4)*d - (7*b^5*c - 40*a*b^3*c^2 + 48*a^2*b*c^3)*e)*f)*sqrt(-c)
*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a
*c)) + 2*(1280*c^6*f^2*x^5 + 1920*b*c^5*d^2 + 128*(24*c^6*e*f + b*c^5*f^2)*
x^4 + 16*(120*c^6*e^2 - (9*b^2*c^4 - 20*a*c^5)*f^2 + 24*(10*c^6*d + b*c^5*e
)*f)*x^3 - 640*(3*b^2*c^4 - 8*a*c^5)*d*e + 40*(15*b^3*c^3 - 52*a*b*c^4)*e^2
 + (315*b^5*c - 1680*a*b^3*c^2 + 1808*a^2*b*c^3)*f^2 + 8*(640*c^6*d*e + 40*
b*c^5*e^2 + (21*b^3*c^3 - 68*a*b*c^4)*f^2 + 8*(10*b*c^5*d - (7*b^2*c^4 - 16
*a*c^5)*e)*f)*x^2 + 8*(10*(15*b^3*c^3 - 52*a*b*c^4)*d - (105*b^4*c^2 - 460*
a*b^2*c^3 + 256*a^2*c^4)*e)*f + 2*(1920*c^6*d^2 + 640*b*c^5*d*e - 40*(5*b^2
*c^4 - 12*a*c^5)*e^2 - (105*b^4*c^2 - 448*a*b^2*c^3 + 240*a^2*c^4)*f^2 - 8*
(10*(5*b^2*c^4 - 12*a*c^5)*d - (35*b^3*c^3 - 116*a*b*c^4)*e)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^6]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + bx + cx^2} \left(d + ex + fx^2\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)**2, x)
```

---

**Giac [A]** time = 1.30159, size = 861, normalized size = 1.97

$$\frac{1}{7680}\sqrt{cx^2+bx+a}\left(2\left(4\left(2\left(8\left(10\,f^2x+\frac{bc^4f^2+24\,c^5fe}{c^5}\right)x+\frac{240\,c^5df-9\,b^2c^3f^2+20\,ac^4f^2+24\,bc^4fe+120\,c^5e^2}{c^5}\right)\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^2*x + (b*c^4*f^2 + 24*c^5*f*
e)/c^5)*x + (240*c^5*d*f - 9*b^2*c^3*f^2 + 20*a*c^4*f^2 + 24*b*c^4*f*e + 12
0*c^5*e^2)/c^5)*x + (80*b*c^4*d*f + 21*b^3*c^2*f^2 - 68*a*b*c^3*f^2 + 640*c
^5*d*e - 56*b^2*c^3*f*e + 128*a*c^4*f*e + 40*b*c^4*e^2)/c^5)*x + (1920*c^5*
d^2 - 400*b^2*c^3*d*f + 960*a*c^4*d*f - 105*b^4*c*f^2 + 448*a*b^2*c^2*f^2 -
 240*a^2*c^3*f^2 + 640*b*c^4*d*e + 280*b^3*c^2*f*e - 928*a*b*c^3*f*e - 200*
b^2*c^3*e^2 + 480*a*c^4*e^2)/c^5)*x + (1920*b*c^4*d^2 + 1200*b^3*c^2*d*f -
4160*a*b*c^3*d*f + 315*b^5*f^2 - 1680*a*b^3*c*f^2 + 1808*a^2*b*c^2*f^2 - 19
20*b^2*c^3*d*e + 5120*a*c^4*d*e - 840*b^4*c*f*e + 3680*a*b^2*c^2*f*e - 2048
*a^2*c^3*f*e + 600*b^3*c^2*e^2 - 2080*a*b*c^3*e^2)/c^5) + 1/1024*(128*b^2*c
^4*d^2 - 512*a*c^5*d^2 + 80*b^4*c^2*d*f - 384*a*b^2*c^3*d*f + 256*a^2*c^4*d
*f + 21*b^6*f^2 - 140*a*b^4*c*f^2 + 240*a^2*b^2*c^2*f^2 - 64*a^3*c^3*f^2 -
128*b^3*c^3*d*e + 512*a*b*c^4*d*e - 56*b^5*c*f*e + 320*a*b^3*c^2*f*e - 384*
a^2*b*c^3*f*e + 40*b^4*c^2*e^2 - 192*a*b^2*c^3*e^2 + 128*a^2*c^4*e^2)*log(a
bs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

# 3.101 $\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx$

**Optimal**. Leaf size=175

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8bce+16c^2d\right)}{64c^3}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16c\right.}{128c^{7/2}}$$

```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```

---

**Rubi [A]** time = 0.163777, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}\left(-4acf+5b^2f-8bce+16c^2d\right)}{64c^3}-\frac{\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+2be)+5b^2f+16c\right.}{128c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```


```
[Out] ((16*c^2*d - 8*b*c*e + 5*b^2*f - 4*a*c*f)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2]
)/(64*c^3) + ((8*c*e - 5*b*f)*(a + b*x + c*x^2)^(3/2))/(24*c^2) + (f*x*(a +
 b*x + c*x^2)^(3/2))/(4*c) - ((b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*
e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(128*c^(7
/2))
```


Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\int \left(4cd-af+\frac{1}{2}(8ce-5bf)x\right)\sqrt{a+bx+cx^2}\,dx}{4c}$$

$$= \frac{(8ce-5bf)\left(a+bx+cx^2\right)^{3/2}}{24c^2} + \frac{fx\left(a+bx+cx^2\right)^{3/2}}{4c} + \frac{\left(16c^2d-8bce+5b^2f-\right.}{1}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+bx+\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+bx+\right.}{24c^2}$$

$$= \frac{\left(16c^2d-8bce+5b^2f-4acf\right)(b+2cx)\sqrt{a+bx+cx^2}}{64c^3} + \frac{(8ce-5bf)\left(a+bx+\right.}{24c^2}$$

**Mathematica [A]** time = 0.292534, size = 173, normalized size = 0.99

$$\frac{2\sqrt{c}\sqrt{a+x(b+cx)}\left(4bc\left(2c\left(6d+2ex+fx^2\right)-13af\right)+8c^2\left(a(8e+3fx)+2cx\left(6d+4ex+3fx^2\right)\right)-2b^2c(12e+5fx\right.}{384c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(15*b^3*f - 2*b^2*c*(12*e + 5*f*x) + 4*b*c
*(-13*a*f + 2*c*(6*d + 2*e*x + f*x^2)) + 8*c^2*(a*(8*e + 3*f*x) + 2*c*x*(6*
d + 4*e*x + 3*f*x^2))) - 3*(b^2 - 4*a*c)*(16*c^2*d + 5*b^2*f - 4*c*(2*b*e +
 a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(384*c^(7/2)
)
```

---

**Maple [B]** time = 0.052, size = 453, normalized size = 2.6

$$\frac{fx}{4c}\left(cx^2+bx+a\right)^{\frac{3}{2}} - \frac{5bf}{24c^2}\left(cx^2+bx+a\right)^{\frac{3}{2}} + \frac{5b^2fx}{32c^2}\sqrt{cx^2+bx+a} + \frac{5b^3f}{64c^3}\sqrt{cx^2+bx+a} + \frac{3b^2fa}{16}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/4*f*x*(c*x^2+b*x+a)^(3/2)/c-5/24*f*b/c^2*(c*x^2+b*x+a)^(3/2)+5/32*f*b^2/c
^2*x*(c*x^2+b*x+a)^(1/2)+5/64*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)+3/16*f*b^2/c^(5
```

```
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-5/128*f*b^4/c^(7/2)*ln((1
/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/8*f*a/c*x*(c*x^2+b*x+a)^(1/2)-1/16
*f*a/c^2*(c*x^2+b*x+a)^(1/2)*b-1/8*f*a^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+1/3*e*(c*x^2+b*x+a)^(3/2)/c-1/4*e*b/c*x*(c*x^2+b*x+a)^(1/
2)-1/8*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)-1/4*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a+1/16*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))+1/2*d*x*(c*x^2+b*x+a)^(1/2)+1/4*d/c*(c*x^2+b*x+a)^(1/2)*b+1/2*d
/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a-1/8*d/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.49495, size = 1061, normalized size = 6.06

$$\left[\frac{3\left(16\left(b^{2}c^{2}-4\,ac^{3}\right)d-8\left(b^{3}c-4\,abc^{2}\right)e+\left(5\,b^{4}-24\,ab^{2}c+16\,a^{2}c^{2}\right)f\right)\sqrt{c}\log\left(-8\,c^{2}x^{2}-8\,bcx-b^{2}+4\,\sqrt{cx^{2}+bx+a}(2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c^2)*e + (5*b^4 - 24
*a*b^2*c + 16*a^2*c^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f*x^3 + 48*b*c^3*d
 + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15*b^3*c - 52*a
*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f)*x)*sqrt(c*x
^2 + b*x + a))/c^4, 1/384*(3*(16*(b^2*c^2 - 4*a*c^3)*d - 8*(b^3*c - 4*a*b*c
^2)*e + (5*b^4 - 24*a*b^2*c + 16*a^2*c^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2
 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(48*c^4*f*x^3
 + 48*b*c^3*d + 8*(8*c^4*e + b*c^3*f)*x^2 - 8*(3*b^2*c^2 - 8*a*c^3)*e + (15
*b^3*c - 52*a*b*c^2)*f + 2*(48*c^4*d + 8*b*c^3*e - (5*b^2*c^2 - 12*a*c^3)*f
)*x)*sqrt(c*x^2 + b*x + a))/c^4]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+bx+cx^{2}}\left(d+ex+fx^{2}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2), x)
```

---

**Giac [A]** time = 1.15989, size = 286, normalized size = 1.63

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(6fx+\frac{bc^2f+8c^3e}{c^3}\right)x+\frac{48c^3d-5b^2cf+12ac^2f+8bc^2e}{c^3}\right)x+\frac{48bc^2d+15b^3f-52abcf}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f*x + (b*c^2*f + 8*c^3*e)/c^3)*x + (48
*c^3*d - 5*b^2*c*f + 12*a*c^2*f + 8*b*c^2*e)/c^3)*x + (48*b*c^2*d + 15*b^3*
f - 52*a*b*c*f - 24*b^2*c*e + 64*a*c^2*e)/c^3) + 1/128*(16*b^2*c^2*d - 64*a
*c^3*d + 5*b^4*f - 24*a*b^2*c*f + 16*a^2*c^2*f - 8*b^3*c*e + 32*a*b*c^2*e)*
log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

# 3.102 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=431

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\dots$$

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

---

**Rubi [A]** time = 1.05133, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {989, 621, 206, 1032, 724}

$$-\frac{\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}+\dots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (Sqrt[
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2 - 4*d*f])
 + (Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

Rule 989

```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f,
Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),
x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 -
4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx}{f} + \frac{c\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f}$$

$$= \frac{(2c)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{}{\left(e-\sqrt{e^2-4df}\ldots\right.}}{f\sqrt{e^2-4df}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\left(2\left(2f(cd-af)-(ce-bf)\left(e-\sqrt{e^2-4df}\right)\right)\right)\mathrm{Subst}\left(\int \frac{}{16af^2-8bf\ldots}\right.}{f\sqrt{\ldots}}$$

$$= \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} - \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\tanh^{-1}\ldots}{\sqrt{2}f\sqrt{e^2-4df}}$$

**Mathematica [A]** time = 1.25489, size = 417, normalized size = 0.97

$$\frac{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

```
[Out] (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/f + (Sqrt[
c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]
))]*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*
f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a
*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])] - Sqrt[f*(-(b*e) +
```

```
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcT
anh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2
*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 -
2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*f*Sqrt[e^2
- 4*d*f])
```

---

**Maple [B]** time = 0.433, size = 6019, normalized size = 14.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] result too large to display
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.103 $\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx$

**Optimal**. Leaf size=488

$$-\frac{\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(f(be-4af)-\left(\sqrt{e^2\right.\right.}{\sqrt{2}\left(e\right.}$$

```
[Out] -(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) -
((f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b
*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((f*(b*e - 4*a*f) - (c
*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
 - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]])
```

---

**Rubi [A]** time = 2.92941, antiderivative size = 488, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {971, 1032, 724, 206}

$$-\frac{\left(f(be-4af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(f(be-4af)-\left(\sqrt{e^2\right.\right.}{\sqrt{2}\left(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] -(((e + 2*f*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*(d + e*x + f*x^2))) -
((f*(b*e - 4*a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b
*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((f*(b*e - 4*a*f) - (c
*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
 - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
```

```
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}\,dx = -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{\frac{1}{2}(be-4af)+(ce-bf)x}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx}{-e^2+4df}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\int \frac{\cdots}{\left(e-\sqrt{e^2-\cdots}\right.\cdots}}{\left(e^2-4df\right)^{3/2}}\cdots$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(2\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\right)\text{Subs}\cdots}{\cdots}$$

$$= -\frac{(e+2fx)\sqrt{a+bx+cx^2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(ce\left(e-\sqrt{e^2-4df}\right)+f\left(4af-b\left(2e-\sqrt{e^2-4df}\right)\right)\right)\tanh^{-1}\left(\cdots\right.}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{ce^2-2cdf-bef+2a\cdots}}$$

**Mathematica [A]** time = 5.4495, size = 555, normalized size = 1.14

$$\frac{4f(e+2fx)\sqrt{a+x(b+cx)}}{\left(e^2-4df\right)\left(\sqrt{e^2-4df}-e-2fx\right)\left(\sqrt{e^2-4df}+e+2fx\right)} + \frac{\left(ce\left(\sqrt{e^2-4df}-e\right)-f\left(4af+b\left(\sqrt{e^2-4df}-2e\right)\right)\right)\tanh\cdots}{\sqrt{2}\left(e^2-4df\right)^{3/2}\sqrt{f\left(2af+b\left(\sqrt{e^2-4\cdots}\right.\right.}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2)^2,x]
```

```
[Out] (4*f*(e + 2*f*x)*Sqrt[a + x*(b + c*x)])/((e^2 - 4*d*f)*(-e + Sqrt[e^2 - 4*d
*f] - 2*f*x)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + ((c*e*(-e + Sqrt[e^2 - 4*d*
```

```
f]) - f*(4*a*f + b*(-2*e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(-4*a*f + 2*c*(e -
Sqrt[e^2 - 4*d*f])*x + b*(e - Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c
*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]
))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]) - ((c*
e*(e + Sqrt[e^2 - 4*d*f]) + f*(4*a*f - b*(2*e + Sqrt[e^2 - 4*d*f])))*ArcTan
h[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x
))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e
+ Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(Sqrt[2]*(e^2 - 4*d*f)^(3/2
)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 -
 4*d*f]))])
```

---

**Maple [B]** time = 0.371, size = 22287, normalized size = 45.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{\left(fx^2 + ex + d\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + b*x + a)/(f*x^2 + e*x + d)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

## 3.104 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right)^2 \, dx$

**Optimal**. Leaf size=564

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)}{6144c^5}$$

```
[Out] -((b^2 - 4*a*c)*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 12
8*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2
*(e^2 + 2*d*f)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16384*c^6) + ((768*c^4
*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2
+ 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*(b + 2*
c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^5) + ((5376*c^3*d*e - 693*b^3*f^2 + 3
6*b*c*f*(56*b*e + 31*a*f) - 32*c^2*(48*a*e*f + 49*b*(e^2 + 2*d*f)))*(a + b*
x + c*x^2)^(5/2))/(13440*c^4) + ((99*b^2*f^2 - 12*c*f*(24*b*e + 7*a*f) + 22
4*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(5/2))/(1344*c^3) + (f*(32*c*e - 1
1*b*f)*x^2*(a + b*x + c*x^2)^(5/2))/(112*c^2) + (f^2*x^3*(a + b*x + c*x^2)^
(5/2))/(8*c) + ((b^2 - 4*a*c)^2*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b
*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*
a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(32768*c^(13/2))
```

---

**Rubi [A]** time = 0.934847, antiderivative size = 564, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(16c^2\left(3a^2f^2+24abef+14b^2\left(2df+e^2\right)\right)-72b^2cf(3af+4be)-128c^3\left(a\left(2df+e^2\right)+6bde\right)}{6144c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] -((b^2 - 4*a*c)*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 12
8*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2
*(e^2 + 2*d*f)))*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(16384*c^6) + ((768*c^4
*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2
+ 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*(b + 2*
c*x)*(a + b*x + c*x^2)^(3/2))/(6144*c^5) + ((5376*c^3*d*e - 693*b^3*f^2 + 3
6*b*c*f*(56*b*e + 31*a*f) - 32*c^2*(48*a*e*f + 49*b*(e^2 + 2*d*f)))*(a + b*
x + c*x^2)^(5/2))/(13440*c^4) + ((99*b^2*f^2 - 12*c*f*(24*b*e + 7*a*f) + 22
4*c^2*(e^2 + 2*d*f))*x*(a + b*x + c*x^2)^(5/2))/(1344*c^3) + (f*(32*c*e - 1
1*b*f)*x^2*(a + b*x + c*x^2)^(5/2))/(112*c^2) + (f^2*x^3*(a + b*x + c*x^2)^
(5/2))/(8*c) + ((b^2 - 4*a*c)^2*(768*c^4*d^2 + 99*b^4*f^2 - 72*b^2*c*f*(4*b
*e + 3*a*f) - 128*c^3*(6*b*d*e + a*(e^2 + 2*d*f)) + 16*c^2*(24*a*b*e*f + 3*
a^2*f^2 + 14*b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(32768*c^(13/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
```

```
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)^2\,dx = \frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c} + \frac{\int \left(a+bx+cx^2\right)^{3/2}\left(8cd^2+16cdex-\left(3af^2-8c\left(\right.\right.\right.}{8c}$$

$$= \frac{f(32ce-11bf)x^2\left(a+bx+cx^2\right)^{5/2}}{112c^2} + \frac{f^2x^3\left(a+bx+cx^2\right)^{5/2}}{8c} + \frac{\int \left(a+bx+\right.}{}$$

$$= \frac{\left(99b^2f^2-12cf(24be+7af)+224c^2\left(e^2+2df\right)\right)x\left(a+bx+cx^2\right)^{5/2}}{1344c^3} + \frac{f(3}{}$$

$$= \frac{\left(5376c^3de-693b^3f^2+36bcf(56be+31af)-32c^2\left(48aef+49b\left(e^2+2df\right.\right.\right.}{13440c^4}$$

$$= \frac{\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+a\left(e^2+2df\right)\right)+16\right.}{6144c^5}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+a\left(e^2\right.\right.\right.}{163}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+a\left(e^2\right.\right.\right.}{163}$$

$$= -\frac{\left(b^2-4ac\right)\left(768c^4d^2+99b^4f^2-72b^2cf(4be+3af)-128c^3\left(6bde+a\left(e^2\right.\right.\right.}{163}$$

**Mathematica [A]** time = 1.8282, size = 829, normalized size = 1.47

$$\frac{430080f^2(a+x(b+cx))^{5/2}x^3+983040ef(a+x(b+cx))^{5/2}x^2+573440\left(e^2+2df\right)(a+x(b+cx))^{5/2}x+1376256de(a+x\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)^2,x]
```

```
[Out] (430080*d^2*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 1376256*d*e*(a + x*(b + c
*x))^(5/2) + 573440*(e^2 + 2*d*f)*x*(a + x*(b + c*x))^(5/2) + 983040*e*f*x^
2*(a + x*(b + c*x))^(5/2) + 430080*f^2*x^3*(a + x*(b + c*x))^(5/2) + (80640
*(b^2 - 4*a*c)*d^2*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4
*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - (2
6880*b*d*e*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c
)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b +
 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])))/c^(5/2) + (96*e*f*(-256*c^(5/
2)*(-21*b^2 + 16*a*c + 30*b*c*x)*(a + x*(b + c*x))^(5/2) - 35*b*(3*b^2 - 4*
a*c)*(16*c^(3/2)*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*S
qrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x
)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]))))/c^(9/2) - (224*(e^2 + 2*d*f)*(1792
*b*c^(5/2)*(a + x*(b + c*x))^(5/2) - 5*(7*b^2 - 4*a*c)*(16*c^(3/2)*(b + 2*c
*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt[a
 + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(
b + c*x)])]))))/c^(7/2) - (3*f^2*(112640*b*c^(9/2)*x^2*(a + x*(b + c*x))^(5
/2) + 256*c^(5/2)*(231*b^3 - 372*a*b*c - 330*b^2*c*x + 280*a*c^2*x)*(a + x*
(b + c*x))^(5/2) - 35*(33*b^4 - 72*a*b^2*c + 16*a^2*c^2)*(16*c^(3/2)*(b + 2
*c*x)*(a + x*(b + c*x))^(3/2) - 3*(b^2 - 4*a*c)*(2*Sqrt[c]*(b + 2*c*x)*Sqrt
[a + x*(b + c*x)] - (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])]))))/c^(11/2))/(3440640*c)
```

---

**Maple [B]** time = 0.063, size = 2458, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x)
```

```
[Out] -7/512*b^5/c^4*(c*x^2+b*x+a)^(1/2)*e^2+7/1024*b^6/c^(9/2)*ln((1/2*b+c*x)/c^
(1/2)+(c*x^2+b*x+a)^(1/2))*e^2-1/16*a^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*e^2+1/8*d^2/c*(c*x^2+b*x+a)^(3/2)*b+3/8*d^2*(c*x^2+b*x+a)^
(1/2)*x*a-3/64*d^2/c^2*(c*x^2+b*x+a)^(1/2)*b^3+3/8*d^2/c^(1/2)*ln((1/2*b+c*
x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3/128*d^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x+a)^(1/2))*b^4+3/128*f^2*a^4/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))-99/16384*f^2*b^7/c^6*(c*x^2+b*x+a)^(1/2)+33/2048*f^2*b^5/
c^5*(c*x^2+b*x+a)^(3/2)-33/640*f^2*b^3/c^4*(c*x^2+b*x+a)^(5/2)+1/6*x*(c*x^2
+b*x+a)^(5/2)/c*e^2-7/60*b/c^2*(c*x^2+b*x+a)^(5/2)*e^2+7/192*b^3/c^3*(c*x^2
+b*x+a)^(3/2)*e^2+3/32*e*f*b^2/c^3*a^2*(c*x^2+b*x+a)^(1/2)-3/16*d*e*b^2/c^2
*(c*x^2+b*x+a)^(1/2)*a-3/8*d*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+
a)^(1/2))*a^2+3/16*d*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/
2))*a-1/4*d*e*b/c*x*(c*x^2+b*x+a)^(3/2)+3/32*d*e*b^3/c^2*(c*x^2+b*x+a)^(1/2
)*x-9/128*f^2*b^2/c^3*a*x*(c*x^2+b*x+a)^(3/2)-57/512*f^2*b^2/c^3*a^2*(c*x^2
+b*x+a)^(1/2)*x+153/2048*f^2*b^4/c^4*(c*x^2+b*x+a)^(1/2)*x*a-1/8*a^2/c*(c*x
^2+b*x+a)^(1/2)*x*d*f-1/16*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b*d*f-1/24*a/c^2*(c*
```

```
x^2+b*x+a)^(3/2)*b*d*f-1/12*a/c*x*(c*x^2+b*x+a)^(3/2)*d*f+7/48*b^2/c^2*x*(c
*x^2+b*x+a)^(3/2)*d*f+1/8*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a*e^2-7/128*b^4/c^3
*(c*x^2+b*x+a)^(1/2)*x*d*f+1/8*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*d*f+9/32*b^2/c
^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2*d*f-15/128*b^4/c^(7/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d*f+1/4*b^2/c^2*(c*x^2+b*x
+a)^(1/2)*x*a*d*f-3/8*d*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+3/16*e*f*b/c^2*a^2*(c
*x^2+b*x+a)^(1/2)*x-3/16*e*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)*x*a+1/8*e*f*b/c^2*
a*x*(c*x^2+b*x+a)^(3/2)+21/256*e*f*b^5/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^
2+b*x+a)^(1/2))*a-3/32*e*f*b^3/c^3*x*(c*x^2+b*x+a)^(3/2)+9/256*e*f*b^5/c^4*
(c*x^2+b*x+a)^(1/2)*x-3/32*e*f*b^4/c^4*(c*x^2+b*x+a)^(1/2)*a+1/16*e*f*b^2/c
^3*a*(c*x^2+b*x+a)^(3/2)+1/4*d^2*x*(c*x^2+b*x+a)^(3/2)-7/30*b/c^2*(c*x^2+b*
x+a)^(5/2)*d*f+7/96*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)*e^2+99/32768*f^2*b^8/c^(1
3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+2/5*d*e*(c*x^2+b*x+a)^(5/2
)/c-15/128*f^2*b^2/c^(7/2)*a^3*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-
1/16*f^2*a/c^2*x*(c*x^2+b*x+a)^(5/2)+1/64*f^2*a^2/c^2*x*(c*x^2+b*x+a)^(3/2)
+1/128*f^2*a^2/c^3*(c*x^2+b*x+a)^(3/2)*b-9/1024*e*f*b^7/c^(11/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/20*e*f*b^2/c^3*(c*x^2+b*x+a)^(5/2)+2/7*
e*f*x^2*(c*x^2+b*x+a)^(5/2)/c+3/128*f^2*a^3/c^2*(c*x^2+b*x+a)^(1/2)*x+3/256
*f^2*a^3/c^3*(c*x^2+b*x+a)^(1/2)*b+33/1024*f^2*b^4/c^4*x*(c*x^2+b*x+a)^(3/2
)-99/8192*f^2*b^6/c^5*(c*x^2+b*x+a)^(1/2)*x+153/4096*f^2*b^5/c^5*(c*x^2+b*x
+a)^(1/2)*a-9/256*f^2*b^3/c^4*a*(c*x^2+b*x+a)^(3/2)-57/1024*f^2*b^3/c^4*a^2
*(c*x^2+b*x+a)^(1/2)+33/448*f^2*b^2/c^3*x*(c*x^2+b*x+a)^(5/2)-11/112*f^2*b/
c^2*x^2*(c*x^2+b*x+a)^(5/2)+93/1120*f^2*b/c^3*a*(c*x^2+b*x+a)^(5/2)+105/102
4*f^2*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-63/2048*f
^2*b^6/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+1/3*x*(c*x^2+
b*x+a)^(5/2)/c*d*f-3/64*e*f*b^4/c^4*(c*x^2+b*x+a)^(3/2)+9/512*e*f*b^6/c^5*(
c*x^2+b*x+a)^(1/2)-4/35*e*f*a/c^2*(c*x^2+b*x+a)^(5/2)-3/16*d^2/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a-1/8*d*e*b^2/c^2*(c*x^2+b*x+a)
^(3/2)+3/64*d*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)-3/128*d*e*b^5/c^(7/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/32*d^2/c*(c*x^2+b*x+a)^(1/2)*x*b^2+3/1
6*d^2/c*(c*x^2+b*x+a)^(1/2)*b*a+9/64*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*a^2*e^2-15/256*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))*a*e^2+7/512*b^6/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*d*f-1/24*a/c*x*(c*x^2+b*x+a)^(3/2)*e^2-1/48*a/c^2*(c*x^2+b*x+a)^(3/2
)*b*e^2-1/16*a^2/c*(c*x^2+b*x+a)^(1/2)*x*e^2-1/32*a^2/c^2*(c*x^2+b*x+a)^(1/
2)*b*e^2-1/8*a^3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f+7/
96*b^3/c^3*(c*x^2+b*x+a)^(3/2)*d*f-7/256*b^4/c^3*(c*x^2+b*x+a)^(1/2)*x*e^2+
1/16*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a*e^2-7/256*b^5/c^4*(c*x^2+b*x+a)^(1/2)*d*
f-3/14*e*f*b/c^2*x*(c*x^2+b*x+a)^(5/2)+3/16*e*f*b/c^(5/2)*a^3*ln((1/2*b+c*x
)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-15/64*e*f*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))*a^2+1/8*f^2*x^3*(c*x^2+b*x+a)^(5/2)/c
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 9.81723, size = 5115, normalized size = 9.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] [1/6881280*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d^2 - 768*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^
2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c + 1120*a^2*b^4*c^2 -
1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2 - 60*a*b^4*c^3 + 144
*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c^2 + 80*a^2*b^3*c^3 -
 64*a^3*b*c^4)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(215040*c^8*f^2*x^7 + 15360*(32
*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^6 + 84*a*c^7)*f
^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e + 2912*b*c^7*e^2
 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^2*c^6 + 64*a*c^
7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*b^2*c^6 + 140*a
*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2 + 32*(14*(3*b^
2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3 - 26880*(3*b^3*
c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 128*a^2*c^6)*d*e
 - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 - 3*(3465*b^7*c -
 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2 + 8*(80640*b*c^
7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*a*b*c^6)*e^2 -
3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(14*(7*b^3*c^5 -
 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6)*e)*f)*x^2 - 3
2*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(315*b^6*c^2 - 2
520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(26880*(b^2*c^6 +
 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35*b^4*c^4 - 216*
a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*c^3 + 18896*a^2
*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2
*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e)*f)*x)*sqrt(c*
x^2 + b*x + a))/c^7, -1/3440640*(105*(768*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c
^6)*d^2 - 768*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d*e + 32*(7*b^6*c^2 -
60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*e^2 + 3*(33*b^8 - 336*a*b^6*c
+ 1120*a^2*b^4*c^2 - 1280*a^3*b^2*c^3 + 256*a^4*c^4)*f^2 + 32*(2*(7*b^6*c^2
 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*d - 3*(3*b^7*c - 28*a*b^5*c
^2 + 80*a^2*b^3*c^3 - 64*a^3*b*c^4)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 +
b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(215040*c^8*f^2*
x^7 + 15360*(32*c^8*e*f + 17*b*c^7*f^2)*x^6 + 1280*(224*c^8*e^2 + 3*(b^2*c^
6 + 84*a*c^7)*f^2 + 32*(14*c^8*d + 15*b*c^7*e)*f)*x^5 + 128*(5376*c^8*d*e +
 2912*b*c^7*e^2 - 3*(11*b^3*c^5 - 52*a*b*c^6)*f^2 + 32*(182*b*c^7*d + 3*(b^
2*c^6 + 64*a*c^7)*e)*f)*x^4 + 16*(26880*c^8*d^2 + 59136*b*c^7*d*e + 224*(3*
b^2*c^6 + 140*a*c^7)*e^2 + 3*(99*b^4*c^4 - 568*a*b^2*c^5 + 560*a^2*c^6)*f^2
 + 32*(14*(3*b^2*c^6 + 140*a*c^7)*d - 3*(9*b^3*c^5 - 44*a*b*c^6)*e)*f)*x^3
- 26880*(3*b^3*c^5 - 20*a*b*c^6)*d^2 + 5376*(15*b^4*c^4 - 100*a*b^2*c^5 + 1
28*a^2*c^6)*d*e - 224*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*e^2 -
3*(3465*b^7*c - 30660*a*b^5*c^2 + 81648*a^2*b^3*c^3 - 58816*a^3*b*c^4)*f^2
+ 8*(80640*b*c^7*d^2 + 5376*(b^2*c^6 + 32*a*c^7)*d*e - 224*(7*b^3*c^5 - 36*
a*b*c^6)*e^2 - 3*(231*b^5*c^3 - 1560*a*b^3*c^4 + 2416*a^2*b*c^5)*f^2 - 32*(
14*(7*b^3*c^5 - 36*a*b*c^6)*d - 3*(21*b^4*c^4 - 124*a*b^2*c^5 + 128*a^2*c^6
)*e)*f)*x^2 - 32*(14*(105*b^5*c^3 - 760*a*b^3*c^4 + 1296*a^2*b*c^5)*d - 3*(
315*b^6*c^2 - 2520*a*b^4*c^3 + 5488*a^2*b^2*c^4 - 2048*a^3*c^5)*e)*f + 2*(2
6880*(b^2*c^6 + 20*a*c^7)*d^2 - 5376*(5*b^3*c^5 - 28*a*b*c^6)*d*e + 224*(35
*b^4*c^4 - 216*a*b^2*c^5 + 240*a^2*c^6)*e^2 + 3*(1155*b^6*c^2 - 8988*a*b^4*
c^3 + 18896*a^2*b^2*c^4 - 6720*a^3*c^5)*f^2 + 32*(14*(35*b^4*c^4 - 216*a*b^
2*c^5 + 240*a^2*c^6)*d - 3*(105*b^5*c^3 - 728*a*b^3*c^4 + 1168*a^2*b*c^5)*e
)*f)*x)*sqrt(c*x^2 + b*x + a))/c^7]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d)**2,x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)**2, x)
```

---

**Giac [B]** time = 1.33874, size = 1553, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] 1/1720320*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*(12*(14*c*f^2*x + (17*b*c^7
*f^2 + 32*c^8*f*e)/c^7)*x + (448*c^8*d*f + 3*b^2*c^6*f^2 + 252*a*c^7*f^2 +
480*b*c^7*f*e + 224*c^8*e^2)/c^7)*x + (5824*b*c^7*d*f - 33*b^3*c^5*f^2 + 15
6*a*b*c^6*f^2 + 5376*c^8*d*e + 96*b^2*c^6*f*e + 6144*a*c^7*f*e + 2912*b*c^7
*e^2)/c^7)*x + (26880*c^8*d^2 + 1344*b^2*c^6*d*f + 62720*a*c^7*d*f + 297*b^
4*c^4*f^2 - 1704*a*b^2*c^5*f^2 + 1680*a^2*c^6*f^2 + 59136*b*c^7*d*e - 864*b
^3*c^5*f*e + 4224*a*b*c^6*f*e + 672*b^2*c^6*e^2 + 31360*a*c^7*e^2)/c^7)*x +
 (80640*b*c^7*d^2 - 3136*b^3*c^5*d*f + 16128*a*b*c^6*d*f - 693*b^5*c^3*f^2
+ 4680*a*b^3*c^4*f^2 - 7248*a^2*b*c^5*f^2 + 5376*b^2*c^6*d*e + 172032*a*c^7
*d*e + 2016*b^4*c^4*f*e - 11904*a*b^2*c^5*f*e + 12288*a^2*c^6*f*e - 1568*b^
3*c^5*e^2 + 8064*a*b*c^6*e^2)/c^7)*x + (26880*b^2*c^6*d^2 + 537600*a*c^7*d^
2 + 15680*b^4*c^4*d*f - 96768*a*b^2*c^5*d*f + 107520*a^2*c^6*d*f + 3465*b^6
*c^2*f^2 - 26964*a*b^4*c^3*f^2 + 56688*a^2*b^2*c^4*f^2 - 20160*a^3*c^5*f^2
- 26880*b^3*c^5*d*e + 150528*a*b*c^6*d*e - 10080*b^5*c^3*f*e + 69888*a*b^3*
c^4*f*e - 112128*a^2*b*c^5*f*e + 7840*b^4*c^4*e^2 - 48384*a*b^2*c^5*e^2 + 5
3760*a^2*c^6*e^2)/c^7)*x - (80640*b^3*c^5*d^2 - 537600*a*b*c^6*d^2 + 47040*
b^5*c^3*d*f - 340480*a*b^3*c^4*d*f + 580608*a^2*b*c^5*d*f + 10395*b^7*c*f^2
 - 91980*a*b^5*c^2*f^2 + 244944*a^2*b^3*c^3*f^2 - 176448*a^3*b*c^4*f^2 - 80
640*b^4*c^4*d*e + 537600*a*b^2*c^5*d*e - 688128*a^2*c^6*d*e - 30240*b^6*c^2
*f*e + 241920*a*b^4*c^3*f*e - 526848*a^2*b^2*c^4*f*e + 196608*a^3*c^5*f*e +
 23520*b^5*c^3*e^2 - 170240*a*b^3*c^4*e^2 + 290304*a^2*b*c^5*e^2)/c^7) - 1/
32768*(768*b^4*c^4*d^2 - 6144*a*b^2*c^5*d^2 + 12288*a^2*c^6*d^2 + 448*b^6*c
^2*d*f - 3840*a*b^4*c^3*d*f + 9216*a^2*b^2*c^4*d*f - 4096*a^3*c^5*d*f + 99*
b^8*f^2 - 1008*a*b^6*c*f^2 + 3360*a^2*b^4*c^2*f^2 - 3840*a^3*b^2*c^3*f^2 +
768*a^4*c^4*f^2 - 768*b^5*c^3*d*e + 6144*a*b^3*c^4*d*e - 12288*a^2*b*c^5*d*
e - 288*b^7*c*f*e + 2688*a*b^5*c^2*f*e - 7680*a^2*b^3*c^3*f*e + 6144*a^3*b*
c^4*f*e + 224*b^6*c^2*e^2 - 1920*a*b^4*c^3*e^2 + 4608*a^2*b^2*c^4*e^2 - 204
8*a^3*c^5*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))
/c^(13/2)
```

## 3.105 $\int \left(a+bx+cx^2\right)^{3/2} \left(d+ex+fx^2\right) dx$

**Optimal**. Leaf size=236

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)+7b^2f\right.}{512c^4}$$

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(512*c^4) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

---

**Rubi [A]** time = 0.230253, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {1661, 640, 612, 621, 206}

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}\left(-4acf+7b^2f-12bce+24c^2d\right)}{192c^3}-\frac{\left(b^2-4ac\right)(b+2cx)\sqrt{a+bx+cx^2}\left(-4c(af+3be)+7b^2f\right.}{512c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] -((b^2 - 4*a*c)*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*(b + 2*c*x)*Sqrt[a
 + b*x + c*x^2])/(512*c^4) + ((24*c^2*d - 12*b*c*e + 7*b^2*f - 4*a*c*f)*(b
+ 2*c*x)*(a + b*x + c*x^2)^(3/2))/(192*c^3) + ((12*c*e - 7*b*f)*(a + b*x +
c*x^2)^(5/2))/(60*c^2) + (f*x*(a + b*x + c*x^2)^(5/2))/(6*c) + ((b^2 - 4*a*
c)^2*(24*c^2*d + 7*b^2*f - 4*c*(3*b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c
]*Sqrt[a + b*x + c*x^2])])/(1024*c^(9/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 612

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2
*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)dx = \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} + \frac{\int \left(6cd-af+\frac{1}{2}(12ce-7bf)x\right)\left(a+bx+cx^2\right)^{3/2}dx}{6c}$$

$$= \frac{(12ce-7bf)\left(a+bx+cx^2\right)^{5/2}}{60c^2} + \frac{fx\left(a+bx+cx^2\right)^{5/2}}{6c} + \frac{\left(2c(6cd-af)-\frac{1}{2}b(\ldots\right.}{\ldots}$$

$$= \frac{\left(24c^2d-12bce+7b^2f-4acf\right)(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{192c^3} + \frac{(12ce-7bf)\left(a\ldots\right.}{60\ldots}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} + \frac{\left(24c^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} + \frac{\left(24c^2\ldots\right.}{\ldots}$$

$$= -\frac{\left(b^2-4ac\right)\left(24c^2d+7b^2f-4c(3be+af)\right)(b+2cx)\sqrt{a+bx+cx^2}}{512c^4} + \frac{\left(24c^2\ldots\right.}{\ldots}$$

**Mathematica [A]** time = 0.703481, size = 392, normalized size = 1.66

$$\frac{\frac{360d\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+cx)}\right)}{c^{3/2}} - 60be\left(\frac{3\left(b^2-4ac\right)\left(\left(b^2-4ac\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-2\sqrt{c}(b+2cx)\sqrt{a+x(b+c\ldots}\right.\right.}{c^{5/2}}\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2),x]
```

```
[Out] (1920*d*(b + 2*c*x)*(a + x*(b + c*x))^(3/2) + 3072*e*(a + x*(b + c*x))^(5/2
) + 2560*f*x*(a + x*(b + c*x))^(5/2) + (360*(b^2 - 4*a*c)*d*(-2*Sqrt[c]*(b
+ 2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[
c]*Sqrt[a + x*(b + c*x)])]))/c^(3/2) - 60*b*e*((16*(b + 2*c*x)*(a + x*(b +
c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b + 2*c*x)*Sqrt[a + x*(b + c
*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]
))/c^(5/2)) + (f*(-1792*b*(a + x*(b + c*x))^(5/2) + 5*(7*b^2 - 4*a*c)*((16*
(b + 2*c*x)*(a + x*(b + c*x))^(3/2))/c + (3*(b^2 - 4*a*c)*(-2*Sqrt[c]*(b +
2*c*x)*Sqrt[a + x*(b + c*x)] + (b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])]))/c^(5/2))))/c)/(15360*c)
```

**Maple [B]** time = 0.053, size = 862, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x)
```

```
[Out] 1/8*f*b^2/c^2*(c*x^2+b*x+a)^(1/2)*x*a-3/16*e*b/c*(c*x^2+b*x+a)^(1/2)*x*a+1/
4*d*x*(c*x^2+b*x+a)^(3/2)+1/5*e*(c*x^2+b*x+a)^(5/2)/c-7/256*f*b^4/c^3*(c*x^
2+b*x+a)^(1/2)*x+1/16*f*b^3/c^3*(c*x^2+b*x+a)^(1/2)*a+9/64*f*b^2/c^(5/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2-3/32*d/c*(c*x^2+b*x+a)^(1/2)*
x*b^2+3/16*d/c*(c*x^2+b*x+a)^(1/2)*b*a-1/48*f*a/c^2*(c*x^2+b*x+a)^(3/2)*b-1
/16*f*a^2/c*(c*x^2+b*x+a)^(1/2)*x-1/32*f*a^2/c^2*(c*x^2+b*x+a)^(1/2)*b+7/96
*f*b^2/c^2*x*(c*x^2+b*x+a)^(3/2)-3/16*e*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*a^2+3/32*e*b^3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*a-1/8*e*b/c*x*(c*x^2+b*x+a)^(3/2)-3/16*d/c^(3/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2*a+3/64*e*b^3/c^2*(c*x^2+b*x+a)^(1/2)*x-3/3
2*e*b^2/c^2*(c*x^2+b*x+a)^(1/2)*a-15/256*f*b^4/c^(7/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))*a-1/24*f*a/c*x*(c*x^2+b*x+a)^(3/2)-1/16*f*a^3/c^(3/
2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/60*f*b/c^2*(c*x^2+b*x+a)^(
5/2)+7/192*f*b^3/c^3*(c*x^2+b*x+a)^(3/2)-7/512*f*b^5/c^4*(c*x^2+b*x+a)^(1/2
)+7/1024*f*b^6/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/8*d/c*
(c*x^2+b*x+a)^(3/2)*b+3/8*d*(c*x^2+b*x+a)^(1/2)*x*a-3/64*d/c^2*(c*x^2+b*x+a
)^(1/2)*b^3+3/8*d/c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a^2+3
/128*d/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^4-1/16*e*b^2/c
^2*(c*x^2+b*x+a)^(3/2)+3/128*e*b^4/c^3*(c*x^2+b*x+a)^(1/2)-3/256*e*b^5/c^(7
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/6*f*x*(c*x^2+b*x+a)^(5/2)
/c
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 2.63385, size = 1947, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] [-1/30720*(15*(24*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*
b^3*c^2 + 16*a^2*b*c^3)*e + (7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*
c^3)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2
*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f
)*x^4 + 16*(120*c^6*d + 132*b*c^5*e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(3
60*b*c^5*d + 12*(b^2*c^4 + 32*a*c^5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 -
```

```
120*(3*b^3*c^3 - 20*a*b*c^4)*d + 12*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c
^4)*e - (105*b^5*c - 760*a*b^3*c^2 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 +
20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 +
 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b*x + a))/c^5, -1/15360*(15*(24*(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4)*d - 12*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e +
(7*b^6 - 60*a*b^4*c + 144*a^2*b^2*c^2 - 64*a^3*c^3)*f)*sqrt(-c)*arctan(1/2*
sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(12
80*c^6*f*x^5 + 128*(12*c^6*e + 13*b*c^5*f)*x^4 + 16*(120*c^6*d + 132*b*c^5*
e + (3*b^2*c^4 + 140*a*c^5)*f)*x^3 + 8*(360*b*c^5*d + 12*(b^2*c^4 + 32*a*c^
5)*e - (7*b^3*c^3 - 36*a*b*c^4)*f)*x^2 - 120*(3*b^3*c^3 - 20*a*b*c^4)*d + 1
2*(15*b^4*c^2 - 100*a*b^2*c^3 + 128*a^2*c^4)*e - (105*b^5*c - 760*a*b^3*c^2
 + 1296*a^2*b*c^3)*f + 2*(120*(b^2*c^4 + 20*a*c^5)*d - 12*(5*b^3*c^3 - 28*a
*b*c^4)*e + (35*b^4*c^2 - 216*a*b^2*c^3 + 240*a^2*c^4)*f)*x)*sqrt(c*x^2 + b
*x + a))/c^5]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \left(a + bx + cx^2\right)^{\frac{3}{2}} \left(d + ex + fx^2\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)*(f*x**2+e*x+d),x)
```

```
[Out] Integral((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2), x)
```

---

**Giac [A]** time = 1.29431, size = 563, normalized size = 2.39

$$\frac{1}{7680}\sqrt{cx^2 + bx + a}\left(2\left(4\left(2\left(8\left(10\,cfx + \frac{13\,bc^5f + 12\,c^6e}{c^5}\right)x + \frac{120\,c^6d + 3\,b^2c^4f + 140\,ac^5f + 132\,bc^5e}{c^5}\right)x + \frac{360\,bc^5d}{}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)*(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*c*f*x + (13*b*c^5*f + 12*c^6*e
)/c^5)*x + (120*c^6*d + 3*b^2*c^4*f + 140*a*c^5*f + 132*b*c^5*e)/c^5)*x + (
360*b*c^5*d - 7*b^3*c^3*f + 36*a*b*c^4*f + 12*b^2*c^4*e + 384*a*c^5*e)/c^5)
*x + (120*b^2*c^4*d + 2400*a*c^5*d + 35*b^4*c^2*f - 216*a*b^2*c^3*f + 240*a
^2*c^4*f - 60*b^3*c^3*e + 336*a*b*c^4*e)/c^5)*x - (360*b^3*c^3*d - 2400*a*b
*c^4*d + 105*b^5*c*f - 760*a*b^3*c^2*f + 1296*a^2*b*c^3*f - 180*b^4*c^2*e +
 1200*a*b^2*c^3*e - 1536*a^2*c^4*e)/c^5) - 1/1024*(24*b^4*c^2*d - 192*a*b^2
*c^3*d + 384*a^2*c^4*d + 7*b^6*f - 60*a*b^4*c*f + 144*a^2*b^2*c^2*f - 64*a^
3*c^3*f - 12*b^5*c*e + 96*a*b^3*c^2*e - 192*a^2*b*c^3*e)*log(abs(-2*(sqrt(c
)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2)
```

# 3.106 $\int \frac{(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

**Optimal**. Leaf size=679

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-2f\left(-f^2\left(b^2d-a^2f\right)+2cdf(be-af)+c^2(-d)\left(e^2-df\right)\right)\right)\tanh}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*f^2) + ((3*b^2*f^2 -
12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(8*Sqrt[c]*f^3) + (((c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]
)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2
*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]
) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])
/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (((c*e - b*f)*(e + Sqrt[e^2 - 4*d*f])*(f*(b
*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2*d - a^
2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(
b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[
2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*
f)*Sqrt[e^2 - 4*d*f]])
```

---

**Rubi [A]** time = 11.0327, antiderivative size = 678, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {977, 1076, 621, 206, 1032, 724}

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-2f\left(-f^2\left(b^2d-a^2f\right)+2cdf(be-af)+c^2(-d)\left(e^2-df\right)\right)\right)\tanh}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] -((4*c*e - 5*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(4*f^2) + ((3*b^2*f^2 -
12*c*f*(b*e - a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt
[a + b*x + c*x^2])])/(8*Sqrt[c]*f^3) + (((c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]
)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) - 2*f*(2*c*d*f*(b*e - a*f) - f^2*(b^2
*d - a^2*f) - c^2*d*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]
) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])
/(Sqrt[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c
*e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((4*c*d*f^2*(b*e - a*f) - 2*f^3*(b^2*d - a^
2*f) - 2*c^2*d*f*(e^2 - d*f) - (c*e - b*f)*(e + Sqrt[e^2 - 4*d*f])*(f*(b*e
- 2*a*f) - c*(e^2 - 2*d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2
*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e
*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqr
t[2]*f^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 977

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b*f*(3*p + 2*q) - c*e*(2*p + q) + 2*c*f*(p
+ q)*x)*(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^(q + 1))/(2*f^2*(p + q)
*(2*p + 2*q + 1)), x] - Dist[1/(2*f^2*(p + q)*(2*p + 2*q + 1)), Int[(a + b*
x + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Simp[(b*d - a*e)*(c*e - b*f)*(1 - p)
*(2*p + q) - (p + q)*(b^2*d*f*(1 - p) - a*(f*(b*e - 2*a*f)*(2*p + 2*q + 1)
+ c*(2*d*f - e^2*(2*p + q)))) + (2*(c*d - a*f)*(c*e - b*f)*(1 - p)*(2*p + q
) - (p + q)*((b^2 - 4*a*c)*e*f*(1 - p) + b*(c*(e^2 - 4*d*f)*(2*p + q) + f*(
2*c*d - b*e + 2*a*f)*(2*p + 2*q + 1))))*x + ((c*e - b*f)^2*(1 - p)*p + c*(p
 + q)*(f*(b*e - 2*a*f)*(4*p + 2*q - 1) - c*(2*d*f*(1 - 2*p) + e^2*(3*p + q
- 1))))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*
q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}\left(-4bcde+5b^2df+4af(cd-2af)\right)-\frac{1}{4}\left(8c^2de-4acef-bf(5be-16af)+4bc\left(e^2 \cdots\right.\right.}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)\cdots}}{2f^2}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} - \frac{\int \frac{\frac{1}{4}f\left(-4bcde+5b^2df+4af(cd-2af)\right)-\frac{1}{4}d\left(-3b^2f^2+12cf(be-af)-8c^2\left(e^2-d\cdots\right.\right.}{\cdots}}{\cdots}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\cdots\right.}{4f^3}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{b+2\cdots}{2\sqrt{c}\sqrt{a+\cdots}}\right.}{8\sqrt{c}f^3}$$

$$= -\frac{(4ce-5bf-2cfx)\sqrt{a+bx+cx^2}}{4f^2} + \frac{\left(3b^2f^2-12cf(be-af)+8c^2\left(e^2-df\right)\right)\tanh^{-1}\left(\frac{b+2\cdots}{2\sqrt{c}\sqrt{a+\cdots}}\right.}{8\sqrt{c}f^3}$$

**Mathematica [A]** time = 4.85734, size = 1232, normalized size = 1.81

$$\frac{\sqrt{a+x(b+cx)}\left(-4\left(e+\sqrt{e^2-4df}\right)^2c^2+4f\left(e+\sqrt{e^2-4df}\right)xc^2-16af^2c+10bf\left(e+\sqrt{e^2-4df}\right)c-4bf^2xc-2b^2f^2\right)\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

```
[Out] ((-2*b^2*f^2 - 16*a*c*f^2 + 10*b*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*c^2*(e + S
qrt[e^2 - 4*d*f])^2 - 4*b*c*f^2*x + 4*c^2*f*(e + Sqrt[e^2 - 4*d*f])*x)*Sqrt
[a + x*(b + c*x)] + 2*Sqrt[a + x*(b + c*x)]*(b^2*f^2 - 2*c^2*(-2*e^2 + 4*d*
f + 2*e*Sqrt[e^2 - 4*d*f] + e*f*x - f*Sqrt[e^2 - 4*d*f]*x) + c*f*(8*a*f + b
*(-5*e + 5*Sqrt[e^2 - 4*d*f] + 2*f*x))) - ((b*f + c*(-e + Sqrt[e^2 - 4*d*f]
))*(b^2*f^2 + 4*c^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f + b
*(-e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b +
c*x)])])/(Sqrt[c]*f) + ((-(b*f) + c*(e + Sqrt[e^2 - 4*d*f]))*(-(b^2*f^2) +
4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(-3*a*f + b*(e + Sqrt[e^2
 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(Sqrt[
c]*f) + (8*Sqrt[2]*c*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2 + e^3*Sqrt[e^2 - 4*d
*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f^2*(2*a^2*f^2 - 2*a*b*f*(e + Sqrt[e^2 -
 4*d*f]) + b^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])) + 2*c*f*(a*f*(e^2 - 2*d
*f + e*Sqrt[e^2 - 4*d*f]) - b*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*
Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e
+ Sqrt[e^2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 -
 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/
(f*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
- 4*d*f]))]) + (8*Sqrt[2]*c*(c^2*(-e^4 + 4*d*e^2*f - 2*d^2*f^2 + e^3*Sqrt[e
^2 - 4*d*f] - 2*d*e*f*Sqrt[e^2 - 4*d*f]) + f^2*(-2*a^2*f^2 + 2*a*b*f*(e - S
qrt[e^2 - 4*d*f]) + b^2*(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f])) + 2*c*f*(a*f*
(-e^2 + 2*d*f + e*Sqrt[e^2 - 4*d*f]) + b*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*
d*f] + d*f*Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f
])*x + b*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f -
```

```
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*
(b + c*x)])])/(f*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*
(-e + Sqrt[e^2 - 4*d*f]))]))/(16*c*f^2*Sqrt[e^2 - 4*d*f])
```

---

**Maple [B]** time = 0.329, size = 22523, normalized size = 33.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)

[Out] result too large to display
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")

[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a + bx + cx^2\right)^{\frac{3}{2}}}{d + ex + fx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)

[Out] Integral((a + b*x + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

# 3.107 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^2}\,dx$

**Optimal**. Leaf size=704

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2\right.\right.\right.}{2\sqrt{2}f^2\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2c}}$$

```
[Out] -(((c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(f*(e^2 - 4*d*f))) - ((e
+ 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)) + (c^(3
/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f^2 - (((c*e -
b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2
*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d + 3*a*f)
)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*f^2*(e^2 - 4*d*f)^
(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f
]]) + (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^2 - 4
*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*
e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*f^2
*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*S
qrt[e^2 - 4*d*f]])
```

---

**Rubi [A]** time = 11.9496, antiderivative size = 704, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {971, 1066, 1076, 621, 206, 1032, 724}

$$-\frac{\left(\left(e-\sqrt{e^2-4df}\right)(ce-bf)\left(f(be-2af)+2c\left(e^2-5df\right)\right)-2f\left(f\left(-be(3af+cd)+4af(af+cd)+2b^2df\right)+2c^2d\left(e^2\right.\right.\right.}{2\sqrt{2}f^2\left(e^2-4df\right)^{3/2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] -(((c*e - 2*b*f - 2*c*f*x)*Sqrt[a + b*x + c*x^2])/(f*(e^2 - 4*d*f))) - ((e
+ 2*f*x)*(a + b*x + c*x^2)^(3/2))/((e^2 - 4*d*f)*(d + e*x + f*x^2)) + (c^(3
/2)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f^2 - (((c*e -
b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2
*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*e*(c*d + 3*a*f)
)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*f^2*(e^2 - 4*d*f)^
(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f
]]) + (((c*e - b*f)*(f*(b*e - 2*a*f) + 2*c*(e^2 - 5*d*f))*(e + Sqrt[e^2 - 4
*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(2*b^2*d*f + 4*a*f*(c*d + a*f) - b*
e*(c*d + 3*a*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*f^2
*(e^2 - 4*d*f)^(3/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*S
```

```
qrt[e^2 - 4*d*f]])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1066

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((A_.) + (B_.)*(x_) + (C_.)*(x_
)^2)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((B*c*f*(2*
p + 2*q + 3) + C*(b*f*p - c*e*(2*p + q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a +
b*x + c*x^2)^p*(d + e*x + f*x^2)^(q + 1))/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3)), x] - Dist[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3)), Int[(a + b*x + c*x
^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) -
 c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f
- e^2*(2*p + q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)
*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e
*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e +
2*A*f)*(2*p + 2*q + 3))))*x + (p*(c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*
e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^
2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, q}, x] && NeQ[b^2 - 4*a*c,
0] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*
q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

Rule 1076

```
Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)
*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[C/c, Int[1/Sqr
t[d + e*x + f*x^2], x], x] + Dist[1/c, Int[(A*c - a*C + (B*c - b*C)*x)/((a
+ b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, A
, B, C}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^2}\,dx = -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{\sqrt{a+bx+cx^2}\left(\frac{1}{2}(3be-4af)+(3ce+bf)x+4cfx^2\right)}{d+ex+fx^2}\,dx}{-e^2+4df}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} - \frac{\int \frac{cf\left(2b^2df+4af(cd+af)-be(cd\right.}{}}{}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^2\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{f^2} - \frac{\int \frac{2c^3}{}}{}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{\left(2c^2\right)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,\right.}{f^2}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^{3/2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.}{f^2}$$

$$= -\frac{(ce-2bf-2cfx)\sqrt{a+bx+cx^2}}{f\left(e^2-4df\right)} - \frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{\left(e^2-4df\right)\left(d+ex+fx^2\right)} + \frac{c^{3/2}\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.}{f^2}$$

**Mathematica [B]** time = 6.81996, size = 2843, normalized size = 4.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^2,x]
```

```
[Out] (-2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e - Sqrt[e^2 - 4*d*f] + 2*f*
x)) - (2*f*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)*(e + Sqrt[e^2 - 4*d*f] +
 2*f*x)) - (3*f*(a + x*(b + c*x))^(3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e +
Sqrt[e^2 - 4*d*f])) - 4*c^2*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((2*Sqr
t[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*
(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x
^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2
 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - Sqrt[e^
2 - 4*d*f]) - 4*a*c*f*(e - Sqrt[e^2 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*Sqrt
[e^2 - 4*d*f])) + 4*c*(-e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTan
h[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 -
4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b
```

```
*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))
)/((e^2 - 4*d*f)*(a + b*x + c*x^2)^(3/2)) + (f*(a + x*(b + c*x))^(3/2)*(((-
4*c*f*(4*a*f - b*(e - Sqrt[e^2 - 4*d*f])) - 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2
- 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*
a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[c]*f) + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*
f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(-4*(-e + Sqrt[e^2 - 4
*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*
f*(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*
(e - Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f])))
)*ArcTanh[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^
2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqr
t[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f
^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16
*c*f^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) - (f*(a + x*(b + c*
x))^(3/2)*(((4*c*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e + S
qrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*
(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f -
c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(4*(e
+ Sqrt[e^2 - 4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f])
)) + 4*f*(2*c*f*(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e^2 - 4*d
*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + Sqrt[e
^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*
(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f
^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f]
)^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f*(a
 + x*(b + c*x))^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f]
)) + 4*c^2*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*Sqrt[c]*(b^2*f^2 +
4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 -
4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*
Sqrt[e^2 - 4*d*f]]*(4*c*(e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d
*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*c*
f*(3*b^2*f*(e + Sqrt[e^2 - 4*d*f]) + 4*a*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*b*
(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 -
4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)*(a +
 b*x + c*x^2)^(3/2))
```

---

**Maple [B]** time = 0.369, size = 72576, normalized size = 103.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(fx^2 + ex + d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^2, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.108 $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx$

**Optimal**. Leaf size=671

$$-\frac{3\left(2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-f\left(4be(3af+cd)-4a\left(4af^2+ce^2\right)+b^2\left(-\left(4df+e^2\right)\right)\right)\right)\tanh^{-1}\left(\frac{}{2\sqrt{2}\sqrt{}}\right.}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

```
[Out] -((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/(2*(e^2 - 4*d*f)*(d + e*x + f*x^2)^2
) + (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^2)
*x)*Sqrt[a + b*x + c*x^2])/(4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) - (3*(2*(2
*c*d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]) - f*(4*b*e*(c*d + 3
*a*f) - b^2*(e^2 + 4*d*f) - 4*a*(c*e^2 + 4*a*f^2)))*ArcTanh[(4*a*f - b*(e -
 Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
 + b*x + c*x^2])])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*
e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) + (3*(2*(2*c*d - b*e + 2*a*
f)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - f*(4*b*e*(c*d + 3*a*f) - b^2*(e^2
+ 4*d*f) - 4*a*(c*e^2 + 4*a*f^2)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f
]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
 - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])]
)/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

---

**Rubi [A]** time = 11.597, antiderivative size = 669, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {971, 1013, 1032, 724, 206}

$$\frac{3\left(-2\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)+4bef(3af+cd)-4af\left(4af^2+ce^2\right)+b^2(-f)\left(4df+e^2\right)\right)\tanh^{-1}\left(\frac{}{2\sqrt{2}\sqrt{}}\right.}{4\sqrt{2}\left(e^2-4df\right)^{5/2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] -((e + 2*f*x)*(a + b*x + c*x^2)^(3/2))/(2*(e^2 - 4*d*f)*(d + e*x + f*x^2)^2
) + (3*(4*c*d*e + 4*a*e*f - b*(e^2 + 4*d*f) + 2*(c*e^2 - 2*b*e*f + 4*a*f^2)
*x)*Sqrt[a + b*x + c*x^2])/(4*(e^2 - 4*d*f)^2*(d + e*x + f*x^2)) + (3*(4*b*
e*f*(c*d + 3*a*f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*
d - b*e + 2*a*f)*(c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e
 - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt
[a + b*x + c*x^2])])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (3*(4*b*e*f*(c*d + 3*a*
f) - b^2*f*(e^2 + 4*d*f) - 4*a*f*(c*e^2 + 4*a*f^2) - 2*(2*c*d - b*e + 2*a*f
)*(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4
*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^
2])])/(4*Sqrt[2]*(e^2 - 4*d*f)^(5/2)*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 971

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1)*(d + e
*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1))
, Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^(q - 1)*Simp[2*c*d*(2*p +
 3) + b*e*q + (2*b*f*q + 2*c*e*(2*p + q + 3))*x + 2*c*f*(2*p + 2*q + 3)*x^2
, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0] &&  !IGtQ[q, 0]
```

Rule 1013

```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[((g*b - 2*a*h - (b*h - 2*g
*c)*x)*(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q)/((b^2 - 4*a*c)*(p + 1
)), x] - Dist[1/((b^2 - 4*a*c)*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d +
 e*x + f*x^2)^(q - 1)*Simp[e*q*(g*b - 2*a*h) - d*(b*h - 2*g*c)*(2*p + 3) +
(2*f*q*(g*b - 2*a*h) - e*(b*h - 2*g*c)*(2*p + q + 3))*x - f*(b*h - 2*g*c)*(
2*p + 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{\left(d+ex+fx^2\right)^3}\,dx = -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{\int \frac{\left(\frac{3}{2}(be-4af)+3(ce-bf)x\right)\sqrt{a+bx+cx^2}}{\left(d+ex+fx^2\right)^2}dx}{2\left(e^2-4df\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right)x\right)\sqrt{a}}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right)x\right)\sqrt{a}}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right)x\right)\sqrt{a}}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

$$= -\frac{(e+2fx)\left(a+bx+cx^2\right)^{3/2}}{2\left(e^2-4df\right)\left(d+ex+fx^2\right)^2} + \frac{3\left(4cde+4aef-b\left(e^2+4df\right)+2\left(ce^2-2bef+4af^2\right)x\right)\sqrt{a}}{4\left(e^2-4df\right)^2\left(d+ex+fx^2\right)}$$

**Mathematica [B]** time = 7.29595, size = 4727, normalized size = 7.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x + f*x^2)^3,x]
```

```
[Out] (-2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^(3/2)*(e - Sqrt[e^2 - 4*d*f
] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^2*(e - Sqrt[
e^2 - 4*d*f] + 2*f*x)) + (2*f^2*(a + x*(b + c*x))^(3/2))/((e^2 - 4*d*f)^(3/
2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)^2) + (6*f^2*(a + x*(b + c*x))^(3/2))/((e
^2 - 4*d*f)^2*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + (9*f^2*(a + x*(b + c*x))^(
3/2)*(((-4*b*c*f - 2*c*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])) - 4*c^2*f*x)*Sq
rt[a + b*x + c*x^2])/(8*c*f^2) - ((2*Sqrt[c]*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f
- e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(
b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 -
2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*
(4*c*f*(8*a*b*f^2 - 3*b^2*f*(e - Sqrt[e^2 - 4*d*f]) - 4*a*c*f*(e - Sqrt[e^2
 - 4*d*f]) + 4*b*c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])) + 4*c*(-e + Sqrt[e^
2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a
*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b*(-e + Sqrt[e^2 - 4*d*
f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2]])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2 - 4*d*f]) + 4*c*(
-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2
)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((-4*c*f*(4*a*f - b*(e - Sqrt[e^
2 - 4*d*f])) - 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b*f + 2*c*(-e + Sqrt[e^
2 - 4*d*f])) - 4*c*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*
x^2])/(8*c*f^2) - ((-2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e - Sqrt[e^2 - 4*d*f
])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) +
(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] +
 b*f*Sqrt[e^2 - 4*d*f]]*(-4*(-e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3
```

*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*(4*a*f - b*(e - Sqrt[e^2 -
4*d*f]))^2 - (e - Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b^2
*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b*(-e + S
qrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sq
rt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2
 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt[e^2 - 4
*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^(5/2
)*(a + b*x + c*x^2)^(3/2)) + (3*f^2*(a + x*(b + c*x))^(3/2)*(((-2*b*f - 2*c
*(-e + Sqrt[e^2 - 4*d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 - 2*b*f*(-e
+ Sqrt[e^2 - 4*d*f]) - c*(-e + Sqrt[e^2 - 4*d*f])^2)*(-e + Sqrt[e^2 - 4*d*f
] - 2*f*x)) + (((-4*c*f*(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f])) -
 4*c*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))
 - 8*c^2*f*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c
*f^2) - ((16*c^(3/2)*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f - e*
Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f + (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(32*c^
2*(-e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))) + 16*c*f*
(b^2*f + 4*a*c*f - 2*b*c*(e - Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f - e*Sqrt[
e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))))*ArcTanh[(-4*a*f - b
*(-e + Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sq
rt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 + 8*b*f*(-e + Sqrt
[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))/(-4*a*f^2 -
2*b*f*(-e + Sqrt[e^2 - 4*d*f]) - c*(-e + Sqrt[e^2 - 4*d*f])^2)))/((e^2 - 4*
d*f)^(3/2)*(a + b*x + c*x^2)^(3/2)) + (3*f^2*(a + x*(b + c*x))^(3/2)*(((4*c
*f*(-4*a*f + b*(e + Sqrt[e^2 - 4*d*f])) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]
))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) - 4*c*f*(b*f - c*(e + Sqrt[e^2 -
4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*(b*f - c*(e + Sqrt[e^2
- 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) - 4*c*f*(3*
a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[c]*f) - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*
f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(4*(e + Sqrt[e^2 - 4*d
*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(b^2*f^2 - 4*c^2*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f]) - 4*c*f*(3*a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*f*(2*c*f*
(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]))^2 - (e + Sqrt[e^2 - 4*d*f])*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*(b^2*f + 4*a*c*f - 2*b*c*(e + Sqrt[e^2 - 4*d*f]))))*
ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 -
4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^
2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(f*(16*a*f^2 -
 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2
)))/((e^2 - 4*d*f)^(5/2)*(a + b*x + c*x^2)^(3/2)) - (9*f^2*(a + x*(b + c*x)
)^(3/2)*(((4*b*c*f - 2*c*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) + 4*c^2*f*x
)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((-2*Sqrt[c]*(b^2*f^2 + 4*c^2*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (2*Sqrt[2]*Sqrt[c*e
^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d
*f]]*(4*c*(e + Sqrt[e^2 - 4*d*f])*(b^2*f^2 + 4*c^2*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f]) + 4*c*f*(a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 4*c*f*(3*b^2*f*(e
+ Sqrt[e^2 - 4*d*f]) + 4*a*c*f*(e + Sqrt[e^2 - 4*d*f]) - 4*b*(2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*
d*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2
*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*S
qrt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*
(e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2)))/((e^2 - 4*d*f)^2*(a + b*x + c*x^2
)^(3/2)) - (3*f^2*(a + x*(b + c*x))^(3/2)*(((2*b*f - 2*c*(e + Sqrt[e^2 - 4*
d*f]))*(a + b*x + c*x^2)^(3/2))/((-4*a*f^2 + 2*b*f*(e + Sqrt[e^2 - 4*d*f])
- c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 - 4*d*f] + 2*f*x)) + (((4*c*(b

```
*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-(b*f) + 2*c*(e + Sqrt[e^2 - 4*d*f])) + 4*
c*f*(-(b^2*f) - 4*a*c*f + 2*b*c*(e + Sqrt[e^2 - 4*d*f])) - 8*c^2*f*(b*f - c
*(e + Sqrt[e^2 - 4*d*f]))*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^2) - ((16*c^(3/2
)*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqr
t[a + b*x + c*x^2])])/f - (2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*(-32*c^2*(e + Sqrt[e^2 -
4*d*f])*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))) + 16*c*f*(b^2*f + 4*a*c*f -
2*b*c*(e + Sqrt[e^2 - 4*d*f]))*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d
*f]) - (-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sq
rt[a + b*x + c*x^2])])/(f*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(
e + Sqrt[e^2 - 4*d*f])^2)))/(16*c*f^2))/(-4*a*f^2 + 2*b*f*(e + Sqrt[e^2 - 4
*d*f]) - c*(e + Sqrt[e^2 - 4*d*f])^2)))/((e^2 - 4*d*f)^(3/2)*(a + b*x + c*x
^2)^(3/2))
```

---

**Maple [B]** time = 0.36, size = 178044, normalized size = 265.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(cx^2 + bx + a\right)^{\frac{3}{2}}}{\left(fx^2 + ex + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^(3/2)/(f*x^2 + e*x + d)^3, x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d)**3,x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

## 3.109 $\int \frac{\left(d+ex+fx^2\right)^3}{\sqrt{a+bx+cx^2}}\,dx$

**Optimal**. Leaf size=717

$$\frac{\sqrt{a+bx+cx^2}\left(-504bc^2f\left(22a^2f^2+70abef+25b^2\left(df+e^2\right)\right)+96c^3\left(128a^2ef^2+275abf\left(df+e^2\right)+50b^2\left(6def+e^3\right)\right)}{7680c^6}$$

```
[Out] ((23040*c^5*d^2*e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*
c^2*f*(70*a*b*e*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2
 + d*f) + 8*a*e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d
*f) + 50*b^2*(e^3 + 6*d*e*f)))*Sqrt[a + b*x + c*x^2])/(7680*c^6) + ((1155*b
^4*f^3 - 252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*
f*(322*a*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 +
 d*f) + 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(3840*c^5) - ((231*
b^3*f^3 - 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f
*(32*a*e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/(960*c^4) + (f*(
99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c^2*(e^2 + d*f))*x^3*Sqrt[a + b*
x + c*x^2])/(480*c^3) + (f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a + b*x + c*x^2])/(
60*c^2) + (f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((1024*c^6*d^3 + 231*b^6*
f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) +
840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*
(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2
*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(13/2))
```

---

**Rubi [A]** time = 2.70964, antiderivative size = 717, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 621, 206}


$$\frac{\sqrt{a+bx+cx^2}\left(-504bc^2f\left(22a^2f^2+70abef+25b^2\left(df+e^2\right)\right)+96c^3\left(128a^2ef^2+275abf\left(df+e^2\right)+50b^2\left(6def+e^3\right)\right)}{7680c^6}$$


Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```


```
[Out] ((23040*c^5*d^2*e - 3465*b^5*f^3 + 420*b^3*c*f^2*(27*b*e + 34*a*f) - 504*b*
c^2*f*(70*a*b*e*f + 22*a^2*f^2 + 25*b^2*(e^2 + d*f)) - 640*c^4*(27*b*d*(e^2
 + d*f) + 8*a*e*(e^2 + 6*d*f)) + 96*c^3*(128*a^2*e*f^2 + 275*a*b*f*(e^2 + d
*f) + 50*b^2*(e^3 + 6*d*e*f)))*Sqrt[a + b*x + c*x^2])/(7680*c^6) + ((1155*b
^4*f^3 - 252*b^2*c*f^2*(15*b*e + 14*a*f) + 5760*c^4*d*(e^2 + d*f) + 24*c^2*
f*(322*a*b*e*f + 50*a^2*f^2 + 175*b^2*(e^2 + d*f)) - 160*c^3*(27*a*f*(e^2 +
 d*f) + 10*b*(e^3 + 6*d*e*f)))*x*Sqrt[a + b*x + c*x^2])/(3840*c^5) - ((231*
b^3*f^3 - 36*b*c*f^2*(21*b*e + 13*a*f) - 320*c^3*(e^3 + 6*d*e*f) + 24*c^2*f
*(32*a*e*f + 35*b*(e^2 + d*f)))*x^2*Sqrt[a + b*x + c*x^2])/(960*c^4) + (f*(
99*b^2*f^2 - 4*c*f*(81*b*e + 25*a*f) + 360*c^2*(e^2 + d*f))*x^3*Sqrt[a + b*
x + c*x^2])/(480*c^3) + (f^2*(36*c*e - 11*b*f)*x^4*Sqrt[a + b*x + c*x^2])/(
60*c^2) + (f^3*x^5*Sqrt[a + b*x + c*x^2])/(6*c) + ((1024*c^6*d^3 + 231*b^6*
f^3 - 252*b^4*c*f^2*(3*b*e + 5*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) +
840*b^2*c^2*f*(4*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*
(e^2 + d*f) + 3*a^2*f*(e^2 + d*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2
*b*e*f^2 + a^3*f^3 + 9*a*b^2*f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[
(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(1024*c^(13/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\sqrt{a+bx+cx^2}}\,dx = \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{6cd^3+18cd^2ex+18cd\left(e^2+df\right)x^2+6ce\left(e^2+6df\right)x^3-f\left(5af^2-18c\left(e^2+df\right)\right)x^4+\frac{1}{2}f^2(36ce-11bf)\cdots}{\sqrt{a+bx+cx^2}}\cdots}{6c}\cdots$$

$$= \frac{f^2(36ce-11bf)x^4\sqrt{a+bx+cx^2}}{60c^2} + \frac{f^3x^5\sqrt{a+bx+cx^2}}{6c} + \frac{\int \frac{30c^2d^3+90c^2d^2ex+90c^2d\left(e^2+df\right)x^2-2(36a\cdots}{\cdots}}{\cdots}$$

$$= \frac{f\left(99b^2f^2-4cf(81be+25af)+360c^2\left(e^2+df\right)\right)x^3\sqrt{a+bx+cx^2}}{480c^3} + \frac{f^2(36ce-11bf)x^4\sqrt{a\cdots}}{60c^2}\cdots$$

$$= -\frac{\left(231b^3f^3-36bcf^2(21be+13af)-320c^3\left(e^3+6def\right)+24c^2f\left(32aef+35b\left(e^2+df\right)\right)\right)x^2\cdots}{960c^4}\cdots$$

$$= \frac{\left(1155b^4f^3-252b^2cf^2(15be+14af)+5760c^4d\left(e^2+df\right)+24c^2f\left(322abef+50a^2f^2+175b\cdots\right.\right.}{3840c^5}\cdots$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+25b^2\left(e^2\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+25b^2\left(e^2\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{\left(23040c^5d^2e-3465b^5f^3+420b^3cf^2(27be+34af)-504bc^2f\left(70abef+22a^2f^2+25b^2\left(e^2\cdots\right.\right.\right.}{\cdots}$$

**Mathematica [A]** time = 1.40472, size = 615, normalized size = 0.86

$$\sqrt{a+x(b+cx)}\left(-168bc^2f\left(66a^2f^2+42abf(5e+fx)+b^2\left(75df+75e^2+45efx+11f^2x^2\right)\right)+48c^3\left(2a^2f^2(128e+25fx\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-3465*b^5*f^3 + 210*b^3*c*f^2*(54*b*e + 68*a*f + 11
*b*f*x) - 168*b*c^2*f*(66*a^2*f^2 + 42*a*b*f*(5*e + f*x) + b^2*(75*e^2 + 75
*d*f + 45*e*f*x + 11*f^2*x^2)) + 128*c^5*(90*d^2*(2*e + f*x) + 15*d*x*(6*e^
2 + 8*e*f*x + 3*f^2*x^2) + x^2*(20*e^3 + 45*e^2*f*x + 36*e*f^2*x^2 + 10*f^3
*x^3)) + 48*c^3*(2*a^2*f^2*(128*e + 25*f*x) + b^2*(100*e^3 + 175*e^2*f*x +
6*e*f*(100*d + 21*f*x^2) + f^2*x*(175*d + 33*f*x^2)) + 2*a*b*f*(275*e^2 + 1
61*e*f*x + f*(275*d + 39*f*x^2))) - 64*c^4*(a*(80*e^3 + 135*e^2*f*x + 96*e*
f*(5*d + f*x^2) + 5*f^2*x*(27*d + 5*f*x^2)) + b*(270*d^2*f + 15*d*(18*e^2 +
 20*e*f*x + 7*f^2*x^2) + x*(50*e^3 + 105*e^2*f*x + 81*e*f^2*x^2 + 22*f^3*x^
3)))))/(7680*c^6) + ((1024*c^6*d^3 + 231*b^6*f^3 - 252*b^4*c*f^2*(3*b*e + 5
*a*f) - 1536*c^5*d*(b*d*e + a*(e^2 + d*f)) + 840*b^2*c^2*f*(4*a*b*e*f + 2*a
^2*f^2 + b^2*(e^2 + d*f)) + 384*c^4*(3*b^2*d*(e^2 + d*f) + 3*a^2*f*(e^2 + d
*f) + 2*a*b*e*(e^2 + 6*d*f)) - 320*c^3*(9*a^2*b*e*f^2 + a^3*f^3 + 9*a*b^2*f
*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)])])/(1024*c^(13/2))
```

---

**Maple [B]** time = 0.073, size = 1930, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -45/16*b^2/c^(7/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f+55/1
6*b/c^3*a*(c*x^2+b*x+a)^(1/2)*d*f^2+55/16*b/c^3*a*(c*x^2+b*x+a)^(1/2)*e^2*f
-9/8*a/c^2*x*(c*x^2+b*x+a)^(1/2)*d*f^2-45/16*e*f^2*b/c^(7/2)*a^2*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4/5*e*f^2*a/c^2*x^2*(c*x^2+b*x+a)^(1/2)-7
/8*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)*d*f^2-7/8*b/c^2*x^2*(c*x^2+b*x+a)^(1/2)*e^
2*f-27/40*e*f^2*b/c^2*x^3*(c*x^2+b*x+a)^(1/2)+63/80*e*f^2*b^2/c^3*x^2*(c*x^
2+b*x+a)^(1/2)-63/64*e*f^2*b^3/c^4*x*(c*x^2+b*x+a)^(1/2)+105/32*e*f^2*b^3/c
^(9/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+161/80*e*f^2*b/c^3*a*x
*(c*x^2+b*x+a)^(1/2)-5/2*b/c^2*x*(c*x^2+b*x+a)^(1/2)*d*e*f+9/2*b/c^(5/2)*a*
ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f+35/32*b^2/c^3*x*(c*x^2+b*
x+a)^(1/2)*e^2*f+2*x^2/c*(c*x^2+b*x+a)^(1/2)*d*e*f+15/4*b^2/c^3*(c*x^2+b*x+
a)^(1/2)*d*e*f-15/8*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
*d*e*f-4*a/c^2*(c*x^2+b*x+a)^(1/2)*d*e*f-45/16*b^2/c^(7/2)*a*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+3/5*e*f^2*x^4/c*(c*x^2+b*x+a)^(1/2)+3/4
*x^3/c*(c*x^2+b*x+a)^(1/2)*d*f^2-9/8*a/c^2*x*(c*x^2+b*x+a)^(1/2)*e^2*f-147/
160*f^3*b^2/c^4*a*x*(c*x^2+b*x+a)^(1/2)+39/80*f^3*b/c^3*a*x^2*(c*x^2+b*x+a)
^(1/2)+35/32*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)*d*f^2+d^3*ln((1/2*b+c*x)/c^(1/2)
+(c*x^2+b*x+a)^(1/2))/c^(1/2)-147/32*e*f^2*b^2/c^4*a*(c*x^2+b*x+a)^(1/2)+1/
3*x^2/c*(c*x^2+b*x+a)^(1/2)*e^3+5/8*b^2/c^3*(c*x^2+b*x+a)^(1/2)*e^3-5/16*b^
3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^3-2/3*a/c^2*(c*x^2+
b*x+a)^(1/2)*e^3-231/512*f^3*b^5/c^6*(c*x^2+b*x+a)^(1/2)+231/1024*f^3*b^6/c
^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-5/16*f^3*a^3/c^(7/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3*d^2*e/c*(c*x^2+b*x+a)^(1/2)+9/8
```

```
*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f*d^2+9/8*b^2/c^(5
/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e^2+3/2*x/c*(c*x^2+b*x+a)
^(1/2)*f*d^2+33/160*f^3*b^2/c^3*x^3*(c*x^2+b*x+a)^(1/2)+8/5*e*f^2*a^2/c^3*(
c*x^2+b*x+a)^(1/2)+189/128*e*f^2*b^4/c^5*(c*x^2+b*x+a)^(1/2)-189/256*e*f^2*
b^5/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*a/c^(3/2)*ln((
1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e^2-3/2*d^2*e*b/c^(3/2)*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))*f*d^2+3/2*x/c*(c*x^2+b*x+a)^(1/2)*d*e^2-9/4*b/c^2*(c*x^2+
b*x+a)^(1/2)*f*d^2-9/4*b/c^2*(c*x^2+b*x+a)^(1/2)*d*e^2+3/4*x^3/c*(c*x^2+b*x
+a)^(1/2)*e^2*f-105/64*b^3/c^4*(c*x^2+b*x+a)^(1/2)*d*f^2-105/64*b^3/c^4*(c*
x^2+b*x+a)^(1/2)*e^2*f+105/128*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*
x+a)^(1/2))*d*f^2+105/128*b^4/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^
(1/2))*e^2*f+9/8*a^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*
f^2+9/8*a^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f-5/12*
b/c^2*x*(c*x^2+b*x+a)^(1/2)*e^3+3/4*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*e^3-77/320*f^3*b^3/c^4*x^2*(c*x^2+b*x+a)^(1/2)+77/256*f^3*
b^4/c^5*x*(c*x^2+b*x+a)^(1/2)-315/256*f^3*b^4/c^(11/2)*a*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))+119/64*f^3*b^3/c^5*a*(c*x^2+b*x+a)^(1/2)+105/64*f
^3*b^2/c^(9/2)*a^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-231/160*f^3*
b/c^4*a^2*(c*x^2+b*x+a)^(1/2)-5/24*f^3*a/c^2*x^3*(c*x^2+b*x+a)^(1/2)+5/16*f
^3*a^2/c^3*x*(c*x^2+b*x+a)^(1/2)-11/60*f^3*b/c^2*x^4*(c*x^2+b*x+a)^(1/2)+1/
6*f^3*x^5*(c*x^2+b*x+a)^(1/2)/c
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 7.39034, size = 3661, normalized size = 5.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")

[Out] [-1/30720*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*a*c^5)*
d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c + 1680*a^
2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 48*a^2*c
^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(3*b^2*c
^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 - 120*a*b
^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqr
t(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(1280*c^6*f^3*x^5 + 230
40*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3)*x^4 +
320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*b
*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36*(10*c^6*d
 - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2
- 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 -
156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(1
6*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16*
```

```
a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600*
b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(35
*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*d
^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a
))/c^7, -1/15360*(15*(1024*c^6*d^3 - 1536*b*c^5*d^2*e + 384*(3*b^2*c^4 - 4*
a*c^5)*d*e^2 - 64*(5*b^3*c^3 - 12*a*b*c^4)*e^3 + (231*b^6 - 1260*a*b^4*c +
1680*a^2*b^2*c^2 - 320*a^3*c^3)*f^3 + 12*(2*(35*b^4*c^2 - 120*a*b^2*c^3 + 4
8*a^2*c^4)*d - (63*b^5*c - 280*a*b^3*c^2 + 240*a^2*b*c^3)*e)*f^2 + 24*(16*(
3*b^2*c^4 - 4*a*c^5)*d^2 - 16*(5*b^3*c^3 - 12*a*b*c^4)*d*e + (35*b^4*c^2 -
120*a*b^2*c^3 + 48*a^2*c^4)*e^2)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x +
a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(1280*c^6*f^3*x^5 + 23
040*c^6*d^2*e - 17280*b*c^5*d*e^2 + 128*(36*c^6*e*f^2 - 11*b*c^5*f^3)*x^4 +
 320*(15*b^2*c^4 - 16*a*c^5)*e^3 - 21*(165*b^5*c - 680*a*b^3*c^2 + 528*a^2*
b*c^3)*f^3 + 16*(360*c^6*e^2*f + (99*b^2*c^4 - 100*a*c^5)*f^3 + 36*(10*c^6*
d - 9*b*c^5*e)*f^2)*x^3 - 12*(50*(21*b^3*c^3 - 44*a*b*c^4)*d - (945*b^4*c^2
 - 2940*a*b^2*c^3 + 1024*a^2*c^4)*e)*f^2 + 8*(320*c^6*e^3 - 3*(77*b^3*c^3 -
 156*a*b*c^4)*f^3 - 12*(70*b*c^5*d - (63*b^2*c^4 - 64*a*c^5)*e)*f^2 + 120*(
16*c^6*d*e - 7*b*c^5*e^2)*f)*x^2 - 120*(144*b*c^5*d^2 - 16*(15*b^2*c^4 - 16
*a*c^5)*d*e + 5*(21*b^3*c^3 - 44*a*b*c^4)*e^2)*f + 2*(5760*c^6*d*e^2 - 1600
*b*c^5*e^3 + 3*(385*b^4*c^2 - 1176*a*b^2*c^3 + 400*a^2*c^4)*f^3 + 12*(10*(3
5*b^2*c^4 - 36*a*c^5)*d - 7*(45*b^3*c^3 - 92*a*b*c^4)*e)*f^2 + 120*(48*c^6*
d^2 - 80*b*c^5*d*e + (35*b^2*c^4 - 36*a*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x +
a))/c^7]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d + ex + fx^2\right)^3}{\sqrt{a + bx + cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**3/sqrt(a + b*x + c*x**2), x)
```

---

**Giac [A]** time = 1.36933, size = 1112, normalized size = 1.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/7680*sqrt(c*x^2 + b*x + a)*(2*(4*(2*(8*(10*f^3*x/c - (11*b*c^4*f^3 - 36*c
^5*f^2*e)/c^6)*x + (360*c^5*d*f^2 + 99*b^2*c^3*f^3 - 100*a*c^4*f^3 - 324*b*
c^4*f^2*e + 360*c^5*f*e^2)/c^6)*x - (840*b*c^4*d*f^2 + 231*b^3*c^2*f^3 - 46
8*a*b*c^3*f^3 - 1920*c^5*d*f*e - 756*b^2*c^3*f^2*e + 768*a*c^4*f^2*e + 840*
b*c^4*f*e^2 - 320*c^5*e^3)/c^6)*x + (5760*c^5*d^2*f + 4200*b^2*c^3*d*f^2 -
4320*a*c^4*d*f^2 + 1155*b^4*c*f^3 - 3528*a*b^2*c^2*f^3 + 1200*a^2*c^3*f^3 -
 9600*b*c^4*d*f*e - 3780*b^3*c^2*f^2*e + 7728*a*b*c^3*f^2*e + 5760*c^5*d*e^
2 + 4200*b^2*c^3*f*e^2 - 4320*a*c^4*f*e^2 - 1600*b*c^4*e^3)/c^6)*x - (17280
*b*c^4*d^2*f + 12600*b^3*c^2*d*f^2 - 26400*a*b*c^3*d*f^2 + 3465*b^5*f^3 - 1
4280*a*b^3*c*f^3 + 11088*a^2*b*c^2*f^3 - 23040*c^5*d^2*e - 28800*b^2*c^3*d*
f*e + 30720*a*c^4*d*f*e - 11340*b^4*c*f^2*e + 35280*a*b^2*c^2*f^2*e - 12288
```

```
*a^2*c^3*f^2*e + 17280*b*c^4*d*e^2 + 12600*b^3*c^2*f*e^2 - 26400*a*b*c^3*f*
e^2 - 4800*b^2*c^3*e^3 + 5120*a*c^4*e^3)/c^6) - 1/1024*(1024*c^6*d^3 + 1152
*b^2*c^4*d^2*f - 1536*a*c^5*d^2*f + 840*b^4*c^2*d*f^2 - 2880*a*b^2*c^3*d*f^
2 + 1152*a^2*c^4*d*f^2 + 231*b^6*f^3 - 1260*a*b^4*c*f^3 + 1680*a^2*b^2*c^2*
f^3 - 320*a^3*c^3*f^3 - 1536*b*c^5*d^2*e - 1920*b^3*c^3*d*f*e + 4608*a*b*c^
4*d*f*e - 756*b^5*c*f^2*e + 3360*a*b^3*c^2*f^2*e - 2880*a^2*b*c^3*f^2*e + 1
152*b^2*c^4*d*e^2 - 1536*a*c^5*d*e^2 + 840*b^4*c^2*f*e^2 - 2880*a*b^2*c^3*f
*e^2 + 1152*a^2*c^4*f*e^2 - 320*b^3*c^3*e^3 + 768*a*b*c^4*e^3)*log(abs(-2*(
sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(13/2)
```

# 3.110 $\int \frac{(d+ex+fx^2)^2}{\sqrt{a+bx+cx^2}} \, dx$

**Optimal**. Leaf size=316

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2\right.}{128c^{9/2}}$$

```
[Out] ((384*c^3*d*e - 105*b^3*f^2 + 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f
 + 9*b*(e^2 + 2*d*f)))*Sqrt[a + b*x + c*x^2])/(192*c^4) + ((35*b^2*f^2 - 4*
c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(96*c
^3) + (f*(16*c*e - 7*b*f)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (f^2*x^3*Sq
rt[a + b*x + c*x^2])/(4*c) + ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e
 + 3*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^
2 + b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(128*c^(9/2))
```

---

**Rubi [A]** time = 0.62621, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b^4f^2\right.}{128c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((384*c^3*d*e - 105*b^3*f^2 + 20*b*c*f*(12*b*e + 11*a*f) - 16*c^2*(16*a*e*f
 + 9*b*(e^2 + 2*d*f)))*Sqrt[a + b*x + c*x^2])/(192*c^4) + ((35*b^2*f^2 - 4*
c*f*(20*b*e + 9*a*f) + 48*c^2*(e^2 + 2*d*f))*x*Sqrt[a + b*x + c*x^2])/(96*c
^3) + (f*(16*c*e - 7*b*f)*x^2*Sqrt[a + b*x + c*x^2])/(24*c^2) + (f^2*x^3*Sq
rt[a + b*x + c*x^2])/(4*c) + ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e
 + 3*a*f) - 64*c^3*(2*b*d*e + a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^
2 + b^2*(e^2 + 2*d*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2
])])/(128*c^(9/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{4cd^2+8cdex-\left(3af^2-4c\left(e^2+2df\right)\right)x^2+\frac{1}{2}f(16ce-7bf)x^3}{\sqrt{a+bx+cx^2}}\,dx}{4c}$$

$$= \frac{f(16ce-7bf)x^2\sqrt{a+bx+cx^2}}{24c^2} + \frac{f^2x^3\sqrt{a+bx+cx^2}}{4c} + \frac{\int \frac{12c^2d^2+\left(24c^2de-16acef+7abf^2\right)x+\frac{1}{4}\left(35b^2f^2\right.}{\sqrt{a+bx+cx}}}{12c^2}$$

$$= \frac{\left(35b^2f^2-4cf(20be+9af)+48c^2\left(e^2+2df\right)\right)x\sqrt{a+bx+cx^2}}{96c^3} + \frac{f(16ce-7bf)x^2\sqrt{a+bx+}}{24c^2}$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+cx^2}}{192c^4} +$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+cx^2}}{192c^4} +$$

$$= \frac{\left(384c^3de-105b^3f^2+20bcf(12be+11af)-16c^2\left(16aef+9b\left(e^2+2df\right)\right)\right)\sqrt{a+bx+cx^2}}{192c^4} +$$

**Mathematica [A]** time = 0.536985, size = 251, normalized size = 0.79

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(48c^2\left(a^2f^2+4abef+b^2\left(2df+e^2\right)\right)-40b^2cf(3af+2be)-64c^3\left(a\left(2df+e^2\right)+2bde\right)+35b\right.}{128c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] (Sqrt[a + x*(b + c*x)]*(-105*b^3*f^2 + 10*b*c*f*(24*b*e + 22*a*f + 7*b*f*x)
 + 16*c^3*(12*d*(2*e + f*x) + x*(6*e^2 + 8*e*f*x + 3*f^2*x^2)) - 8*c^2*(a*f
*(32*e + 9*f*x) + b*(18*e^2 + 36*d*f + 20*e*f*x + 7*f^2*x^2))))/(192*c^4) +
 ((128*c^4*d^2 + 35*b^4*f^2 - 40*b^2*c*f*(2*b*e + 3*a*f) - 64*c^3*(2*b*d*e
+ a*(e^2 + 2*d*f)) + 48*c^2*(4*a*b*e*f + a^2*f^2 + b^2*(e^2 + 2*d*f)))*ArcT
anh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/(128*c^(9/2))
```

---

**Maple [B]** time = 0.062, size = 706, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -5/6*e*f*b/c^2*x*(c*x^2+b*x+a)^(1/2)+3/2*e*f*b/c^(5/2)*a*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x+a)^(1/2))+3/4*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))*d*f+2*d*e/c*(c*x^2+b*x+a)^(1/2)+1/2*x/c*(c*x^2+b*x+a)^(1/2)*e^2-
3/4*b/c^2*(c*x^2+b*x+a)^(1/2)*e^2+3/8*b^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))*e^2-35/64*f^2*b^3/c^4*(c*x^2+b*x+a)^(1/2)+35/128*f^2*b^4
/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/8*f^2*a^2/c^(5/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*a/c^(3/2)*ln((1/2*b+c*x)/c^(1
/2)+(c*x^2+b*x+a)^(1/2))*e^2+x/c*(c*x^2+b*x+a)^(1/2)*d*f-3/2*b/c^2*(c*x^2+b
*x+a)^(1/2)*d*f+d^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-a/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f-d*e*b/c^(3/2)*ln((1/
2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-7/24*f^2*b/c^2*x^2*(c*x^2+b*x+a)^(1/2
)+35/96*f^2*b^2/c^3*x*(c*x^2+b*x+a)^(1/2)-15/16*f^2*b^2/c^(7/2)*a*ln((1/2*b
+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+55/48*f^2*b/c^3*a*(c*x^2+b*x+a)^(1/2)-3/
8*f^2*a/c^2*x*(c*x^2+b*x+a)^(1/2)+2/3*e*f*x^2/c*(c*x^2+b*x+a)^(1/2)+5/4*e*f
*b^2/c^3*(c*x^2+b*x+a)^(1/2)-5/8*e*f*b^3/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))-4/3*e*f*a/c^2*(c*x^2+b*x+a)^(1/2)+1/4*f^2*x^3*(c*x^2+b*x+
a)^(1/2)/c
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 2.84519, size = 1457, normalized size = 4.61

$$\left[\frac{3\left(128\,c^4d^2 - 128\,bc^3de + 16\left(3\,b^2c^2 - 4\,ac^3\right)e^2 + \left(35\,b^4 - 120\,ab^2c + 48\,a^2c^2\right)f^2 + 16\left(2\left(3\,b^2c^2 - 4\,ac^3\right)d - \left(5\,b^3c - 12\,ab\right.\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/768*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*a*c^3)*e^2 + (35
*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4*a*c^3)*d - (5*b
^3*c - 12*a*b*c^2)*e)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*
x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(48*c^4*f^2*x^3 + 384*c^4*d
*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*f - 7*b*c^
3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(48*c^4*e^2
+ (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sqrt(c*x^2 +
 b*x + a))/c^5, -1/384*(3*(128*c^4*d^2 - 128*b*c^3*d*e + 16*(3*b^2*c^2 - 4*
a*c^3)*e^2 + (35*b^4 - 120*a*b^2*c + 48*a^2*c^2)*f^2 + 16*(2*(3*b^2*c^2 - 4
*a*c^3)*d - (5*b^3*c - 12*a*b*c^2)*e)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b
*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(48*c^4*f^2*x^3 +
 384*c^4*d*e - 144*b*c^3*e^2 - 5*(21*b^3*c - 44*a*b*c^2)*f^2 + 8*(16*c^4*e*
f - 7*b*c^3*f^2)*x^2 - 16*(18*b*c^3*d - (15*b^2*c^2 - 16*a*c^3)*e)*f + 2*(4
8*c^4*e^2 + (35*b^2*c^2 - 36*a*c^3)*f^2 + 16*(6*c^4*d - 5*b*c^3*e)*f)*x)*sq
rt(c*x^2 + b*x + a))/c^5]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+fx^2\right)^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/sqrt(a + b*x + c*x**2), x)
```

**Giac [A]** time = 1.39602, size = 410, normalized size = 1.3

$$\frac{1}{192}\sqrt{cx^2+bx+a}\left(2\left(4\left(\frac{6f^2x}{c}-\frac{7bc^2f^2-16c^3fe}{c^4}\right)x+\frac{96c^3df+35b^2cf^2-36ac^2f^2-80bc^2fe+48c^3e^2}{c^4}\right)x-288\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/192*sqrt(c*x^2 + b*x + a)*(2*(4*(6*f^2*x/c - (7*b*c^2*f^2 - 16*c^3*f*e)/c
^4)*x + (96*c^3*d*f + 35*b^2*c*f^2 - 36*a*c^2*f^2 - 80*b*c^2*f*e + 48*c^3*e
^2)/c^4)*x - (288*b*c^2*d*f + 105*b^3*f^2 - 220*a*b*c*f^2 - 384*c^3*d*e - 2
40*b^2*c*f*e + 256*a*c^2*f*e + 144*b*c^2*e^2)/c^4) - 1/128*(128*c^4*d^2 + 9
6*b^2*c^2*d*f - 128*a*c^3*d*f + 35*b^4*f^2 - 120*a*b^2*c*f^2 + 48*a^2*c^2*f
^2 - 128*b*c^3*d*e - 80*b^3*c*f*e + 192*a*b*c^2*f*e + 48*b^2*c^2*e^2 - 64*a
*c^3*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9
/2)
```

# 3.111 $\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}} \, dx$

**Optimal**. Leaf size=116

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

---

**Rubi [A]** time = 0.110769, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1661, 640, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}}+\frac{\sqrt{a+bx+cx^2}(4ce-3bf)}{4c^2}+\frac{fx\sqrt{a+bx+cx^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^2) + (f*x*Sqrt[a + b*x + c*x^2
])/(2*c) + ((8*c^2*d + 3*b^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sq
rt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(5/2))
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx = \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\int \frac{2cd-af+\frac{1}{2}(4ce-3bf)x}{\sqrt{a+bx+cx^2}}\,dx}{2c}$$
$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\int \frac{1}{\sqrt{a+bx+cx^2}}\,d}{4c^2}$$
$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(2c(2cd-af)-\frac{1}{2}b(4ce-3bf)\right)\mathrm{Subst}\left(\int \frac{1}{4c-}\right.}{2c^2}$$
$$= \frac{(4ce-3bf)\sqrt{a+bx+cx^2}}{4c^2} + \frac{fx\sqrt{a+bx+cx^2}}{2c} + \frac{\left(8c^2d+3b^2f-4c(be+af)\right)\tanh^{-1}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.}{8c^{5/2}}$$

**Mathematica [A]** time = 0.150978, size = 96, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)\left(-4c(af+be)+3b^2f+8c^2d\right)}{8c^{5/2}} + \frac{\sqrt{a+x(b+cx)}(-3bf+4ce+2cfx)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/Sqrt[a + b*x + c*x^2],x]
```

```
[Out] ((4*c*e - 3*b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)])/(4*c^2) + ((8*c^2*d + 3*b
^2*f - 4*c*(b*e + a*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)
])])/(8*c^(5/2))
```

---

**Maple [A]** time = 0.062, size = 185, normalized size = 1.6

$$\frac{fx}{2c}\sqrt{cx^2+bx+a} - \frac{3bf}{4c^2}\sqrt{cx^2+bx+a} + \frac{3b^2f}{8}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx+a}\right)c^{-\frac{5}{2}} - \frac{af}{2}\ln\left(\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] 1/2*f*x*(c*x^2+b*x+a)^(1/2)/c-3/4*f*b/c^2*(c*x^2+b*x+a)^(1/2)+3/8*f*b^2/c^(
5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-1/2*f*a/c^(3/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+e/c*(c*x^2+b*x+a)^(1/2)-1/2*e*b/c^(3/2)*l
n((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+d*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2))/c^(1/2)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.92911, size = 549, normalized size = 4.73

$$\left[-\frac{\left(8\,c^2d-4\,bce+\left(3\,b^2-4\,ac\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2+4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)-4\left(2\,c^2fx+4\,c^2e-\right.}{16\,c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*c^2*d - 4*b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(c)*log(-8*c^2*x^2 - 8*
b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(2*c
^2*f*x + 4*c^2*e - 3*b*c*f)*sqrt(c*x^2 + b*x + a))/c^3, -1/8*((8*c^2*d - 4*
b*c*e + (3*b^2 - 4*a*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
 + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(2*c^2*f*x + 4*c^2*e - 3*b*c*f)
*sqrt(c*x^2 + b*x + a))/c^3]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex+fx^2}{\sqrt{a+bx+cx^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/sqrt(a + b*x + c*x**2), x)
```

---

**Giac [A]** time = 1.29089, size = 132, normalized size = 1.14

$$\frac{1}{4}\sqrt{cx^2+bx+a}\left(\frac{2\,fx}{c}-\frac{3\,bf-4\,ce}{c^2}\right)-\frac{\left(8\,c^2d+3\,b^2f-4\,acf-4\,bce\right)\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{8\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(c*x^2 + b*x + a)*(2*f*x/c - (3*b*f - 4*c*e)/c^2) - 1/8*(8*c^2*d +
3*b^2*f - 4*a*c*f - 4*b*c*e)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))
*sqrt(c) - b))/c^(5/2)
```

## 3.112 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=374

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+c\cdots}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+\cdots}}$$

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

---

**Rubi [A]** time = 0.57883, antiderivative size = 374, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {983, 724, 206}

$$\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+c\cdots}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] -((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - S
qrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) +
(Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqr
t[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f]*Sqr
t[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 983

```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/q, Int[1/((
b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[(2*c)/q, Int[1/((b + q
 + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ[b
^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx = \frac{(2f)\int \frac{1}{\left(e-\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}} - \frac{(2f)\int \frac{1}{\left(e+\sqrt{e^2-4df}+2fx\right)\sqrt{a+bx+cx^2}}\,dx}{\sqrt{e^2-4df}}$$

$$= -\frac{(4f)\,\mathrm{Subst}\left(\int \frac{1}{16af^2-8bf\left(e-\sqrt{e^2-4df}\right)+4c\left(e-\sqrt{e^2-4df}\right)^2-x^2}\,dx, x, \frac{4af-b\left(e-\sqrt{e^2-4df}\right)-\left(-2bf+2c\right.}{\sqrt{a+bx+cx^2}}\right.}{\sqrt{e^2-4df}}$$

$$= -\frac{\sqrt{2}f\tanh^{-1}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}f\tanh^{-1}\left(\frac{}{2\sqrt{2}}\right.}{\sqrt{e^2-4df}\sqrt{ce^2-}}$$

**Mathematica [A]** time = 1.50838, size = 376, normalized size = 1.01

$$\frac{\sqrt{2}f\left(\frac{\tanh^{-1}\left(\frac{4af-b\left(\sqrt{e^2-4df}+e-2fx\right)-2cx\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\tanh^{-1}\left(\frac{4af+b\left(\sqrt{e^2-4df}-e+2fx\right)+2cx\left(\sqrt{e^2-4df}-e\right)}{2\sqrt{2}\sqrt{a+x(b+cx)}\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{f\left(2af+b\sqrt{e^2-4df}+b(-e)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

```
[Out] (Sqrt[2]*f*(ArcTanh[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^
2 - 4*d*f] - 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])
+ f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])]/Sqrt[c*(e^
2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))] -
 ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2 - 4*d*f
] + 2*f*x))/(2*Sqrt[2]*Sqrt[f*(-(b*e) + 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e
^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + x*(b + c*x)])]/Sqrt[f*(-(b*e) +
 2*a*f + b*Sqrt[e^2 - 4*d*f]) + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))/Sq
rt[e^2 - 4*d*f]
```

---

**Maple [B]** time = 0.359, size = 761, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x)
```

```
[Out] -1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*
```

```
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2
)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/
f))+1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e
^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*
e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 45.0179, size = 22873, normalized size = 61.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*
a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^
2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^
2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*
a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*
d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^
3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)
*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^
```

3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d
^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b
*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a
^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*
c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c
)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2
*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a
*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2
 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*
a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a
*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*
d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (
b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)
*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 -
 a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d
^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 +
2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 -
 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b
*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^
3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d
*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*
a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*
a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a
*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e +
 a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*
b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c
*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3
*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^
2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*
e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^
2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c
^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)
*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*
b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*
b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*
c*d + b*e)*f + (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*
e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3
 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^
2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^
2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*
f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b
*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8
*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b
^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c
^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*
d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*
c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*
c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d -
a*b*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*
b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f - (c^3*d^3*e^3 - b*c^2*d
^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3
*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^
4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3
*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e
^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f
+ b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^
2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*

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d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*
c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2
*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2
- 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2
+ (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*
x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f + (c^2*d^2*e^2 - b*c
*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2
)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((
c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d
*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d
*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a
*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d
*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 -
 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2
+ 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2
 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)
*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e
+ a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
 (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^
2 - (4*c^2*d*e - b*c*e^2)*f)*x - (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 -
4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 -
 (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)
*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3
 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 -
(b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f +
 b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 -
4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2
*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d
^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c
^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*
c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 -
 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 +
 (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x) + 1/4
*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3
 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2
- (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^
2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 +
 a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a
^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*
f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*
b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^
3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a
*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4
)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*
e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2
- 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b*e)*f^2 + sqrt(2)*(c^2*d*e^3 - 4*a*b
*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 +
 a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 -
 a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b
^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 - a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3
*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d
^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4 - a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^
2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e
^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*
e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^
3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*
b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2
)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d
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*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*
c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^
2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 -
(2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b
*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*
e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4
*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2
*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^
2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^
2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4
- 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(
a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*
b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^
2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 +
 a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 -
(4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e
- a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 - (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3
*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c
^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^
2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f + (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^
2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 + (4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2
*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^
4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^
3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e
^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3
 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b
^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)
*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*
e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c
*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2
*c - 2*a^2*c^2)*d*e^4)*f)))/x) - 1/4*sqrt(2)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d
 + b*e)*f - (c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e +
 a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 -
(b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 -
 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 +
 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4
 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)
*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b
^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3
- 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*
d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2
*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e
^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2
*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f))*log((2*(b^2*d - a*b
*e)*f^2 - sqrt(2)*(c^2*d*e^3 - 4*a*b*d*f^3 + (4*b*c*d^2 + 4*a*c*d*e + a*b*e
^2)*f^2 - (4*c^2*d^2*e + b*c*d*e^2 + a*c*e^3)*f + (c^3*d^3*e^3 - b*c^2*d^2*
e^4 + a*c^2*d*e^5 + 4*(2*a^2*b*d^2 - a^3*d*e)*f^4 + (2*a^2*b*d*e^2 + a^3*e^
3 + 8*(b^3 - 2*a*b*c)*d^3 - 4*(3*a*b^2 - a^2*c)*d^2*e)*f^3 + (8*b*c^2*d^4 -
 a^2*b*e^4 - 4*(3*b^2*c - a*c^2)*d^3*e - 2*(b^3 - 10*a*b*c)*d^2*e^2 + (3*a*
b^2 - 5*a^2*c)*d*e^3)*f^2 - (4*c^3*d^4*e - 2*b*c^2*d^3*e^2 + 4*a*b*c*d*e^4
- a^2*c*e^5 - (3*b^2*c - 5*a*c^2)*d^2*e^3)*f)*sqrt((c^2*e^2 - 2*b*c*e*f + b
^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*
a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b
^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^
2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2
*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (
b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))*sqrt(c*x^2
```

```
 + b*x + a)*sqrt((c*e^2 + 2*a*f^2 - (2*c*d + b*e)*f - (c^2*d^2*e^2 - b*c*d*
e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f
^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b^2 - 6*a*c)*d*e^2)*f)*sqrt((c^2
*e^2 - 2*b*c*e*f + b^2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^
5 + a^2*c^2*e^6 - 4*a^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e
+ a^4*e^2 - 8*(a^2*b^2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^
2*c + 6*a^2*c^2)*d^3 - 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^
2)*f^3 - (8*(b^2*c^2 - 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20
*a*b^2*c + 22*a^2*c^2)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2
*a^3*c)*e^4)*f^2 - 2*(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 +
6*a*c^3)*d^3*e^2 + (b^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*
e^4)*f)))/(c^2*d^2*e^2 - b*c*d*e^3 + a*c*e^4 - 4*a^2*d*f^3 + (4*a*b*d*e + a
^2*e^2 - 4*(b^2 - 2*a*c)*d^2)*f^2 - (4*c^2*d^3 - 4*b*c*d^2*e + a*b*e^3 - (b
^2 - 6*a*c)*d*e^2)*f)) - 2*(b*c*d*e - a*c*e^2)*f + ((4*b*c*d - b^2*e)*f^2 -
 (4*c^2*d*e - b*c*e^2)*f)*x + (8*a^3*d*f^4 - 2*(4*a^2*b*d*e + a^3*e^2 - 4*(
a*b^2 - 2*a^2*c)*d^2)*f^3 + 2*(4*a*c^2*d^3 - 4*a*b*c*d^2*e + a^2*b*e^3 - (a
*b^2 - 6*a^2*c)*d*e^2)*f^2 - 2*(a*c^2*d^2*e^2 - a*b*c*d*e^3 + a^2*c*e^4)*f
+ (4*a^2*b*d*f^4 - (4*a*b^2*d*e + a^2*b*e^2 - 4*(b^3 - 2*a*b*c)*d^2)*f^3 +
(4*b*c^2*d^3 - 4*b^2*c*d^2*e + a*b^2*e^3 - (b^3 - 6*a*b*c)*d*e^2)*f^2 - (b*
c^2*d^2*e^2 - b^2*c*d*e^3 + a*b*c*e^4)*f)*x)*sqrt((c^2*e^2 - 2*b*c*e*f + b^
2*f^2)/(c^4*d^4*e^2 - 2*b*c^3*d^3*e^3 - 2*a*b*c^2*d*e^5 + a^2*c^2*e^6 - 4*a
^4*d*f^5 + (b^2*c^2 + 2*a*c^3)*d^2*e^4 + (8*a^3*b*d*e + a^4*e^2 - 8*(a^2*b^
2 - 2*a^3*c)*d^2)*f^4 - 2*(a^3*b*e^3 + 2*(b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3
- 4*(a*b^3 - a^2*b*c)*d^2*e + (a^2*b^2 + 6*a^3*c)*d*e^2)*f^3 - (8*(b^2*c^2
- 2*a*c^3)*d^4 - 8*(b^3*c - a*b*c^2)*d^3*e - (b^4 - 20*a*b^2*c + 22*a^2*c^2
)*d^2*e^2 + 2*(a*b^3 - 5*a^2*b*c)*d*e^3 - (a^2*b^2 + 2*a^3*c)*e^4)*f^2 - 2*
(2*c^4*d^5 - 4*b*c^3*d^4*e + a^2*b*c*e^5 + (b^2*c^2 + 6*a*c^3)*d^3*e^2 + (b
^3*c - 5*a*b*c^2)*d^2*e^3 - 2*(a*b^2*c - 2*a^2*c^2)*d*e^4)*f)))/x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

# 3.113 $\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx$

**Optimal**. Leaf size=789

$$\frac{\left(f\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2f\left(f\left(-4a^2f^2+3abef+b^2\left(e^2-6df\right)\right)-c\left(4af\left(e^2-3df\right)+b\left(e^3-5de\right.\right.\right.\right.}{2\sqrt{2}\left(e^2-4df\right)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(}}$$

```
[Out] ((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c*
(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*(2*c*d - b*e + 2*a*f)*(c*e
- b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(3*a*b*e*f
- 4*a^2*f^2 + b^2*(e^2 - 6*d*f)) - c*(4*a*f*(e^2 - 3*d*f) + b*(e^3 - 5*d*e*
f))))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*(e^2 - 4*d*f)^(3
/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((f*(2*c*d - b*e + 2*a*f)*(c*
e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*f*(2*c^2*d*(e^2 - 4*d*f) + f*(3*a*b*e*
f - 4*a^2*f^2 + b^2*(e^2 - 6*d*f)) - c*(4*a*f*(e^2 - 3*d*f) + b*(e^3 - 5*d*
e*f))))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*(e^2 - 4*d*f)^
(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

---

**Rubi [A]** time = 8.20973, antiderivative size = 787, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{\left(f\left(e-\sqrt{e^2-4df}\right)(ce-bf)(2af-be+2cd)-2f\left(-4a^2f^3+3abef^2-4acf\left(e^2-3df\right)+b^2f\left(e^2-6df\right)-bc\left(e^3-5def\right)\right.\right.}{2\sqrt{2}\left(e^2-4df\right)^{3/2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] ((f*(b*e^2 - 2*b*d*f - a*e*f) - c*(e^3 - 3*d*e*f) + f*(f*(b*e - 2*a*f) - c*
(e^2 - 2*d*f))*x)*Sqrt[a + b*x + c*x^2])/((e^2 - 4*d*f)*((c*d - a*f)^2 - (b
*d - a*e)*(c*e - b*f))*(d + e*x + f*x^2)) + ((f*(2*c*d - b*e + 2*a*f)*(c*e
- b*f)*(e - Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^2
- 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d*e
*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2
 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*(e^2 - 4*d*f)^(3
/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f
+ 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((f*(2*c*d - b*e + 2*a*f)*(c*
e - b*f)*(e + Sqrt[e^2 - 4*d*f]) - 2*f*(3*a*b*e*f^2 - 4*a^2*f^3 + b^2*f*(e^
2 - 6*d*f) + 2*c^2*d*(e^2 - 4*d*f) - 4*a*c*f*(e^2 - 3*d*f) - b*c*(e^3 - 5*d
*e*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[2]*(e^2 - 4*d*f)^
```

```
(3/2)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*
f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)^2}\,dx = \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right)x\right)\sqrt{a+bx}}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+fx^2\right)}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right)x\right)\sqrt{a+bx}}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+fx^2\right)}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right)x\right)\sqrt{a+bx}}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+fx^2\right)}$$

$$= \frac{\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)+f\left(f(be-2af)-c\left(e^2-2df\right)\right)x\right)\sqrt{a+bx}}{\left(e^2-4df\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(d+ex+fx^2\right)}$$

**Mathematica [A]** time = 6.73391, size = 1377, normalized size = 1.75

$$-\frac{8\left(cx^2+bx+a\right)f^3}{\left(e^2-4df\right)\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{a+x(b+cx)}} - \frac{}{\left(e^2-4df\right)\left(4af\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)^2),x]
```

```
[Out] (-8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e - Sqrt[e^2 -
4*d*f]) + c*(e - Sqrt[e^2 - 4*d*f])^2)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt
[a + x*(b + c*x)]) - (8*f^3*(a + b*x + c*x^2))/((e^2 - 4*d*f)*(4*a*f^2 - 2*
b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2 - 4*d*f])^2)*(e + Sqrt[e^2 -
4*d*f] + 2*f*x)*Sqrt[a + x*(b + c*x)]) + (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^
2]*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*
f - b*(e - Sqrt[e^2 - 4*d*f]))]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*d*f)^(3/
2)*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2
- 4*d*f]))]*Sqrt[a + x*(b + c*x)]) - (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[e^2 - 4*d*f]]*(2*b*f + 2
*c*(-e + Sqrt[e^2 - 4*d*f]))*Sqrt[a + b*x + c*x^2]*ArcTanh[(-4*a*f - b*(-e
+ Sqrt[e^2 - 4*d*f]) - (2*b*f + 2*c*(-e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]
*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - c*e*Sqrt[e^2 - 4*d*f] + b*f*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/((e^2 - 4*d*f)*(4*a*f^2 + 2*b*f*(-e
+ Sqrt[e^2 - 4*d*f]) + c*(-e + Sqrt[e^2 - 4*d*f])^2)*(16*a*f^2 + 8*b*f*(-e
+ Sqrt[e^2 - 4*d*f]) + 4*c*(-e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)
]) - (2*Sqrt[2]*f^2*Sqrt[a + b*x + c*x^2]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2
- 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt
[a + b*x + c*x^2])])/((e^2 - 4*d*f)^(3/2)*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2
- 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)]) -
 (8*Sqrt[2]*f^2*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + c*e*Sqrt[e^2 - 4*d
*f] - b*f*Sqrt[e^2 - 4*d*f]]*(-2*b*f + 2*c*(e + Sqrt[e^2 - 4*d*f]))*Sqrt[a
+ b*x + c*x^2]*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) - (-2*b*f + 2*c*(
e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2 + c*e*Sqrt[e^2 - 4*d*f] - b*f*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])
/((e^2 - 4*d*f)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) + c*(e + Sqrt[e^2
```

```
- 4*d*f])^2)*(16*a*f^2 - 8*b*f*(e + Sqrt[e^2 - 4*d*f]) + 4*c*(e + Sqrt[e^2
- 4*d*f])^2)*Sqrt[a + x*(b + c*x)])
```

---

**Maple [B]** time = 0.329, size = 3858, normalized size = 4.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x)
```

```
[Out] -2*f/(4*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*
f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4
*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*
f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f))+2/(4*d*f-e^2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/(x+1/2*(-4*d*f+e^2)^(1/2)/f+1/2*e/f)*
((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/(4*d*f-e^2)*f/(-(-4*d*f+
e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*2^(1/2)/
((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)
)/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))*c*(-4*d*f+e^2)^(1/2)-1
/(4*d*f-e^2)*f^2/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*
e*f-2*c*d*f+c*e^2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/
2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1
/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f))*b+1/(4*d*f-e^2)*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f
+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^
2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e
^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*
e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f))*c*e+2/(4*d*f-e^2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/(x-1/2*(-4*d*f+e^2)^(1/2)/f+1/2*e/f)*((x-
1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*
(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-1/(4*d*f-e^2)*f/((-4*d*f+e^2)^(1
/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*2^(1/2)/(((-4*d
```

```
*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^
(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*
(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d
*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^
(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*(-4*d*f+e^2)^(1/2)-1/(4*d*f-e^2
)*f^2/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/
2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e
^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*b+1/(4*d*f-
e^2)*f/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f
+c*e^2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e
*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1
/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*
f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+
e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e
*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f))*c*e+2*f/(4
*d*f-e^2)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)
*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*
f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d
*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^
(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*
f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)^2), x)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**2,x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

# 3.114 $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=649

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^2c^2df+b^2c^2e^2-2b^3cef+b^4f^2-b\right.\right.$$

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(c^5*(b^2 -
 4*a*c)*Sqrt[a + b*x + c*x^2]) - ((187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*
f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*Sqrt[
a + b*x + c*x^2])/(64*c^5) + (f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) + 48*c
^2*(e^2 + d*f))*x*Sqrt[a + b*x + c*x^2])/(32*c^4) + (f^2*(8*c*e - 5*b*f)*x^
2*Sqrt[a + b*x + c*x^2])/(8*c^3) + (f^3*x^3*Sqrt[a + b*x + c*x^2])/(4*c^2)
+ (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*
c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f)
 + b*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(128*c^(11/2))
```

---

**Rubi [A]** time = 2.10557, antiderivative size = 649, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 621, 206}

$$2\left(-x\left(-2acf+b^2f-bce+2c^2d\right)\left(a^2c^2f^2-4ab^2cf^2+7abc^2ef-2ac^3df-3ac^3e^2+b^2c^2df+b^2c^2e^2-2b^3cef+b^4f^2-b\right.\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(c^5*(b^2 -
 4*a*c)*Sqrt[a + b*x + c*x^2]) - ((187*b^3*f^3 - 4*b*c*f^2*(114*b*e + 73*a*
f) - 64*c^3*(e^3 + 6*d*e*f) + 16*c^2*f*(20*a*e*f + 21*b*(e^2 + d*f)))*Sqrt[
a + b*x + c*x^2])/(64*c^5) + (f*(41*b^2*f^2 - 4*c*f*(22*b*e + 7*a*f) + 48*c
^2*(e^2 + d*f))*x*Sqrt[a + b*x + c*x^2])/(32*c^4) + (f^2*(8*c*e - 5*b*f)*x^
2*Sqrt[a + b*x + c*x^2])/(8*c^3) + (f^3*x^3*Sqrt[a + b*x + c*x^2])/(4*c^2)
+ (3*(105*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*
c^2*f*(6*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f)
 + b*(e^3 + 6*d*e*f)))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2]
)])/(128*c^(11/2))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+df\right)-9a\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+df\right)-9a\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+df\right)-9a\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+df\right)-9a\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+df\right)-9a\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+df\right)-9a\right.}{}$$

$$= \frac{2\left(3ab^4cef^2-ab^5f^3+ab^3cf\left(5af^2-3c\left(e^2+df\right)\right)-bc^2\left(c^3d^3+5a^3f^3+3ac^2d\left(e^2+df\right)-9a\right.}{}$$

**Mathematica [A]** time = 1.6847, size = 745, normalized size = 1.15

$$\frac{-16b^2c^2\left(-a^2f^2(230e+169fx)+ac\left(2ef\left(36d-43fx^2\right)+f^2x\left(186d-13fx^2\right)+186e^2fx+12e^3\right)+c^2x\left(-24d^2f+6d\left(-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (315*b^6*f^3*x + 105*b^5*f^2*(3*a*f + c*x*(-8*e + f*x)) - 2*b^4*c*f*(105*a*
f*(4*e + 9*f*x) + c*x*(-360*e^2 - 360*d*f + 140*e*f*x + 21*f^2*x^2)) - 8*b^
3*c*(210*a^2*f^3 - c^2*x*(-24*e^3 + 30*e^2*f*x + 3*f^2*x*(10*d + f*x^2) + 2
*e*f*(-72*d + 7*f*x^2)) + a*c*f*(-90*e^2 - 530*e*f*x + f*(-90*d + 77*f*x^2)
)) - 16*b^2*c^2*(-(a^2*f^2*(230*e + 169*f*x)) + a*c*(12*e^3 + 186*e^2*f*x +
 2*e*f*(36*d - 43*f*x^2) + f^2*x*(186*d - 13*f*x^2)) + c^2*x*(-24*d^2*f + 6
*d*(-4*e^2 + 4*e*f*x + f^2*x^2) + x*(4*e^3 + 6*e^2*f*x + 4*e*f^2*x^2 + f^3*
x^3))) + 32*c^3*(8*c^3*d^3*x - a^3*f^2*(64*e + 15*f*x) + a^2*c*(16*e^3 + 36
*e^2*f*x + f^2*x*(36*d - 5*f*x^2) - 32*e*f*(-3*d + f*x^2)) + 2*a*c^2*(-12*d
^2*(e + f*x) + 6*d*x*(-2*e^2 + 4*e*f*x + f^2*x^2) + x^2*(4*e^3 + 6*e^2*f*x
+ 4*e*f^2*x^2 + f^3*x^3))) + 16*b*c^2*(113*a^3*f^3 + 8*c^3*d^2*(d - 3*e*x)
+ a^2*c*f*(-156*e^2 - 244*e*f*x + f*(-156*d + 49*f*x^2)) + 2*a*c^2*(12*d^2*
f + 6*d*(2*e^2 + 20*e*f*x - 5*f^2*x^2) - x*(-20*e^3 + 30*e^2*f*x + 14*e*f^2
*x^2 + 3*f^3*x^3))))/(64*c^5*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + (3*(10
5*b^4*f^3 - 280*b^2*c*f^2*(b*e + a*f) + 128*c^4*d*(e^2 + d*f) + 80*c^2*f*(6
*a*b*e*f + a^2*f^2 + 3*b^2*(e^2 + d*f)) - 64*c^3*(3*a*f*(e^2 + d*f) + b*(e^
3 + 6*d*e*f)))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(128*c^(11
/2))
```

**Maple [B]** time = 0.069, size = 2827, normalized size = 4.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 3/2*b/c^2*x/(c*x^2+b*x+a)^(1/2)*e^3-3/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)*e^3+115/8*e*f^2*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-45/4*e*f^2*b
/c^3*a*x/(c*x^2+b*x+a)^(1/2)+113/8*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a
)^(1/2)*x+9*b/c^2*x/(c*x^2+b*x+a)^(1/2)*d*e*f-9/2*b^4/c^3/(4*a*c-b^2)/(c*x^
2+b*x+a)^(1/2)*d*e*f-8*e*f^2*a^2/c^3*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+45
/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f^2+45/8*b^4/c^3/(4*a*c-b^2)
/(c*x^2+b*x+a)^(1/2)*x*e^2*f-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
*d*f^2-39/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^2*f+3*b^2/c/(4*a*c-
b^2)/(c*x^2+b*x+a)^(1/2)*x*d*e^2+4*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*
e^3+3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*f*d^2+24*a/c*b/(4*a*c-b^2)/(c
*x^2+b*x+a)^(1/2)*x*d*e*f+6*x^2/c/(c*x^2+b*x+a)^(1/2)*d*e*f-9/2*b^2/c^3/(c*
x^2+b*x+a)^(1/2)*d*e*f-3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^3-9*
b/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*e*f+12*a/c^2/(c*x^2
+b*x+a)^(1/2)*d*e*f-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*f^2-
16*e*f^2*a^2/c^2*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+115/4*e*f^2*b^3/c^3*a/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-39/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)*x*e^2*f+12*a/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*e*f-9*b^3/c^2/
(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*d*e*f-105/32*e*f^2*b^4/c^5/(c*x^2+b*x+a)^
(1/2)-105/16*e*f^2*b^3/c^(9/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-
8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^(1/2)-5/8*f^3*a/c^2*x^3/(c*x^2+b*x+a)^(1/2)-3
/8*f^3*b/c^2*x^4/(c*x^2+b*x+a)^(1/2)+21/32*f^3*b^2/c^3*x^3/(c*x^2+b*x+a)^(1
/2)-105/64*f^3*b^3/c^4*x^2/(c*x^2+b*x+a)^(1/2)-315/128*f^3*b^4/c^5*x/(c*x^2
+b*x+a)^(1/2)+315/256*f^3*b^7/c^6/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-105/16*f^
3*b^3/c^5*a/(c*x^2+b*x+a)^(1/2)+113/16*f^3*b/c^4*a^2/(c*x^2+b*x+a)^(1/2)-10
5/16*f^3*b^2/c^(9/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-15/8*f^3
*a^2/c^3*x/(c*x^2+b*x+a)^(1/2)-3*x/c/(c*x^2+b*x+a)^(1/2)*f*d^2-3*x/c/(c*x^2
+b*x+a)^(1/2)*d*e^2+3/2*b/c^2/(c*x^2+b*x+a)^(1/2)*f*d^2+3/2*b/c^2/(c*x^2+b*
x+a)^(1/2)*d*e^2+3/2*x^3/c/(c*x^2+b*x+a)^(1/2)*d*f^2+3/2*x^3/c/(c*x^2+b*x+a
)^(1/2)*e^2*f+45/16*b^3/c^4/(c*x^2+b*x+a)^(1/2)*d*f^2+45/16*b^3/c^4/(c*x^2+
b*x+a)^(1/2)*e^2*f+45/8*b^2/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1
/2))*d*f^2+45/8*b^2/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2
*f-9/2*a/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2-9/2*a/c^
(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^2*f+e*f^2*x^4/c/(c*x^2+
b*x+a)^(1/2)-39/4*b/c^3*a/(c*x^2+b*x+a)^(1/2)*d*f^2-39/4*b/c^3*a/(c*x^2+b*x
+a)^(1/2)*e^2*f+9/2*a/c^2*x/(c*x^2+b*x+a)^(1/2)*d*f^2+9/2*a/c^2*x/(c*x^2+b*
x+a)^(1/2)*e^2*f+115/8*e*f^2*b^2/c^4*a/(c*x^2+b*x+a)^(1/2)+45/4*e*f^2*b/c^(
7/2)*a*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-4*e*f^2*a/c^2*x^2/(c*x^2
+b*x+a)^(1/2)+3/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*f*d^2+3/2*b^3/c^2
/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*e^2+45/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x
+a)^(1/2)*d*f^2+45/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^2*f+x^2/c/(
c*x^2+b*x+a)^(1/2)*e^3-3/4*b^2/c^3/(c*x^2+b*x+a)^(1/2)*e^3-3/2*b/c^(5/2)*ln
((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*e^3+2*a/c^2/(c*x^2+b*x+a)^(1/2)*e
^3+315/256*f^3*b^5/c^6/(c*x^2+b*x+a)^(1/2)+315/128*f^3*b^4/c^(11/2)*ln((1/2
*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/4*f^3*x^5/c/(c*x^2+b*x+a)^(1/2)+2*a/
c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^3+15/8*f^3*a^2/c^(7/2)*ln((1/2*b+
c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*f*d^2+3/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*d*
e^2-3*d^2*e/c/(c*x^2+b*x+a)^(1/2)+2*d^3*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)
^(1/2)+105/16*f^3*b^2/c^4*a*x/(c*x^2+b*x+a)^(1/2)+49/16*f^3*b/c^3*a*x^2/(c*
x^2+b*x+a)^(1/2)+113/16*f^3*b^3/c^4*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-15/
4*b/c^2*x^2/(c*x^2+b*x+a)^(1/2)*d*f^2-15/4*b/c^2*x^2/(c*x^2+b*x+a)^(1/2)*e^
```

```
2*f-45/8*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)*d*f^2-45/8*b^2/c^3*x/(c*x^2+b*x+a)^(
1/2)*e^2*f-105/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-105/8*f^3
*b^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-7/4*e*f^2*b/c^2*x^3/(c*x^2+b*x
+a)^(1/2)+35/8*e*f^2*b^2/c^3*x^2/(c*x^2+b*x+a)^(1/2)+105/16*e*f^2*b^3/c^4*x
/(c*x^2+b*x+a)^(1/2)-105/32*e*f^2*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+3
15/128*f^3*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-105/16*f^3*b^5/c^5*a/(
4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-6*d^2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-
3*d^2*e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 15.911, size = 6743, normalized size = 10.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/256*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(a*b^3*c^3 - 4*a^2*b*c^4)
*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^2 - 64*a^4*c^3)*f^3 + 8*
(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d - 5*(7*a*b^5*c - 40*a^2*b^
3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*c^6)*d*e^2 - 64*(b^3*c^4
 - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 240*a^2*b^2*c^3 - 64*a^3*
c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*d - 5*(7*b^5*c^2 -
40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^5 - 4*a*c^6)*d^2 - 24*(b
^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*c^5)*e^2)*f)
*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e
 + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^2)*f + (128*(b^3*c^4 - 4
*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 + 5*(21*b^7 - 140*a*b^5*c
+ 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5*c^2 - 24*a*b^3*c^3 + 16
*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*b^2*c^3)*e)*f^2 + 16*(8*
(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c^4)*d*e + 3*(5*b^5*c^2 -
 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x -
 b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(128*b*c^6*
d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 - 4*a*c^6)*f^3*x^5
- 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5)*f^3)*x^4 - 64*(3
*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3*c^2 + 1808*a^3*b*
c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4*c^3 - 104*a*b^2*c^4 +
80*a^2*c^5)*f^3 + 8*(6*(b^2*c^5 - 4*a*c^6)*d - 7*(b^3*c^4 - 4*a*b*c^5)*e)*f
^2)*x^3 + 8*(6*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d - (105*a*b^4*c^2 - 460*a^2*b
^2*c^3 + 256*a^3*c^4)*e)*f^2 - (64*(b^2*c^5 - 4*a*c^6)*e^3 - 7*(15*b^5*c^2
- 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3 - 8*(30*(b^3*c^4 - 4*a*b*c^5)*d - (35*b
^4*c^3 - 172*a*b^2*c^4 + 128*a^2*c^5)*e)*f^2 + 48*(8*(b^2*c^5 - 4*a*c^6)*d*
e - 5*(b^3*c^4 - 4*a*b*c^5)*e^2)*f)*x^2 + 48*(8*a*b*c^5*d^2 - 8*(3*a*b^2*c^
4 - 8*a^2*c^5)*d*e + (15*a*b^3*c^3 - 52*a^2*b*c^4)*e^2)*f + (256*c^7*d^3 -
384*b*c^6*d^2*e + 384*(b^2*c^5 - 2*a*c^6)*d*e^2 - 64*(3*b^3*c^4 - 10*a*b*c^
```

```
5)*e^3 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 - 480*a^3*c^4)*f^3
+ 8*(6*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*d - (105*b^5*c^2 - 530*a*b^
3*c^3 + 488*a^2*b*c^4)*e)*f^2 + 48*(8*(b^2*c^5 - 2*a*c^6)*d^2 - 8*(3*b^3*c^
4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*e^2)*f)*x)*s
qrt(c*x^2 + b*x + a))/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 - 4*a*c^8)*x^2 + (b
^3*c^6 - 4*a*b*c^7)*x), -1/128*(3*(128*(a*b^2*c^4 - 4*a^2*c^5)*d*e^2 - 64*(
a*b^3*c^3 - 4*a^2*b*c^4)*e^3 + 5*(21*a*b^6 - 140*a^2*b^4*c + 240*a^3*b^2*c^
2 - 64*a^4*c^3)*f^3 + 8*(6*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*d -
5*(7*a*b^5*c - 40*a^2*b^3*c^2 + 48*a^3*b*c^3)*e)*f^2 + (128*(b^2*c^5 - 4*a*
c^6)*d*e^2 - 64*(b^3*c^4 - 4*a*b*c^5)*e^3 + 5*(21*b^6*c - 140*a*b^4*c^2 + 2
40*a^2*b^2*c^3 - 64*a^3*c^4)*f^3 + 8*(6*(5*b^4*c^3 - 24*a*b^2*c^4 + 16*a^2*
c^5)*d - 5*(7*b^5*c^2 - 40*a*b^3*c^3 + 48*a^2*b*c^4)*e)*f^2 + 16*(8*(b^2*c^
5 - 4*a*c^6)*d^2 - 24*(b^3*c^4 - 4*a*b*c^5)*d*e + 3*(5*b^4*c^3 - 24*a*b^2*c
^4 + 16*a^2*c^5)*e^2)*f)*x^2 + 16*(8*(a*b^2*c^4 - 4*a^2*c^5)*d^2 - 24*(a*b^
3*c^3 - 4*a^2*b*c^4)*d*e + 3*(5*a*b^4*c^2 - 24*a^2*b^2*c^3 + 16*a^3*c^4)*e^
2)*f + (128*(b^3*c^4 - 4*a*b*c^5)*d*e^2 - 64*(b^4*c^3 - 4*a*b^2*c^4)*e^3 +
5*(21*b^7 - 140*a*b^5*c + 240*a^2*b^3*c^2 - 64*a^3*b*c^3)*f^3 + 8*(6*(5*b^5
*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*d - 5*(7*b^6*c - 40*a*b^4*c^2 + 48*a^2*
b^2*c^3)*e)*f^2 + 16*(8*(b^3*c^4 - 4*a*b*c^5)*d^2 - 24*(b^4*c^3 - 4*a*b^2*c
^4)*d*e + 3*(5*b^5*c^2 - 24*a*b^3*c^3 + 16*a^2*b*c^4)*e^2)*f)*x)*sqrt(-c)*a
rctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c
)) + 2*(128*b*c^6*d^3 - 768*a*c^6*d^2*e + 384*a*b*c^5*d*e^2 - 16*(b^2*c^5 -
 4*a*c^6)*f^3*x^5 - 8*(8*(b^2*c^5 - 4*a*c^6)*e*f^2 - 3*(b^3*c^4 - 4*a*b*c^5
)*f^3)*x^4 - 64*(3*a*b^2*c^4 - 8*a^2*c^5)*e^3 + (315*a*b^5*c - 1680*a^2*b^3
*c^2 + 1808*a^3*b*c^3)*f^3 - 2*(48*(b^2*c^5 - 4*a*c^6)*e^2*f + (21*b^4*c^3
- 104*a*b^2*c^4 + 80*a^2*c^5)*f^3 + 8*(6*(b^2*c^5 - 4*a*c^6)*d - 7*(b^3*c^4
 - 4*a*b*c^5)*e)*f^2)*x^3 + 8*(6*(15*a*b^3*c^3 - 52*a^2*b*c^4)*d - (105*a*b
^4*c^2 - 460*a^2*b^2*c^3 + 256*a^3*c^4)*e)*f^2 - (64*(b^2*c^5 - 4*a*c^6)*e^
3 - 7*(15*b^5*c^2 - 88*a*b^3*c^3 + 112*a^2*b*c^4)*f^3 - 8*(30*(b^3*c^4 - 4*
a*b*c^5)*d - (35*b^4*c^3 - 172*a*b^2*c^4 + 128*a^2*c^5)*e)*f^2 + 48*(8*(b^2
*c^5 - 4*a*c^6)*d*e - 5*(b^3*c^4 - 4*a*b*c^5)*e^2)*f)*x^2 + 48*(8*a*b*c^5*d
^2 - 8*(3*a*b^2*c^4 - 8*a^2*c^5)*d*e + (15*a*b^3*c^3 - 52*a^2*b*c^4)*e^2)*f
 + (256*c^7*d^3 - 384*b*c^6*d^2*e + 384*(b^2*c^5 - 2*a*c^6)*d*e^2 - 64*(3*b
^3*c^4 - 10*a*b*c^5)*e^3 + (315*b^6*c - 1890*a*b^4*c^2 + 2704*a^2*b^2*c^3 -
 480*a^3*c^4)*f^3 + 8*(6*(15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2*c^5)*d - (105*
b^5*c^2 - 530*a*b^3*c^3 + 488*a^2*b*c^4)*e)*f^2 + 48*(8*(b^2*c^5 - 2*a*c^6)
*d^2 - 8*(3*b^3*c^4 - 10*a*b*c^5)*d*e + (15*b^4*c^3 - 62*a*b^2*c^4 + 24*a^2
*c^5)*e^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^6 - 4*a^2*c^7 + (b^2*c^7 -
 4*a*c^8)*x^2 + (b^3*c^6 - 4*a*b*c^7)*x)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.3817, size = 1484, normalized size = 2.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(((2*(4*(2*(b^2*c^4*f^3 - 4*a*c^5*f^3)*x/(b^2*c^5 - 4*a*c^6) - (3*b^3*
c^3*f^3 - 12*a*b*c^4*f^3 - 8*b^2*c^4*f^2*e + 32*a*c^5*f^2*e)/(b^2*c^5 - 4*a
*c^6))*x + (48*b^2*c^4*d*f^2 - 192*a*c^5*d*f^2 + 21*b^4*c^2*f^3 - 104*a*b^2
*c^3*f^3 + 80*a^2*c^4*f^3 - 56*b^3*c^3*f^2*e + 224*a*b*c^4*f^2*e + 48*b^2*c
^4*f*e^2 - 192*a*c^5*f*e^2)/(b^2*c^5 - 4*a*c^6))*x - (240*b^3*c^3*d*f^2 - 9
60*a*b*c^4*d*f^2 + 105*b^5*c*f^3 - 616*a*b^3*c^2*f^3 + 784*a^2*b*c^3*f^3 -
384*b^2*c^4*d*f*e + 1536*a*c^5*d*f*e - 280*b^4*c^2*f^2*e + 1376*a*b^2*c^3*f
^2*e - 1024*a^2*c^4*f^2*e + 240*b^3*c^3*f*e^2 - 960*a*b*c^4*f*e^2 - 64*b^2*
c^4*e^3 + 256*a*c^5*e^3)/(b^2*c^5 - 4*a*c^6))*x - (256*c^6*d^3 + 384*b^2*c^
4*d^2*f - 768*a*c^5*d^2*f + 720*b^4*c^2*d*f^2 - 2976*a*b^2*c^3*d*f^2 + 1152
*a^2*c^4*d*f^2 + 315*b^6*f^3 - 1890*a*b^4*c*f^3 + 2704*a^2*b^2*c^2*f^3 - 48
0*a^3*c^3*f^3 - 384*b*c^5*d^2*e - 1152*b^3*c^3*d*f*e + 3840*a*b*c^4*d*f*e -
 840*b^5*c*f^2*e + 4240*a*b^3*c^2*f^2*e - 3904*a^2*b*c^3*f^2*e + 384*b^2*c^
4*d*e^2 - 768*a*c^5*d*e^2 + 720*b^4*c^2*f*e^2 - 2976*a*b^2*c^3*f*e^2 + 1152
*a^2*c^4*f*e^2 - 192*b^3*c^3*e^3 + 640*a*b*c^4*e^3)/(b^2*c^5 - 4*a*c^6))*x
- (128*b*c^5*d^3 + 384*a*b*c^4*d^2*f + 720*a*b^3*c^2*d*f^2 - 2496*a^2*b*c^3
*d*f^2 + 315*a*b^5*f^3 - 1680*a^2*b^3*c*f^3 + 1808*a^3*b*c^2*f^3 - 768*a*c^
5*d^2*e - 1152*a*b^2*c^3*d*f*e + 3072*a^2*c^4*d*f*e - 840*a*b^4*c*f^2*e + 3
680*a^2*b^2*c^2*f^2*e - 2048*a^3*c^3*f^2*e + 384*a*b*c^4*d*e^2 + 720*a*b^3*
c^2*f*e^2 - 2496*a^2*b*c^3*f*e^2 - 192*a*b^2*c^3*e^3 + 512*a^2*c^4*e^3)/(b^
2*c^5 - 4*a*c^6))/sqrt(c*x^2 + b*x + a) - 3/128*(128*c^4*d^2*f + 240*b^2*c^
2*d*f^2 - 192*a*c^3*d*f^2 + 105*b^4*f^3 - 280*a*b^2*c*f^3 + 80*a^2*c^2*f^3
- 384*b*c^3*d*f*e - 280*b^3*c*f^2*e + 480*a*b*c^2*f^2*e + 128*c^4*d*e^2 + 2
40*b^2*c^2*f*e^2 - 192*a*c^3*f*e^2 - 64*b*c^3*e^3)*log(abs(-2*(sqrt(c)*x -
sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(11/2)
```

## 3.115 $\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx$

**Optimal**. Leaf size=309

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3a^2f^2\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*(8*c*e - 7*b*
f)*Sqrt[a + b*x + c*x^2])/(4*c^3) + (f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) +
 ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

---

**Rubi [A]** time = 0.446654, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{2\left(-x\left(c^2\left(2a^2f^2+6abef+b^2\left(2df+e^2\right)\right)-2b^2cf(2af+be)-2c^3\left(a\left(2df+e^2\right)+bde\right)+b^4f^2+2c^4d^2\right)-bc\left(-3a^2f^2\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(c^3*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*(8*c*e - 7*b*
f)*Sqrt[a + b*x + c*x^2])/(4*c^3) + (f^2*x*Sqrt[a + b*x + c*x^2])/(2*c^2) +
 ((15*b^2*f^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*ArcTanh[(b + 2*
c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*c^(7/2))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
```

```
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-2b\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+b}}$$

$$=\frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-2b\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+b}}$$

$$=\frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-2b\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+b}}$$

$$=\frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-2b\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+b}}$$

$$=\frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-2b\right.\right.}{c^3\left(b^2-4ac\right)\sqrt{a+b}}$$

**Mathematica [A]** time = 0.756993, size = 288, normalized size = 0.93

$$\frac{4bc\left(-13a^2f^2+ac\left(4df+2e^2+20efx-5f^2x^2\right)+2c^2d(d-2ex)\right)+8c^2\left(a^2f(8e+3fx)+ac\left(x\left(-2e^2+4efx+f^2x^2\right)-4\right.\right.}{4c^3\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (15*b^4*f^2*x + b^3*f*(15*a*f + c*x*(-24*e + 5*f*x)) + 4*b*c*(-13*a^2*f^2 +
 2*c^2*d*(d - 2*e*x) + a*c*(2*e^2 + 4*d*f + 20*e*f*x - 5*f^2*x^2)) - 2*b^2*
c*(a*f*(12*e + 31*f*x) + c*x*(-4*e^2 - 8*d*f + 4*e*f*x + f^2*x^2)) + 8*c^2*
(2*c^2*d^2*x + a^2*f*(8*e + 3*f*x) + a*c*(-4*d*(e + f*x) + x*(-2*e^2 + 4*e*
f*x + f^2*x^2))))/(4*c^3*(-b^2 + 4*a*c)*Sqrt[a + x*(b + c*x)]) + ((15*b^2*f
```

```
^2 - 12*c*f*(2*b*e + a*f) + 8*c^2*(e^2 + 2*d*f))*Log[b + 2*c*x + 2*Sqrt[c]*
Sqrt[a + x*(b + c*x)]])/(8*c^(7/2))
```

**Maple [B]** time = 0.067, size = 1011, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] 8*e*f*a/c*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+2*b^2/c/(4*a*c-b^2)/(c*x^2+b*
x+a)^(1/2)*x*d*f-3*e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+4*e*f*a/c^
2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))*e^2-13/2*f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+3
*e*f*b/c^2*x/(c*x^2+b*x+a)^(1/2)-3/2*e*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)-4*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-2*d*e*b^2/c/(4*a*c-b^2)/(c*
x^2+b*x+a)^(1/2)+b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2+b^3/c^2/(4*a*c
-b^2)/(c*x^2+b*x+a)^(1/2)*d*f+15/8*f^2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1
/2)*x-2*x/c/(c*x^2+b*x+a)^(1/2)*d*f+2*e*f*x^2/c/(c*x^2+b*x+a)^(1/2)-5/4*f^2
*b/c^2*x^2/(c*x^2+b*x+a)^(1/2)-15/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^(1/2)+15/16
*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-3/2*e*f*b^2/c^3/(c*x^2+b*x+a)^
(1/2)-3*e*f*b/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+4*e*f*a/c
^2/(c*x^2+b*x+a)^(1/2)+b/c^2/(c*x^2+b*x+a)^(1/2)*d*f+1/2*b^3/c^2/(4*a*c-b^2
)/(c*x^2+b*x+a)^(1/2)*e^2+3/2*f^2*a/c^2*x/(c*x^2+b*x+a)^(1/2)-13/4*f^2*b/c^
3*a/(c*x^2+b*x+a)^(1/2)+2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2
))*d*f-x/c/(c*x^2+b*x+a)^(1/2)*e^2+1/2*b/c^2/(c*x^2+b*x+a)^(1/2)*e^2+15/8*f
^2*b^2/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-3/2*f^2*a/c^(5/2
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*f^2*x^3/c/(c*x^2+b*x+a)^(
1/2)+15/16*f^2*b^3/c^4/(c*x^2+b*x+a)^(1/2)-2*d*e/c/(c*x^2+b*x+a)^(1/2)+2*d^
2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-13/4*f^2*b^3/c^3*a/(4*a*c-b^2)/
(c*x^2+b*x+a)^(1/2)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 9.59512, size = 2750, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((8*(a*b^2*c^2 - 4*a^2*c^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3
*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2
```

```
*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2
+ 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3
*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b
^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(c)*log(-8*c^2
*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c)
 + 4*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^
2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b
^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e
)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c -
62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 -
10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c
^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*c^5)*x), -1/8*((8*(a*b^2*c^2 - 4*a^2*c
^3)*e^2 + 3*(5*a*b^4 - 24*a^2*b^2*c + 16*a^3*c^2)*f^2 + (8*(b^2*c^3 - 4*a*c
^4)*e^2 + 3*(5*b^4*c - 24*a*b^2*c^2 + 16*a^2*c^3)*f^2 + 8*(2*(b^2*c^3 - 4*a
*c^4)*d - 3*(b^3*c^2 - 4*a*b*c^3)*e)*f)*x^2 + 8*(2*(a*b^2*c^2 - 4*a^2*c^3)*
d - 3*(a*b^3*c - 4*a^2*b*c^2)*e)*f + (8*(b^3*c^2 - 4*a*b*c^3)*e^2 + 3*(5*b^
5 - 24*a*b^3*c + 16*a^2*b*c^2)*f^2 + 8*(2*(b^3*c^2 - 4*a*b*c^3)*d - 3*(b^4*
c - 4*a*b^2*c^2)*e)*f)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(8*b*c^4*d^2 - 32*a*c^4*d*e + 8*
a*b*c^3*e^2 - 2*(b^2*c^3 - 4*a*c^4)*f^2*x^3 + (15*a*b^3*c - 52*a^2*b*c^2)*f
^2 - (8*(b^2*c^3 - 4*a*c^4)*e*f - 5*(b^3*c^2 - 4*a*b*c^3)*f^2)*x^2 + 8*(2*a
*b*c^3*d - (3*a*b^2*c^2 - 8*a^2*c^3)*e)*f + (16*c^5*d^2 - 16*b*c^4*d*e + 8*
(b^2*c^3 - 2*a*c^4)*e^2 + (15*b^4*c - 62*a*b^2*c^2 + 24*a^2*c^3)*f^2 + 8*(2
*(b^2*c^3 - 2*a*c^4)*d - (3*b^3*c^2 - 10*a*b*c^3)*e)*f)*x)*sqrt(c*x^2 + b*x
 + a))/(a*b^2*c^4 - 4*a^2*c^5 + (b^2*c^5 - 4*a*c^6)*x^2 + (b^3*c^4 - 4*a*b*
c^5)*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)**2/(a + b*x + c*x**2)**(3/2), x)
```

---

**Giac [A]** time = 1.65767, size = 549, normalized size = 1.78

$$\frac{\Big(\Big(\Big(\frac{2\left(b^2c^2f^2-4ac^3f^2\right)x}{b^2c^3-4ac^4}-\frac{5b^3cf^2-20abc^2f^2-8b^2c^2fe+32ac^3fe}{b^2c^3-4ac^4}\Big)x-\frac{16c^4d^2+16b^2c^2df-32ac^3df+15b^4f^2-62ab^2cf^2+24a^2c^2f^2-16bc^3de-24b^3cfe+80abc}{b^2c^3-4ac^4}}{4\sqrt{cx^2+bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((2*(b^2*c^2*f^2 - 4*a*c^3*f^2)*x/(b^2*c^3 - 4*a*c^4) - (5*b^3*c*f^2 -
 20*a*b*c^2*f^2 - 8*b^2*c^2*f*e + 32*a*c^3*f*e)/(b^2*c^3 - 4*a*c^4))*x - (1
6*c^4*d^2 + 16*b^2*c^2*d*f - 32*a*c^3*d*f + 15*b^4*f^2 - 62*a*b^2*c*f^2 + 2
4*a^2*c^2*f^2 - 16*b*c^3*d*e - 24*b^3*c*f*e + 80*a*b*c^2*f*e + 8*b^2*c^2*e^
2 - 16*a*c^3*e^2)/(b^2*c^3 - 4*a*c^4))*x - (8*b*c^3*d^2 + 16*a*b*c^2*d*f +
```

```
15*a*b^3*f^2 - 52*a^2*b*c*f^2 - 32*a*c^3*d*e - 24*a*b^2*c*f*e + 64*a^2*c^2*
f*e + 8*a*b*c^2*e^2)/(b^2*c^3 - 4*a*c^4))/sqrt(c*x^2 + b*x + a) - 1/8*(16*c
^2*d*f + 15*b^2*f^2 - 12*a*c*f^2 - 24*b*c*f*e + 8*c^2*e^2)*log(abs(-2*(sqrt
(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(7/2)
```

# 3.116 $\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{3/2}} dx$

**Optimal**. Leaf size=111

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

---

**Rubi [A]** time = 0.0797038, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {1660, 12, 621, 206}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
c*(b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]) + (f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*
Sqrt[a + b*x + c*x^2])])/c^(3/2)
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{3/2}}\,dx &= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}-\frac{2\int -\frac{\left(b^2-4ac\right)f}{2c\sqrt{a+bx+cx^2}}\,dx}{b^2-4ac}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx}{c}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{(2f)\,\mathrm{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx}{\sqrt{a+bx+cx^2}}\right)}{c}\\
&= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{c\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{f\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.328344, size = 113, normalized size = 1.02

$$\frac{\frac{2\sqrt{c}\left(abf-2ac(e+fx)+b^2fx+bc(d-ex)+2c^2dx\right)}{\sqrt{a+x(b+cx)}}-f\left(b^2-4ac\right)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}\left(4ac-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(3/2),x]
```

```
[Out] ((2*Sqrt[c]*(a*b*f + 2*c^2*d*x + b^2*f*x + b*c*(d - e*x) - 2*a*c*(e + f*x))
)/Sqrt[a + x*(b + c*x)] - (b^2 - 4*a*c)*f*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a
+ x*(b + c*x)]])/(c^(3/2)*(-b^2 + 4*a*c))
```

---

**Maple [B]** time = 0.052, size = 249, normalized size = 2.2

$$-\frac{fx}{c}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{bf}{2c^2}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{b^2fx}{c\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}}+\frac{b^3f}{2c^2\left(4ac-b^2\right)}\frac{1}{\sqrt{cx^2+bx+a}}+f\ln\left(\left(\frac{b}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x)
```

```
[Out] -f*x/c/(c*x^2+b*x+a)^(1/2)+1/2*f*b/c^2/(c*x^2+b*x+a)^(1/2)+f*b^2/c/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+f/c
^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-e/c/(c*x^2+b*x+a)^(1/2)-
2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)+2*d*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 4.44962, size = 941, normalized size = 8.48

$$\left[\frac{\left(\left(b^2c-4\,ac^2\right)fx^2+\left(b^3-4\,abc\right)fx+\left(ab^2-4\,a^2c\right)f\right)\sqrt{c}\log\left(-8\,c^2x^2-8\,bcx-b^2-4\,\sqrt{cx^2+bx+a}(2\,cx+b)\sqrt{c}-4\,ac\right)}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^2+\left(b^3c^2-4\,ab\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((b^2*c - 4*a*c^2)*f*x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)
*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*(2*c*x +
b)*sqrt(c) - 4*a*c) - 4*(b*c^2*d - 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e
 + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 +
(b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^2 - 4*a*b*c^3)*x), -(((b^2*c - 4*a*c^2)*f*
x^2 + (b^3 - 4*a*b*c)*f*x + (a*b^2 - 4*a^2*c)*f)*sqrt(-c)*arctan(1/2*sqrt(c
*x^2 + b*x + a)*(2*c*x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) + 2*(b*c^2*d
- 2*a*c^2*e + a*b*c*f + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f)*x)*sqrt(c
*x^2 + b*x + a))/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^2 + (b^3*c^
2 - 4*a*b*c^3)*x)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

```
[Out] Integral((d + e*x + f*x**2)/(a + b*x + c*x**2)**(3/2), x)
```

---

**Giac [A]** time = 1.50197, size = 165, normalized size = 1.49

$$-\frac{2\left(\frac{\left(2\,c^2d+b^2f-2\,acf-bce\right)x}{b^2c-4\,ac^2}+\frac{bcd+abf-2\,ace}{b^2c-4\,ac^2}\right)}{\sqrt{cx^2+bx+a}}-\frac{f\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] -2*((2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*x/(b^2*c - 4*a*c^2) + (b*c*d + a*b*
f - 2*a*c*e)/(b^2*c - 4*a*c^2))/sqrt(c*x^2 + b*x + a) - f*log(abs(-2*(sqrt(
c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(3/2)
```

# 3.117 $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

**Optimal**. Leaf size=666

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{\ldots}\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-a\ldots\right.}$$

```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```

---

**Rubi [A]** time = 1.82916, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {974, 1032, 724, 206}

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{f\left(f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{\ldots}\right.\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-a\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```


```
[Out] (2*(b^2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f) - c*(2*c^2*d - b*c*e +
b^2*f - 2*a*c*f)*x))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f
))*Sqrt[a + b*x + c*x^2]) - (f*(c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(
2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*
f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*
f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqr
t[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*
f]))]) + (f*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt
[e^2 - 4*d*f])))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
 + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^
2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f]))])
```


Rule 974

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[((2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*
```

```
a*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*(
d + e*x + f*x^2)^(q + 1))/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
- b*f))*(p + 1)), x] - Dist[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1)), Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Sim
p[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
- c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e - b
^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f +
 b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f*
(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(
2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^
2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (
b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q,
 0]
```

Rule 1032

```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dis
t[(2*c*g - h*(b - q))/q, Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x],
 x] - Dist[(2*c*g - h*(b + q))/q, Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

Rule 724

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}+\frac{2\int \frac{\frac{1}{2}(b}{}}{\left(b^2-\right.}$$

$$=\frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}+\frac{\left(f\left(2f\right.\right.}{}$$

$$=\frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}-\frac{\left(2f\left(2\right.\right.}{}$$

$$=\frac{2\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c\left(2c^2d-bce+b^2f-2acf\right)x\right)}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}-\frac{f\left(c\left(e^2\right.\right.}{}$$

**Mathematica [A]** time = 6.79196, size = 700, normalized size = 1.05

$$2f\left(-\frac{2\left(-2c\left(2af+cx\left(\sqrt{e^2-4df}+e\right)\right)+2b^2f-bc\left(\sqrt{e^2-4df}+e-2fx\right)\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(4af^2-2bf\left(\sqrt{e^2-4df}+e\right)+c\left(\sqrt{e^2-4df}+e\right)^2\right)}+\frac{2c\left(cx\left(\sqrt{e^2-4df}-e\right)-2af\right)+2b^2f+bc\left(\sqrt{e^2-4df}-e+2fx\right)}{\left(b^2-4ac\right)\sqrt{a+x(b+cx)}\left(f\left(2af+b\left(\sqrt{e^2-4df}-e\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)}+\frac{\sqrt{2}\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

```
[Out] (2*f*((2*b^2*f + b*c*(-e + Sqrt[e^2 - 4*d*f] + 2*f*x) + 2*c*(-2*a*f + c*(-e
 + Sqrt[e^2 - 4*d*f])*x))/((b^2 - 4*a*c)*(c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f])))*Sqrt[a + x*(b + c*x)]) - (2*
(2*b^2*f - b*c*(e + Sqrt[e^2 - 4*d*f] - 2*f*x) - 2*c*(2*a*f + c*(e + Sqrt[e
^2 - 4*d*f])*x)))/((b^2 - 4*a*c)*(4*a*f^2 - 2*b*f*(e + Sqrt[e^2 - 4*d*f]) +
 c*(e + Sqrt[e^2 - 4*d*f])^2)*Sqrt[a + x*(b + c*x)]) + (Sqrt[2]*f^2*ArcTanh
[(4*a*f - 2*c*(e + Sqrt[e^2 - 4*d*f])*x - b*(e + Sqrt[e^2 - 4*d*f] - 2*f*x)
)/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e +
 Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(e^2 - 2*d*f + e*Sqrt[e^2
 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2 - 4*d*f])))^(3/2) - (Sqrt[2]*f^2*Sq
rt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f + b*(-e + Sqrt[e^2 - 4*
d*f]))]*ArcTanh[(4*a*f + 2*c*(-e + Sqrt[e^2 - 4*d*f])*x + b*(-e + Sqrt[e^2
- 4*d*f] + 2*f*x))/(2*Sqrt[2]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) +
f*(2*a*f + b*(-e + Sqrt[e^2 - 4*d*f]))]*Sqrt[a + x*(b + c*x)])])/(c*(-e^2 +
 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(-2*a*f + b*(e - Sqrt[e^2 - 4*d*f])))^2))
/Sqrt[e^2 - 4*d*f]
```

---

**Maple [B]** time = 0.355, size = 4099, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x)
```

```
[Out] 2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-b*e*f-2*c*d*f+c*e^2)*f^2/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^
(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-4*
f/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4
*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*c^2-4/(-4*d*f+e^2)^(1/2)*f^2/((-4*d*f+e^2
)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^
2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2
))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f
)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2)^(1/2)*x*b*c+4/(-4*d*f+e^2)^(1/2)*f/((-4*d*f+e^2)^(1/2)*b*f-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^
2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e
```

```
^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*x*c^2*e-2*f/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-
2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x
-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2
*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*b*c-2/(-4*d*f+e^2)^(1/2)*f^2/((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(
4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f
+e^2)^(1/2))/f)^2*c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2
)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*
f-2*c*d*f+c*e^2)/f^2)^(1/2)*b^2+2/(-4*d*f+e^2)^(1/2)*f/((-4*d*f+e^2)^(1/2)*
b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^
2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*
c+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*((
-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2)^(1/2)*b*c*e-2/(-4*d*f+e^2)^(1/2)/((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)
*(((-4*d*f+e^2)^(1/2)*b*f-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2)*(4*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)^2*c+4*(c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)/f*(x-1/2*(-e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*b*f-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2*(-e+
(-4*d*f+e^2)^(1/2))/f))-2/(-4*d*f+e^2)^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2)-4*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e
^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/
2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*c^2+4/(-4*d
*f+e^2)^(1/2)*f^2/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2
)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*b*c-4/(-4*d*f+e^2)^(1/
2)*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*x*c^2*e-2*f/(-(-4*d*f+e^2)^(1/2)*b*f+
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^
2*e^2-1/f^2*(-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f
*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4
*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2)*b*c+2/(-4*d*f+e^2)^(1/2)*f^2/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(
-4*d*f+e^2)*c^2-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/
2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*b^2-2
/(-4*d*f+e^2)^(1/2)*f/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/(4*a*c-4*c^2/f*d+c^2/f^2*e^2-1/f^2*(-4*d*f+e^2)*c^2
-b^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*b*c*e+2/(-4*d*f+e^2)
^(1/2)/(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)*f^2*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
```

```
^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f
-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*b*
f+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+2*(-(-4*d*f+e^2)^(1/2)*b*f+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

```
[Out] Integral(1/((a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

```
[Out] Timed out
```

# 3.118 $\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=891

$$\frac{x\sqrt{cx^2+bx+a}\,f^3}{2c^3}+\frac{(12ce-11bf)\sqrt{cx^2+bx+a}\,f^2}{4c^4}+\frac{\left(24\left(e^2+df\right)c^2-20f(3be+af)c+35b^2f^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{8c^{9/2}}$$

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(3*c^5*(b^2
 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*c*e*f^2 - b^7*f^3 + 3*b^5*c*
f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 + d*f) - 1
0*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*d*(e^2 + d
*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 + 6*d*f)) -
b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2 + 28*a^2*f^
2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*f^2*(7*b*e +
26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*a*b*e*f + 27*a
^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*a^2*f*(e^2 + d*
f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3*f^3 + 84*a*b^2*
f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x))/(3*c^5*(b^2 - 4*a*c)^2*Sqrt[a + b
*x + c*x^2]) + (f^2*(12*c*e - 11*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^4) + (f^3
*x*Sqrt[a + b*x + c*x^2])/(2*c^3) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) +
 24*c^2*(e^2 + d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(9/2))
```

---

**Rubi [A]** time = 1.76833, antiderivative size = 891, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1660, 1661, 640, 621, 206}

$$\frac{x\sqrt{cx^2+bx+a}\,f^3}{2c^3}+\frac{(12ce-11bf)\sqrt{cx^2+bx+a}\,f^2}{4c^4}+\frac{\left(24\left(e^2+df\right)c^2-20f(3be+af)c+35b^2f^2\right)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{8c^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(3*a*b^4*c*e*f^2 - a*b^5*f^3 + a*b^3*c*f*(5*a*f^2 - 3*c*(e^2 + d*f)) - b
*c^2*(c^3*d^3 + 5*a^3*f^3 + 3*a*c^2*d*(e^2 + d*f) - 9*a^2*c*f*(e^2 + d*f))
- a*b^2*c^2*e*(12*a*f^2 - c*(e^2 + 6*d*f)) + 2*a*c^3*e*(3*c^2*d^2 + 3*a^2*f
^2 - a*c*(e^2 + 6*d*f)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*(c^4*d^2 - b*
c^3*d*e + b^2*c^2*e^2 - 3*a*c^3*e^2 + b^2*c^2*d*f - 2*a*c^3*d*f - 2*b^3*c*e
*f + 7*a*b*c^2*e*f + b^4*f^2 - 4*a*b^2*c*f^2 + a^2*c^2*f^2)*x))/(3*c^5*(b^2
 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(3*b^6*c*e*f^2 - b^7*f^3 + 3*b^5*c*
f*(6*a*f^2 - c*(e^2 + d*f)) - 3*b^3*c^2*(29*a^2*f^3 + c^2*d*(e^2 + d*f) - 1
0*a*c*f*(e^2 + d*f)) - 4*b*c^3*(2*c^3*d^3 - 29*a^3*f^3 + 3*a*c^2*d*(e^2 + d
*f) + 24*a^2*c*f*(e^2 + d*f)) - 24*a^2*c^4*e*(6*a*f^2 - c*(e^2 + 6*d*f)) -
b^4*c^2*e*(42*a*f^2 - c*(e^2 + 6*d*f)) + 6*b^2*c^3*e*(2*c^2*d^2 + 28*a^2*f^
2 - a*c*(e^2 + 6*d*f)) - c*(16*c^6*d^3 - 10*b^6*f^3 + 3*b^4*c*f^2*(7*b*e +
26*a*f) - 24*c^5*d*(b*d*e - a*(e^2 + d*f)) - 6*b^2*c^2*f*(25*a*b*e*f + 27*a
```

```
^2*f^2 + 2*b^2*(e^2 + d*f)) + 6*c^4*(b^2*d*(e^2 + d*f) - 16*a^2*f*(e^2 + d*
f) - 2*a*b*e*(e^2 + 6*d*f)) + c^3*(240*a^2*b*e*f^2 + 56*a^3*f^3 + 84*a*b^2*
f*(e^2 + d*f) + b^3*(e^3 + 6*d*e*f)))*x))/(3*c^5*(b^2 - 4*a*c)^2*Sqrt[a + b
*x + c*x^2]) + (f^2*(12*c*e - 11*b*f)*Sqrt[a + b*x + c*x^2])/(4*c^4) + (f^3
*x*Sqrt[a + b*x + c*x^2])/(2*c^3) + (f*(35*b^2*f^2 - 20*c*f*(3*b*e + a*f) +
 24*c^2*(e^2 + d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(9/2))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1661

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x +
 c*x^2)^(p + 1))/(c*(q + 2*p + 1)), x] + Dist[1/(c*(q + 2*p + 1)), Int[(a +
 b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*
e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c,
 p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 640

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^3}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2+df\right) - 9a\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2+df\right) - 9a\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2+df\right) - 9a\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2+df\right) - 9a\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2+df\right) - 9a\right.\right.}{}$$

$$= \frac{2\left(3ab^4cef^2 - ab^5f^3 + ab^3cf\left(5af^2 - 3c\left(e^2+df\right)\right) - bc^2\left(c^3d^3 + 5a^3f^3 + 3ac^2d\left(e^2+df\right) - 9a\right.\right.}{}$$

**Mathematica [A]** time = 2.37583, size = 872, normalized size = 0.98

$$\frac{-105f^3x^2b^7 - 10f^2x(21af + 2cx(7fx-9e))b^6 - 3f\left(35a^2f^2 - 10acx(12e+23fx)f + c^2x^2\left(24e^2 - 80fxe + 7f^2x^2 + 24df\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^3/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-105*b^7*f^3*x^2 - 10*b^6*f^2*x*(21*a*f + 2*c*x*(-9*e + 7*f*x)) + 6*b^4*c*
f*(5*a^2*f*(6*e + 53*f*x) - 6*a*c*x*(4*e^2 + 4*d*f + 30*e*f*x - 31*f^2*x^2)
 + c^2*x^3*(-16*e^2 - 16*d*f + 6*e*f*x + f^2*x^2)) - 3*b^5*f*(35*a^2*f^2 -
10*a*c*f*x*(12*e + 23*f*x) + c^2*x^2*(24*e^2 + 24*d*f - 80*e*f*x + 7*f^2*x^
2)) - 48*b*c^2*(27*a^4*f^3 - 4*c^4*d^2*x^2*(d - e*x) + a^2*c^2*(-4*d^2*f +
4*e^3*x - 64*e*f^2*x^3 + 7*f^3*x^4 - 4*d*e*(e - 6*f*x)) - 2*a*c^3*(d^3 - e^
3*x^3 + 3*d*e*x^2*(e - 2*f*x) + 3*d^2*x*(-e + f*x)) - 2*a^3*c*f*(5*e^2 + 39
*e*f*x + f*(5*d - 14*f*x^2))) - 8*b^3*c*(-95*a^3*f^3 + c^3*(d^3 - e^3*x^3 +
 9*d^2*x*(e - f*x) - 3*d*e*x^2*(3*e + 2*f*x)) - 3*a*c^2*f*x^2*(18*e^2 - 74*
e*f*x + f*(18*d + 7*f*x^2)) + 3*a^2*c*f*(3*e^2 + 105*e*f*x + f*(3*d + 29*f*
x^2))) + 32*c^3*(4*c^4*d^3*x^3 + 3*a^4*f^2*(16*e + 5*f*x) + 6*a*c^3*d*x*(d^
2 + e^2*x^2 + d*f*x^2) - 2*a^3*c*(2*e^3 + 9*e^2*f*x + f^2*x*(9*d - 10*f*x^2
) + 12*e*f*(d - 3*f*x^2)) - 3*a^2*c^2*(2*d^2*e + 4*d*f*x^2*(3*e + 2*f*x) +
x^2*(2*e^3 + 8*e^2*f*x - 6*e*f^2*x^2 - f^3*x^3))) - 48*b^2*c^2*(a^3*f^2*(25
*e + 63*f*x) - c^3*d*x*(d^2 + e^2*x^2 + d*x*(-6*e + f*x)) + a^2*c*f*x*(-21*
e^2 - 12*e*f*x + 7*f*(-3*d + 7*f*x^2)) + a*c^2*(d^2*(e - 6*f*x) - 2*d*x*(3*
e^2 - 3*e*f*x + 7*f^2*x^2) + x^2*(e^3 - 14*e^2*f*x + 6*e*f^2*x^2 + f^3*x^3)
)))/(12*c^4*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f*(35*b^2*f^2 - 20*
c*f*(3*b*e + a*f) + 24*c^2*(e^2 + d*f))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a +
x*(b + c*x)]])/(8*c^(9/2))
```

**Maple [B]** time = 0.071, size = 4635, normalized size = 5.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^2*f+3/c^2*b^2/(4*a*c-b^2)/(
c*x^2+b*x+a)^(1/2)*x*d*f^2+3/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x*e^2*
f+1/4*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*e*f-6*b/c*a/(4*a*c-b^2)/(c*
x^2+b*x+a)^(3/2)*x*d*e*f+8*e*f^2*a^2/c^3/(c*x^2+b*x+a)^(3/2)+3/2*b^2/c^2*a/
(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*f^2+3/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*
x+a)^(3/2)*x*e^2*f+96*e*f^2*a^2/c*b/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-19/
4*e*f^2*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+12*e*f^2*a^2/c^2*b/(4*a
*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+4*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d
*e*f+12*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*f^2+12*b^2/c*a/(4*a*c
-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2*f-38*e*f^2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2
+b*x+a)^(1/2)*x-48*b*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*e*f-24*b^2/c*a
/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*e*f-3*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x
+a)^(3/2)*d*e*f+1/2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*e*f+3/4*b^3
/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2*f+6*b^3/c^2*a/(4*a*c-b^2)^2/(c*x
^2+b*x+a)^(1/2)*d*f^2-8*d^2*e*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)+3*e*f^2
*x^4/c/(c*x^2+b*x+a)^(3/2)-15/2*e*f^2*b/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x
^2+b*x+a)^(1/2))-15/4*e*f^2*b^2/c^4/(c*x^2+b*x+a)^(1/2)+5/32*e*f^2*b^4/c^5/
(c*x^2+b*x+a)^(3/2)-3/2*b/c^2*x/(c*x^2+b*x+a)^(3/2)*d*e*f-16*d^2*e*b*c/(4*a
*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^3
+5/16*e*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+5/2*e*f^2*b^5/c^3/(4*
a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+3*e*f^2*a/c^3*b*x/(c*x^2+b*x+a)^(3/2)+23*f
^3*b^4/c^3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+3/c^(5/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x+a)^(1/2))*d*f^2+3/c^(5/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b
*x+a)^(1/2))*e^2*f-x^2/c/(c*x^2+b*x+a)^(3/2)*e^3+1/24*b^2/c^3/(c*x^2+b*x+a)
^(3/2)*e^3-2/3*a/c^2/(c*x^2+b*x+a)^(3/2)*e^3-d^2*e/c/(c*x^2+b*x+a)^(3/2)+2/
3*d^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b+35/8*f^3*b^2/c^(9/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2*f^3*x^5/c/(c*x^2+b*x+a)^(3/2)-5/2*f^3*a/c
^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+35/16*f^3*b^3/c^5/(c*x^2
+b*x+a)^(1/2)-35/384*f^3*b^5/c^6/(c*x^2+b*x+a)^(3/2)+16/3*d^3*c/(4*a*c-b^2)
^2/(c*x^2+b*x+a)^(1/2)*b-3/2*x/c/(c*x^2+b*x+a)^(3/2)*f*d^2-3/2*x/c/(c*x^2+b
*x+a)^(3/2)*d*e^2+1/4*b/c^2/(c*x^2+b*x+a)^(3/2)*f*d^2+1/4*b/c^2/(c*x^2+b*x+
a)^(3/2)*d*e^2+173/96*f^3*b^3/c^5*a/(c*x^2+b*x+a)^(3/2)-35/8*f^3*b^2/c^4*x/
(c*x^2+b*x+a)^(1/2)+5/2*f^3*a/c^3*x/(c*x^2+b*x+a)^(1/2)-5/4*f^3*a/c^4*b/(c*
x^2+b*x+a)^(1/2)+5/6*f^3*a/c^2*x^3/(c*x^2+b*x+a)^(3/2)-11/2*f^3*b/c^4*a^2/(
c*x^2+b*x+a)^(3/2)+3/2/c^3*b/(c*x^2+b*x+a)^(1/2)*e^2*f-x^3/c/(c*x^2+b*x+a)^
(3/2)*d*f^2-x^3/c/(c*x^2+b*x+a)^(3/2)*e^2*f-1/16*b^3/c^4/(c*x^2+b*x+a)^(3/2
)*d*f^2-1/16*b^3/c^4/(c*x^2+b*x+a)^(3/2)*e^2*f-3/c^2*x/(c*x^2+b*x+a)^(1/2)*
d*f^2-3/c^2*x/(c*x^2+b*x+a)^(1/2)*e^2*f+3/2/c^3*b/(c*x^2+b*x+a)^(1/2)*d*f^2
-7/4*f^3*b/c^2*x^4/(c*x^2+b*x+a)^(3/2)+35/16*f^3*b^5/c^5/(4*a*c-b^2)/(c*x^2
+b*x+a)^(1/2)-35/24*f^3*b^2/c^3*x^3/(c*x^2+b*x+a)^(3/2)+35/16*f^3*b^3/c^4*x
^2/(c*x^2+b*x+a)^(3/2)+35/64*f^3*b^4/c^5*x/(c*x^2+b*x+a)^(3/2)-35/384*f^3*b
^7/c^6/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-35/48*f^3*b^7/c^5/(4*a*c-b^2)^2/(c*x
^2+b*x+a)^(1/2)-1/4*b/c^2*x/(c*x^2+b*x+a)^(3/2)*e^3+1/24*b^4/c^3/(4*a*c-b^2
)/(c*x^2+b*x+a)^(3/2)*e^3+1/3*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^3
+4/3*d^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*c+32/3*d^3*c^2/(4*a*c-b^2)^2/(c*
x^2+b*x+a)^(1/2)*x+1/2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*e^2+a/c/(4
*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b*f*d^2+2*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)
^(1/2)*d*e*f-5/2*f^3*a/c^3*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x+1/2*b^2/c/
(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*f*d^2+a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)
*b*d*e^2-33/4*f^3*b^2/c^3*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-66*f^3*b^2/
c^2*a^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+23/8*f^3*b^4/c^4*a/(4*a*c-b^2)/
```

```
(c*x^2+b*x+a)^(3/2)*x+6*e*f^2*a^2/c^3*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+4
8*e*f^2*a^2/c^2*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-15/2*e*f^2*b^3/c^3/(4
*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-1/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)
*x*d*f^2-1/8*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2*f-b^4/c^2/(4*a*c
-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*f^2-b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/
2)*x*e^2*f+3/4*b^3/c^3*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*f^2-19/8*e*f^2*b
^4/c^4*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-19*e*f^2*b^4/c^3*a/(4*a*c-b^2)^2/(
c*x^2+b*x+a)^(1/2)+16*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*f*d^2+16*a*c/
(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*e^2+3/2*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)
*d*f^2+3/2*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)*e^2*f+3/8*b^2/c^3*x/(c*x^2+b*x+a)^
(3/2)*d*f^2+3/8*b^2/c^3*x/(c*x^2+b*x+a)^(3/2)*e^2*f+5/4*e*f^2*b^6/c^4/(4*a*
c-b^2)^2/(c*x^2+b*x+a)^(1/2)-3*e*f^2*b^2/c^4*a/(c*x^2+b*x+a)^(3/2)+15/2*e*f
^2*b/c^3*x/(c*x^2+b*x+a)^(1/2)+35/8*f^3*b^4/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(
1/2)*x-1/2*b^5/c^3/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*f^2-1/2*b^5/c^3/(4*a
*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^2*f+b/c^3*a/(c*x^2+b*x+a)^(3/2)*d*f^2+b/c^3
*a/(c*x^2+b*x+a)^(3/2)*e^2*f+3/2/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*d*
f^2+3/2/c^3*b^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*e^2*f-6*x^2/c/(c*x^2+b*x+a)
^(3/2)*d*e*f+1/4*b^2/c^3/(c*x^2+b*x+a)^(3/2)*d*e*f+1/12*b^3/c^2/(4*a*c-b^2)
/(c*x^2+b*x+a)^(3/2)*x*e^3+2/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^
3-1/2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^3-8*b*a/(4*a*c-b^2)^2/(c*
x^2+b*x+a)^(1/2)*x*e^3-4*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^3-4*a/
c^2/(c*x^2+b*x+a)^(3/2)*d*e*f-15/4*e*f^2*b^4/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^
(1/2)+12*e*f^2*a/c^2*x^2/(c*x^2+b*x+a)^(3/2)+5/2*e*f^2*b/c^2*x^3/(c*x^2+b*x
+a)^(3/2)-15/4*e*f^2*b^2/c^3*x^2/(c*x^2+b*x+a)^(3/2)-15/16*e*f^2*b^3/c^4*x/
(c*x^2+b*x+a)^(3/2)+5/32*e*f^2*b^6/c^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-d^2*
e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+23/2*f^3*b^5/c^4*a/(4*a*c-b^2)^2/(c
*x^2+b*x+a)^(1/2)-33/8*f^3*b^3/c^4*a^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-33*f
^3*b^3/c^3*a^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)-5/4*f^3*a/c^4*b^3/(4*a*c-b
^2)/(c*x^2+b*x+a)^(1/2)-33/16*f^3*b^2/c^4*a*x/(c*x^2+b*x+a)^(3/2)-1/16*b^5/
c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*d*f^2-1/16*b^5/c^4/(4*a*c-b^2)/(c*x^2+b
*x+a)^(3/2)*e^2*f-33/4*f^3*b/c^3*a*x^2/(c*x^2+b*x+a)^(3/2)-35/192*f^3*b^6/c
^5/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-35/24*f^3*b^6/c^4/(4*a*c-b^2)^2/(c*x^2
+b*x+a)^(1/2)*x+23/16*f^3*b^5/c^5*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/4*b^3
/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*f*d^2+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^2+b
*x+a)^(3/2)*d*e^2+4*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*f*d^2+4*b^2/(4*
a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*e^2+2*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^
(1/2)*f*d^2+2*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*e^2+2*a/(4*a*c-b^2)
/(c*x^2+b*x+a)^(3/2)*x*f*d^2+2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*e^2+8*
a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b*f*d^2+8*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)
^(1/2)*b*d*e^2-2*d^2*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x
```

---

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

---

**Fricas [B]** time = 48.3546, size = 8338, normalized size = 9.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/48*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5*(7*b^6*c^2 -
60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4*c^4 - 8*a*b^2
*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e)*f^2)*x^4
 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7*a^2*b^6 - 60*
a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^3 - 8*a*b^3*c^
4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2*b^3*c^3 - 64*
a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*d - 5*(b^6*c^
2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^4*c^2 - 8*a^3*
b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*e)*f
^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^8 - 46*a*b^6*c
 + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(2*(b^6*c^2 - 6
*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*b*c^4)*e)*f^2)
*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f + 5*(7*a*b^7
- 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*(a*b^5*c^2 - 8
*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 + 16*a^3*b^2*c^
3)*e)*f^2)*x)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x +
 a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*(192*a^2*b*c^5*d*e^2 - 128*a^3*c^5*e^3
 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4 - 8*a*b^
2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*f^3)*x
^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*e - (105
*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3 - 48*b*c
^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*e^3 - (3
5*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12*(2*(b^4
*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64*a^2*b*c^
5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d*e - 2*(b
^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^3 - 20*a
^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f^2 + 3*(
64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 - 16*(a*b^
2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^3 + 448*
a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30*a*b^4*c^
3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6)*d^2 - 4
*(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2 + 24*(8*
a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2)*f + 6*
(48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3 - 12*(b^
3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3*b^2*c^3
- 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d - (5*a*
b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d^2 - 8*a
^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*sqrt(c*x^
2 + b*x + a))/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7 - 8*a*b^
2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*x^3 + (b
^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c^6 + 16*
a^3*b*c^7)*x), -1/24*(3*((24*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*e^2*f + 5
*(7*b^6*c^2 - 60*a*b^4*c^3 + 144*a^2*b^2*c^4 - 64*a^3*c^5)*f^3 + 12*(2*(b^4
*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*d - 5*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^
5)*e)*f^2)*x^4 + 24*(a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*e^2*f + 5*(7
*a^2*b^6 - 60*a^3*b^4*c + 144*a^4*b^2*c^2 - 64*a^5*c^3)*f^3 + 2*(24*(b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*e^2*f + 5*(7*b^7*c - 60*a*b^5*c^2 + 144*a^2
*b^3*c^3 - 64*a^3*b*c^4)*f^3 + 12*(2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)
*d - 5*(b^6*c^2 - 8*a*b^4*c^3 + 16*a^2*b^2*c^4)*e)*f^2)*x^3 + 12*(2*(a^2*b^
4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*d - 5*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a
^4*b*c^3)*e)*f^2 + (24*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*e^2*f + 5*(7*b^
8 - 46*a*b^6*c + 24*a^2*b^4*c^2 + 224*a^3*b^2*c^3 - 128*a^4*c^4)*f^3 + 12*(
2*(b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*d - 5*(b^7*c - 6*a*b^5*c^2 + 32*a^3*
b*c^4)*e)*f^2)*x^2 + 2*(24*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*e^2*f
 + 5*(7*a*b^7 - 60*a^2*b^5*c + 144*a^3*b^3*c^2 - 64*a^4*b*c^3)*f^3 + 12*(2*
(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*d - 5*(a*b^6*c - 8*a^2*b^4*c^2 +
 16*a^3*b^2*c^3)*e)*f^2)*x)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*
```

```
x + b)*sqrt(-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(192*a^2*b*c^5*d*e^2 - 128*a^3
*c^5*e^3 + 6*(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*f^3*x^5 + 3*(12*(b^4*c^4
- 8*a*b^2*c^5 + 16*a^2*c^6)*e*f^2 - 7*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5
)*f^3)*x^4 - 8*(b^3*c^5 - 12*a*b*c^6)*d^3 - 48*(a*b^2*c^5 + 4*a^2*c^6)*d^2*
e - (105*a^2*b^5*c - 760*a^3*b^3*c^2 + 1296*a^4*b*c^3)*f^3 + 4*(32*c^8*d^3
- 48*b*c^7*d^2*e + 12*(b^2*c^6 + 4*a*c^7)*d*e^2 + 2*(b^3*c^5 - 12*a*b*c^6)*
e^3 - (35*b^6*c^2 - 279*a*b^4*c^3 + 588*a^2*b^2*c^4 - 160*a^3*c^5)*f^3 - 12
*(2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*d - (5*b^5*c^3 - 37*a*b^3*c^4 + 64*
a^2*b*c^5)*e)*f^2 + 12*((b^2*c^6 + 4*a*c^7)*d^2 + (b^3*c^5 - 12*a*b*c^6)*d*
e - 2*(b^4*c^4 - 7*a*b^2*c^5 + 8*a^2*c^6)*e^2)*f)*x^3 - 12*(2*(3*a^2*b^3*c^
3 - 20*a^3*b*c^4)*d - (15*a^2*b^4*c^2 - 100*a^3*b^2*c^3 + 128*a^4*c^4)*e)*f
^2 + 3*(64*b*c^7*d^3 - 96*b^2*c^6*d^2*e + 24*(b^3*c^5 + 4*a*b*c^6)*d*e^2 -
16*(a*b^2*c^5 + 4*a^2*c^6)*e^3 - (35*b^7*c - 230*a*b^5*c^2 + 232*a^2*b^3*c^
3 + 448*a^3*b*c^4)*f^3 - 12*(2*(b^5*c^3 - 6*a*b^3*c^4)*d - (5*b^6*c^2 - 30*
a*b^4*c^3 + 16*a^2*b^2*c^4 + 64*a^3*c^5)*e)*f^2 + 24*((b^3*c^5 + 4*a*b*c^6)
*d^2 - 4*(a*b^2*c^5 + 4*a^2*c^6)*d*e - (b^5*c^3 - 6*a*b^3*c^4)*e^2)*f)*x^2
+ 24*(8*a^2*b*c^5*d^2 - 32*a^3*c^5*d*e - (3*a^2*b^3*c^3 - 20*a^3*b*c^4)*e^2
)*f + 6*(48*a*b^2*c^5*d*e^2 - 32*a^2*b*c^5*e^3 + 8*(b^2*c^6 + 4*a*c^7)*d^3
- 12*(b^3*c^5 + 4*a*b*c^6)*d^2*e - (35*a*b^6*c - 265*a^2*b^4*c^2 + 504*a^3*
b^2*c^3 - 80*a^4*c^4)*f^3 - 12*(2*(a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*d
 - (5*a*b^5*c^2 - 35*a^2*b^3*c^3 + 52*a^3*b*c^4)*e)*f^2 + 24*(2*a*b^2*c^5*d
^2 - 8*a^2*b*c^5*d*e - (a*b^4*c^3 - 7*a^2*b^2*c^4 + 4*a^3*c^5)*e^2)*f)*x)*s
qrt(c*x^2 + b*x + a))/(a^2*b^4*c^5 - 8*a^3*b^2*c^6 + 16*a^4*c^7 + (b^4*c^7
- 8*a*b^2*c^8 + 16*a^2*c^9)*x^4 + 2*(b^5*c^6 - 8*a*b^3*c^7 + 16*a^2*b*c^8)*
x^3 + (b^6*c^5 - 6*a*b^4*c^6 + 32*a^3*c^8)*x^2 + 2*(a*b^5*c^5 - 8*a^2*b^3*c
^6 + 16*a^3*b*c^7)*x)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**3/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

---

**Giac [A]** time = 1.54095, size = 1891, normalized size = 2.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^3/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/12*((((3*(2*(b^4*c^3*f^3 - 8*a*b^2*c^4*f^3 + 16*a^2*c^5*f^3)*x/(b^4*c^4 -
 8*a*b^2*c^5 + 16*a^2*c^6) - (7*b^5*c^2*f^3 - 56*a*b^3*c^3*f^3 + 112*a^2*b*
c^4*f^3 - 12*b^4*c^3*f^2*e + 96*a*b^2*c^4*f^2*e - 192*a^2*c^5*f^2*e)/(b^4*c
^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 4*(32*c^7*d^3 + 12*b^2*c^5*d^2*f + 48*a
*c^6*d^2*f - 24*b^4*c^3*d*f^2 + 168*a*b^2*c^4*d*f^2 - 192*a^2*c^5*d*f^2 - 3
5*b^6*c*f^3 + 279*a*b^4*c^2*f^3 - 588*a^2*b^2*c^3*f^3 + 160*a^3*c^4*f^3 - 4
8*b*c^6*d^2*e + 12*b^3*c^4*d*f*e - 144*a*b*c^5*d*f*e + 60*b^5*c^2*f^2*e - 4
44*a*b^3*c^3*f^2*e + 768*a^2*b*c^4*f^2*e + 12*b^2*c^5*d*e^2 + 48*a*c^6*d*e^
2 - 24*b^4*c^3*f*e^2 + 168*a*b^2*c^4*f*e^2 - 192*a^2*c^5*f*e^2 + 2*b^3*c^4*
```

```
e^3 - 24*a*b*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))*x + 3*(64*b*c^6
*d^3 + 24*b^3*c^4*d^2*f + 96*a*b*c^5*d^2*f - 24*b^5*c^2*d*f^2 + 144*a*b^3*c
^3*d*f^2 - 35*b^7*f^3 + 230*a*b^5*c*f^3 - 232*a^2*b^3*c^2*f^3 - 448*a^3*b*c
^3*f^3 - 96*b^2*c^5*d^2*e - 96*a*b^2*c^4*d*f*e - 384*a^2*c^5*d*f*e + 60*b^6
*c*f^2*e - 360*a*b^4*c^2*f^2*e + 192*a^2*b^2*c^3*f^2*e + 768*a^3*c^4*f^2*e
+ 24*b^3*c^4*d*e^2 + 96*a*b*c^5*d*e^2 - 24*b^5*c^2*f*e^2 + 144*a*b^3*c^3*f*
e^2 - 16*a*b^2*c^4*e^3 - 64*a^2*c^5*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^
6))*x + 6*(8*b^2*c^5*d^3 + 32*a*c^6*d^3 + 48*a*b^2*c^4*d^2*f - 24*a*b^4*c^2
*d*f^2 + 168*a^2*b^2*c^3*d*f^2 - 96*a^3*c^4*d*f^2 - 35*a*b^6*f^3 + 265*a^2*
b^4*c*f^3 - 504*a^3*b^2*c^2*f^3 + 80*a^4*c^3*f^3 - 12*b^3*c^4*d^2*e - 48*a*
b*c^5*d^2*e - 192*a^2*b*c^4*d*f*e + 60*a*b^5*c*f^2*e - 420*a^2*b^3*c^2*f^2*
e + 624*a^3*b*c^3*f^2*e + 48*a*b^2*c^4*d*e^2 - 24*a*b^4*c^2*f*e^2 + 168*a^2
*b^2*c^3*f*e^2 - 96*a^3*c^4*f*e^2 - 32*a^2*b*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^
5 + 16*a^2*c^6))*x - (8*b^3*c^4*d^3 - 96*a*b*c^5*d^3 - 192*a^2*b*c^4*d^2*f
+ 72*a^2*b^3*c^2*d*f^2 - 480*a^3*b*c^3*d*f^2 + 105*a^2*b^5*f^3 - 760*a^3*b^
3*c*f^3 + 1296*a^4*b*c^2*f^3 + 48*a*b^2*c^4*d^2*e + 192*a^2*c^5*d^2*e + 768
*a^3*c^4*d*f*e - 180*a^2*b^4*c*f^2*e + 1200*a^3*b^2*c^2*f^2*e - 1536*a^4*c^
3*f^2*e - 192*a^2*b*c^4*d*e^2 + 72*a^2*b^3*c^2*f*e^2 - 480*a^3*b*c^3*f*e^2
+ 128*a^3*c^4*e^3)/(b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6))/(c*x^2 + b*x + a)^
(3/2) - 1/8*(24*c^2*d*f^2 + 35*b^2*f^3 - 20*a*c*f^3 - 60*b*c*f^2*e + 24*c^2
*f*e^2)*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(9/2
)
```

## 3.119 $\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=444

$$-\frac{2\left(-2cx\left(-c^2\left(16a^2f^2+12abef+b^2\left(-\left(2df+e^2\right)\right)\right)+b^2cf(14af+be)-c^3\left(8bde-4a\left(2df+e^2\right)\right)-2b^4f^2+8c^4d^2\right)-\right.}{3c^3\left(b^2-4ac\right)}$$

```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(2*b^4*c*
e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*(10*a*
f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 + 2*d*f)
) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*d*e - 4*
a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f)))*x))/(
3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (f^2*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```

---

**Rubi [A]** time = 0.450728, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {1660, 12, 621, 206}

$$-\frac{2\left(-2cx\left(-c^2\left(16a^2f^2+12abef+b^2\left(-\left(2df+e^2\right)\right)\right)+b^2cf(14af+be)-c^3\left(8bde-4a\left(2df+e^2\right)\right)-2b^4f^2+8c^4d^2\right)-\right.}{3c^3\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```


```
[Out] (2*(2*a*b^2*c*e*f - a*b^3*f^2 + 4*a*c^2*e*(c*d - a*f) - b*c*(c^2*d^2 - 3*a^
2*f^2 + a*c*(e^2 + 2*d*f)) - (2*c^4*d^2 + b^4*f^2 - 2*b^2*c*f*(b*e + 2*a*f)
 - 2*c^3*(b*d*e + a*(e^2 + 2*d*f)) + c^2*(6*a*b*e*f + 2*a^2*f^2 + b^2*(e^2
+ 2*d*f)))*x))/(3*c^3*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) - (2*(2*b^4*c*
e*f + 48*a^2*c^3*e*f - b^5*f^2 + 4*b^2*c^2*e*(2*c*d - 3*a*f) + b^3*c*(10*a*
f^2 - c*(e^2 + 2*d*f)) - 4*b*c^2*(2*c^2*d^2 + 8*a^2*f^2 + a*c*(e^2 + 2*d*f)
) - 2*c*(8*c^4*d^2 - 2*b^4*f^2 + b^2*c*f*(b*e + 14*a*f) - c^3*(8*b*d*e - 4*
a*(e^2 + 2*d*f)) - c^2*(12*a*b*e*f + 16*a^2*f^2 - b^2*(e^2 + 2*d*f)))*x))/(
3*c^3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2]) + (f^2*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/c^(5/2)
```


Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 621

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\left(d+ex+fx^2\right)^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx = \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+\right.}$$

$$= \frac{2\left(2ab^2cef-ab^3f^2+4ac^2e(cd-af)-bc\left(c^2d^2-3a^2f^2+ac\left(e^2+2df\right)\right)-\left(2c^4d^2+b^4f^2-\right.\right.}{3c^3\left(b^2-4ac\right)\left(a+\right.}$$

**Mathematica [A]** time = 1.31537, size = 387, normalized size = 0.87

$$\frac{2\left(2b^2c\left(21a^2f^2x-2ac\left(d(e-6fx)+x\left(-3e^2+3efx-7f^2x^2\right)\right)+c^2x\left(3d^2+2dx(fx-6e)+e^2x^2\right)\right)+b^3\left(-3a^2f^2+18\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)^2/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(-3*b^5*f^2*x^2 - 2*b^4*f^2*x*(3*a + 2*c*x^2) + 4*b*c*(5*a^3*f^2 + 2*c^3
*d*x^2*(3*d - 2*e*x) + 2*a^2*c*(e^2 + 2*d*f - 6*e*f*x) + 3*a*c^2*(d - e*x)*
(d + x*(-e + 2*f*x))) + b^3*(-3*a^2*f^2 + 18*a*c*f^2*x^2 + c^2*(-d^2 + 6*d*
x*(-e + f*x) + e*x^2*(3*e + 2*f*x))) + 8*c^2*(2*c^3*d^2*x^3 - a^3*f*(4*e +
3*f*x) + a*c^2*x*(3*d^2 + e^2*x^2 + 2*d*f*x^2) - 2*a^2*c*(d*e + f*x^2*(3*e
+ 2*f*x))) + 2*b^2*c*(21*a^2*f^2*x + c^2*x*(3*d^2 + e^2*x^2 + 2*d*x*(-6*e +
 f*x)) - 2*a*c*(d*(e - 6*f*x) + x*(-3*e^2 + 3*e*f*x - 7*f^2*x^2)))))/(3*c^2
*(b^2 - 4*a*c)^2*(a + x*(b + c*x))^(3/2)) + (f^2*Log[b + 2*c*x + 2*Sqrt[c]*
```

```
Sqrt[a + x*(b + c*x)]])/c^(5/2)
```

**Maple [B]** time = 0.061, size = 1786, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 1/2*f^2*b/c^2*x^2/(c*x^2+b*x+a)^(3/2)+1/8*f^2*b^2/c^3*x/(c*x^2+b*x+a)^(3/2)
-1/48*f^2*b^5/c^4/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)-1/6*f^2*b^5/c^3/(4*a*c-b^
2)^2/(c*x^2+b*x+a)^(1/2)+1/3*f^2*b/c^3*a/(c*x^2+b*x+a)^(3/2)+1/2*f^2/c^3*b^
3/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-2*e*f*x^2/c/(c*x^2+b*x+a)^(3/2)+1/12*e*f*
b^2/c^3/(c*x^2+b*x+a)^(3/2)-4/3*e*f*a/c^2/(c*x^2+b*x+a)^(3/2)-16*e*f*b*a/(4
*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-8*e*f*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)
^(1/2)-2*e*f*b/c*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x+32/3*d^2*c^2/(4*a*c-b^
2)^2/(c*x^2+b*x+a)^(1/2)*x+16/3*d^2*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*b+4
/3*d^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*c-16/3*d*e*b^2/(4*a*c-b^2)^2/(c*x^
2+b*x+a)^(1/2)-32/3*d*e*b*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x-e*f*b^2/c^2
*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+1/3*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2
)*x*d*f+2/3*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*b*d*f+32/3*a*c/(4*a*c-b^2)^
2/(c*x^2+b*x+a)^(1/2)*x*d*f+1/2*f^2*b^2/c^2*a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/
2)*x-1/48*f^2*b^3/c^4/(c*x^2+b*x+a)^(3/2)-f^2/c^2*x/(c*x^2+b*x+a)^(1/2)+1/2
*f^2/c^3*b/(c*x^2+b*x+a)^(1/2)-2/3*d*e/c/(c*x^2+b*x+a)^(3/2)+2/3*d^2/(4*a*c
-b^2)/(c*x^2+b*x+a)^(3/2)*b-1/2*x/c/(c*x^2+b*x+a)^(3/2)*e^2+1/12*b/c^2/(c*x
^2+b*x+a)^(3/2)*e^2+4/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*d*f+4/3*a/(
4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*d*f+1/3*a/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2
)*b*e^2+16/3*a*c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*e^2+16/3*a/(4*a*c-b^2)
^2/(c*x^2+b*x+a)^(1/2)*b*d*f+f^2/c^2*b^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)*x-
4/3*d*e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x-2/3*d*e*b^2/c/(4*a*c-b^2)/(c*x^
2+b*x+a)^(3/2)-x/c/(c*x^2+b*x+a)^(3/2)*d*f+f^2/c^(5/2)*ln((1/2*b+c*x)/c^(1/
2)+(c*x^2+b*x+a)^(1/2))+4/3*e*f*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+4
*f^2*b^2/c*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/6*b/c^2/(c*x^2+b*x+a)^(3
/2)*d*f-1/3*f^2*x^3/c/(c*x^2+b*x+a)^(3/2)-1/2*e*f*b/c^2*x/(c*x^2+b*x+a)^(3/
2)+1/12*e*f*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2/3*e*f*b^4/c^2/(4*a*c-
b^2)^2/(c*x^2+b*x+a)^(1/2)-1/24*f^2*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)
*x+1/6*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2+1/6*b^3/c^2/(4*a*c-b^2)/
(c*x^2+b*x+a)^(3/2)*d*f+8/3*b^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x*d*f+1/1
2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*e^2+4/3*b^2/(4*a*c-b^2)^2/(c*x^2+
b*x+a)^(1/2)*x*e^2+2/3*b^3/c/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*e^2+2/3*a/(4
*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x*e^2+8/3*a/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)
*b*e^2-1/3*f^2*b^4/c^2/(4*a*c-b^2)^2/(c*x^2+b*x+a)^(1/2)*x+1/4*f^2*b^3/c^3*
a/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)+2*f^2*b^3/c^2*a/(4*a*c-b^2)^2/(c*x^2+b*x+
a)^(1/2)+1/6*e*f*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^(3/2)*x
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 27.422, size = 3252, normalized size = 7.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2 + 2*(a
*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*
c^2)*f^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 - 4*sqrt(c*x^2 + b*x + a)*
(2*c*x + b)*sqrt(c) - 4*a*c) + 4*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b*c^5*
d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2 +
 (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*c^3 - 1
2*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*a^3*b*c^
2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^2 - (b^5*
c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3 + 4*a^2*c^
4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3*e^2 + (b^2
*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*a^2*b^2*c^2
+ 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x +
 a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 -
6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*x), -1/3*(3*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f^2*x^4 + 2*(b^5*c - 8*
a*b^3*c^2 + 16*a^2*b*c^3)*f^2*x^3 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*f^2*x^2
+ 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f^2*x + (a^2*b^4 - 8*a^3*b^2*c + 1
6*a^4*c^2)*f^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(
-c)/(c^2*x^2 + b*c*x + a*c)) - 2*(8*a^2*b*c^3*e^2 + 2*(8*c^6*d^2 - 8*b*c^5*
d*e + (b^2*c^4 + 4*a*c^5)*e^2 - 2*(b^4*c^2 - 7*a*b^2*c^3 + 8*a^2*c^4)*f^2 +
 (2*(b^2*c^4 + 4*a*c^5)*d + (b^3*c^3 - 12*a*b*c^4)*e)*f)*x^3 - (b^3*c^3 - 1
2*a*b*c^4)*d^2 - 4*(a*b^2*c^3 + 4*a^2*c^4)*d*e - (3*a^2*b^3*c - 20*a^3*b*c^
2)*f^2 + 3*(8*b*c^5*d^2 - 8*b^2*c^4*d*e + (b^3*c^3 + 4*a*b*c^4)*e^2 - (b^5*
c - 6*a*b^3*c^2)*f^2 + 2*((b^3*c^3 + 4*a*b*c^4)*d - 2*(a*b^2*c^3 + 4*a^2*c^
4)*e)*f)*x^2 + 16*(a^2*b*c^3*d - 2*a^3*c^3*e)*f + 6*(2*a*b^2*c^3*e^2 + (b^2
*c^4 + 4*a*c^5)*d^2 - (b^3*c^3 + 4*a*b*c^4)*d*e - (a*b^4*c - 7*a^2*b^2*c^2
+ 4*a^3*c^3)*f^2 + 4*(a*b^2*c^3*d - 2*a^2*b*c^3*e)*f)*x)*sqrt(c*x^2 + b*x +
 a))/(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5 + (b^4*c^5 - 8*a*b^2*c^6 + 1
6*a^2*c^7)*x^4 + 2*(b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^3 + (b^6*c^3 -
6*a*b^4*c^4 + 32*a^3*c^6)*x^2 + 2*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5
)*x)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)**2/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.38829, size = 792, normalized size = 1.78

$$-\frac{f^2\log\left(\left|-2\left(\sqrt{c}x-\sqrt{cx^2+bx+a}\right)\sqrt{c}-b\right|\right)}{c^{\frac{5}{2}}}+\frac{2\left(\left(\left(\frac{2\left(8c^5d^2+2b^2c^3df+8ac^4df-2b^4cf^2+14ab^2c^2f^2-16a^2c^3f^2-8bc^4de+b^3c^2fe-12abc^3fe+b\right.}{b^4c^2-8ab^2c^3+16a^2c^4}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)^2/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] -f^2*log(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x + a))*sqrt(c) - b))/c^(5/2) +
 2/3*(((2*(8*c^5*d^2 + 2*b^2*c^3*d*f + 8*a*c^4*d*f - 2*b^4*c*f^2 + 14*a*b^2
*c^2*f^2 - 16*a^2*c^3*f^2 - 8*b*c^4*d*e + b^3*c^2*f*e - 12*a*b*c^3*f*e + b^
2*c^3*e^2 + 4*a*c^4*e^2)*x/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4) + 3*(8*b*c^
4*d^2 + 2*b^3*c^2*d*f + 8*a*b*c^3*d*f - b^5*f^2 + 6*a*b^3*c*f^2 - 8*b^2*c^3
*d*e - 4*a*b^2*c^2*f*e - 16*a^2*c^3*f*e + b^3*c^2*e^2 + 4*a*b*c^3*e^2)/(b^4
*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x + 6*(b^2*c^3*d^2 + 4*a*c^4*d^2 + 4*a*b^
2*c^2*d*f - a*b^4*f^2 + 7*a^2*b^2*c*f^2 - 4*a^3*c^2*f^2 - b^3*c^2*d*e - 4*a
*b*c^3*d*e - 8*a^2*b*c^2*f*e + 2*a*b^2*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16
*a^2*c^4))*x - (b^3*c^2*d^2 - 12*a*b*c^3*d^2 - 16*a^2*b*c^2*d*f + 3*a^2*b^3
*f^2 - 20*a^3*b*c*f^2 + 4*a*b^2*c^2*d*e + 16*a^2*c^3*d*e + 32*a^3*c^2*f*e -
 8*a^2*b*c^2*e^2)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))/(c*x^2 + b*x + a)^(
3/2)
```

## 3.120 $\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^{5/2}}\,dx$

**Optimal**. Leaf size=131

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f + (b
^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

---

**Rubi [A]** time = 0.0852556, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {1660, 12, 613}

$$\frac{2\left(c\left(2ae-b\left(\frac{af}{c}+d\right)\right)-x\left(-2acf+b^2f-bce+2c^2d\right)\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{2(b+2cx)\left(4af+\frac{b^2f}{c}-4be+8cd\right)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (2*(c*(2*a*e - b*(d + (a*f)/c)) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x))/(
3*c*(b^2 - 4*a*c)*(a + b*x + c*x^2)^(3/2)) + (2*(8*c*d - 4*b*e + 4*a*f + (b
^2*f)/c)*(b + 2*c*x))/(3*(b^2 - 4*a*c)^2*Sqrt[a + b*x + c*x^2])
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 613

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[(-2*(b +
2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{d+ex+fx^2}{(a+bx+cx^2)^{5/2}}\,dx = \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{2\int \frac{8cd-4be+4af+\frac{b^2f}{c}}{2(a+bx+cx^2)^{3/2}}dx}{3\left(b^2-4ac\right)}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} - \frac{\left(8cd-4be+4af+\frac{b^2f}{c}\right)\int \frac{1}{(a+bx+cx^2)^{3/2}}$$

$$= \frac{2\left(c\left(2ae-b\left(d+\frac{af}{c}\right)\right)-\left(2c^2d-bce+b^2f-2acf\right)x\right)}{3c\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}} + \frac{2\left(8cd-4be+4af+\frac{b^2f}{c}\right)(b+2cx)}{3\left(b^2-4ac\right)^2\sqrt{a+bx+cx^2}}$$

**Mathematica [A]** time = 0.394814, size = 147, normalized size = 1.12

$$\frac{8b\left(2a^2f+3ac\left(d-ex+fx^2\right)-2c^2x^2(ex-3d)\right)+16c\left(-a^2e+acx\left(3d+fx^2\right)+2c^2dx^3\right)-4b^2\left(a(e-6fx)-cx\left(3d-6ex\right.\right.}{3\left(b^2-4ac\right)^2(a+x(b+cx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*x^2)/(a + b*x + c*x^2)^(5/2),x]
```

```
[Out] (-2*b^3*(d + 3*x*(e - f*x)) + 16*c*(-(a^2*e) + 2*c^2*d*x^3 + a*c*x*(3*d + f
*x^2)) - 4*b^2*(a*(e - 6*f*x) - c*x*(3*d - 6*e*x + f*x^2)) + 8*b*(2*a^2*f -
 2*c^2*x^2*(-3*d + e*x) + 3*a*c*(d - e*x + f*x^2)))/(3*(b^2 - 4*a*c)^2*(a +
 x*(b + c*x))^(3/2))
```

**Maple [A]** time = 0.052, size = 185, normalized size = 1.4

$$\frac{16\,ac^2fx^3+4\,b^2cfx^3-16\,bc^2ex^3+32\,c^3dx^3+24\,abcfx^2+6\,b^3fx^2-24\,b^2cex^2+48\,bc^2dx^2+24\,ab^2fx-24\,abcex+48\,a}{48\,a^2c^2-24\,acb^2+3\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x)
```

```
[Out] 2/3/(c*x^2+b*x+a)^(3/2)*(8*a*c^2*f*x^3+2*b^2*c*f*x^3-8*b*c^2*e*x^3+16*c^3*d
*x^3+12*a*b*c*f*x^2+3*b^3*f*x^2-12*b^2*c*e*x^2+24*b*c^2*d*x^2+12*a*b^2*f*x-
12*a*b*c*e*x+24*a*c^2*d*x-3*b^3*e*x+6*b^2*c*d*x+8*a^2*b*f-8*a^2*c*e-2*a*b^2
*e+12*a*b*c*d-b^3*d)/(16*a^2*c^2-8*a*b^2*c+b^4)
```

**Maxima [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [B]** time = 23.2333, size = 614, normalized size = 4.69

$$\frac{2\left(8a^2bf+2\left(8c^3d-4bc^2e+\left(b^2c+4ac^2\right)f\right)x^3+3\left(8bc^2d-4b^2ce+\left(b^3+4abc\right)f\right)x^2-\left(b^3-12abc\right)d-2\left(ab^2+4\right.\right.}{3\left(a^2b^4-8a^3b^2c+16a^4c^2+\left(b^4c^2-8ab^2c^3+16a^2c^4\right)x^4+2\left(b^5c-8ab^3c^2+16a^2bc^3\right)x^3+\left(b^6-6\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(8*a^2*b*f + 2*(8*c^3*d - 4*b*c^2*e + (b^2*c + 4*a*c^2)*f)*x^3 + 3*(8*b
*c^2*d - 4*b^2*c*e + (b^3 + 4*a*b*c)*f)*x^2 - (b^3 - 12*a*b*c)*d - 2*(a*b^2
 + 4*a^2*c)*e + 3*(4*a*b^2*f + 2*(b^2*c + 4*a*c^2)*d - (b^3 + 4*a*b*c)*e)*x
)*sqrt(c*x^2 + b*x + a)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^4*c^2 - 8*
a*b^2*c^3 + 16*a^2*c^4)*x^4 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^3 +
(b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^2 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**2+e*x+d)/(c*x**2+b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.36368, size = 356, normalized size = 2.72

$$\frac{\left(\left(\frac{2\left(8c^3d+b^2cf+4ac^2f-4bc^2e\right)x}{b^4c^2-8ab^2c^3+16a^2c^4}+\frac{3\left(8bc^2d+b^3f+4abcf-4b^2ce\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x+\frac{3\left(2b^2cd+8ac^2d+4ab^2f-b^3e-4abce\right)}{b^4c^2-8ab^2c^3+16a^2c^4}\right)x-\frac{b^3d-12abcd-8a^2bf+2ab^2e+8a^2ce}{b^4c^2-8ab^2c^3+16a^2c^4}}{3\left(cx^2+bx+a\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^2+e*x+d)/(c*x^2+b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(((2*(8*c^3*d + b^2*c*f + 4*a*c^2*f - 4*b*c^2*e)*x/(b^4*c^2 - 8*a*b^2*c
^3 + 16*a^2*c^4) + 3*(8*b*c^2*d + b^3*f + 4*a*b*c*f - 4*b^2*c*e)/(b^4*c^2 -
 8*a*b^2*c^3 + 16*a^2*c^4))*x + 3*(2*b^2*c*d + 8*a*c^2*d + 4*a*b^2*f - b^3*
e - 4*a*b*c*e)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4))*x - (b^3*d - 12*a*b*c*
d - 8*a^2*b*f + 2*a*b^2*e + 8*a^2*c*e)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
)/(c*x^2 + b*x + a)^(3/2)
```

## 3.121 $\int \frac{1}{\sqrt{-7+2x+5x^2}\left(8+12x+5x^2\right)}\,dx$

**Optimal**. Leaf size=51

$$\frac{1}{10}\tan^{-1}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

```
[Out] ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sqr
t[-7 + 2*x + 5*x^2]]/5
```

---

**Rubi [A]** time = 0.0656829, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {986, 1029, 203, 207}

$$\frac{1}{10}\tan^{-1}\left(\frac{5(x+2)}{2\sqrt{5x^2+2x-7}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(x+1)}{\sqrt{5x^2+2x-7}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

```
[Out] ArcTan[(5*(2 + x))/(2*Sqrt[-7 + 2*x + 5*x^2])]/10 + ArcTanh[(5*(1 + x))/Sqr
t[-7 + 2*x + 5*x^2]]/5
```

Rule 986

```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Dist[1/(2*q), Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2)
*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[(c*d - a*f - q + (c*e -
 b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
 c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e -
 b*f, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1029

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) +
(e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Dist[-2*g*(g*b - 2*a*h), Subst[In
t[1/Simp[g*(g*b - 2*a*h)*(b^2 - 4*a*c) - (b*d - a*e)*x^2, x], x], x, Simp[g
*b - 2*a*h - (b*h - 2*g*c)*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b,
 c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ
[b*d - a*e, 0] && EqQ[h^2*(b*d - a*e) - 2*g*h*(c*d - a*f) + g^2*(c*e - b*f)
, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\sqrt{-7+2x+5x^2}\left(8+12x+5x^2\right)}dx = -\left(\frac{1}{50}\int \frac{-100-50x}{\sqrt{-7+2x+5x^2}\left(8+12x+5x^2\right)}dx\right)+\frac{1}{50}\int \frac{-50-50}{\sqrt{-7+2x+5x^2}\left(8\right.}$$

$$= 400\,\text{Subst}\left(\int \frac{1}{160000+100x^2}dx, x, \frac{200+100x}{\sqrt{-7+2x+5x^2}}\right)+1600\,\text{Subst}\left(\int\right.$$

$$= \frac{1}{10}\tan^{-1}\left(\frac{5(2+x)}{2\sqrt{-7+2x+5x^2}}\right)+\frac{1}{5}\tanh^{-1}\left(\frac{5(1+x)}{\sqrt{-7+2x+5x^2}}\right)$$

**Mathematica [C]** time = 0.0402311, size = 81, normalized size = 1.59

$$\left(\frac{1}{10}-\frac{i}{20}\right)\tanh^{-1}\left(\frac{\left(\frac{1}{100}+\frac{i}{50}\right)((100-40i)x+(164-8i))}{\sqrt{5x^2+2x-7}}\right)-\left(\frac{1}{20}-\frac{i}{10}\right)\tan^{-1}\left(\frac{\left(\frac{1}{50}+\frac{i}{100}\right)((-100-40i)x-(164+8i}{\sqrt{5x^2+2x-7}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[-7 + 2*x + 5*x^2]*(8 + 12*x + 5*x^2)),x]
```

```
[Out] (-1/20 + I/10)*ArcTan[((1/50 + I/100)*((-164 - 8*I) - (100 + 40*I)*x))/Sqrt
[-7 + 2*x + 5*x^2]] + (1/10 - I/20)*ArcTanh[((1/100 + I/50)*((164 - 8*I) +
(100 - 40*I)*x))/Sqrt[-7 + 2*x + 5*x^2]]
```

---

**Maple [B]** time = 0.108, size = 144, normalized size = 2.8

$$-\frac{1}{10}\sqrt{-4\frac{(2+x)^2}{(-1-x)^2}+9}\left(2\,\text{Artanh}\left(1/5\sqrt{-4\frac{(2+x)^2}{(-1-x)^2}+9}\right)+\arctan\left(\frac{10+5x}{-2-2x}\sqrt{-4\frac{(2+x)^2}{(-1-x)^2}+9}\left(4\frac{(2+x)^2}{(-1-x)^2}-9\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x)
```

```
[Out] -1/10*(-4*(2+x)^2/(-1-x)^2+9)^(1/2)*(2*arctanh(1/5*(-4*(2+x)^2/(-1-x)^2+9)^
(1/2))+arctan(5/2*(-4*(2+x)^2/(-1-x)^2+9)^(1/2)/(4*(2+x)^2/(-1-x)^2-9)*(2+x
)/(-1-x)))/(-(4*(2+x)^2/(-1-x)^2-9)/(1+(2+x)/(-1-x))^2)^(1/2)/(1+(2+x)/(-1-
x))
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(5x^2+12x+8\right)\sqrt{5x^2+2x-7}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((5*x^2 + 12*x + 8)*sqrt(5*x^2 + 2*x - 7)), x)
```

**Fricas [B]** time = 1.91039, size = 424, normalized size = 8.31

$$\frac{1}{20}\arctan\left(\frac{27x^2+20\sqrt{5x^2+2x-7}(x+2)+36x}{31x^2+16x-56}\right)+\frac{1}{20}\arctan\left(-\frac{27x^2-20\sqrt{5x^2+2x-7}(x+2)+36x}{31x^2+16x-56}\right)+\frac{1}{20}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/20*arctan((27*x^2 + 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)/(31*x^2 + 16
*x - 56)) + 1/20*arctan(-(27*x^2 - 20*sqrt(5*x^2 + 2*x - 7)*(x + 2) + 36*x)
/(31*x^2 + 16*x - 56)) + 1/20*log((15*x^2 + 5*sqrt(5*x^2 + 2*x - 7)*(x + 1)
 + 26*x + 9)/x^2) - 1/20*log((15*x^2 - 5*sqrt(5*x^2 + 2*x - 7)*(x + 1) + 26
*x + 9)/x^2)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{(x-1)(5x+7)}\left(5x^2+12x+8\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+12*x+8)/(5*x**2+2*x-7)**(1/2),x)
```

```
[Out] Integral(1/(sqrt((x - 1)*(5*x + 7))*(5*x**2 + 12*x + 8)), x)
```

**Giac [B]** time = 1.49367, size = 277, normalized size = 5.43

$$-\frac{1}{10}\arctan\left(-\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5x^2+2x-7}+5}{2\left(\sqrt{5}+5\right)}\right)-\frac{1}{10}\arctan\left(\frac{5\sqrt{5}x+6\sqrt{5}-5\sqrt{5x^2+2x-7}-5}{2\left(\sqrt{5}-5\right)}\right)+\frac{1}{10}\log\left(5\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+12*x+8)/(5*x^2+2*x-7)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*arctan(-1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2*x - 7) + 5)/(
sqrt(5) + 5)) - 1/10*arctan(1/2*(5*sqrt(5)*x + 6*sqrt(5) - 5*sqrt(5*x^2 + 2
*x - 7) - 5)/(sqrt(5) - 5)) + 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7)
)^2 + 2*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))*(6*sqrt(5) + 5) + 20*sqrt(5) +
65) - 1/10*log(5*(sqrt(5)*x - sqrt(5*x^2 + 2*x - 7))^2 + 2*(sqrt(5)*x - sqr
t(5*x^2 + 2*x - 7))*(6*sqrt(5) - 5) - 20*sqrt(5) + 65)
```

# 3.122 $\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx$

**Optimal**. Leaf size=1432

result too large to display

```
[Out] -(((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e
- 2*a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[
b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^2*(d + e*x + f*x
^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*
f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f -
 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)
)/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c
]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - ((b + Sqrt[b^2 -
4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/((b^2*d +
b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e
+ (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b +
Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - S
qrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*
d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)
]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[((2*c^2*d - b*c*e
 + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[2*a + (b + S
qrt[b^2 - 4*a*c])*x])/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d +
Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])],
(2 + ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f))/(Sqrt[b^2*d + b*(Sqrt[
b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[2*c^2
*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)]))
/4])/((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - b*c*e + b^2*f - 2*a*c*
f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x
 + f*x^2]*Sqrt[1 - ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b
 + Sqrt[b^2 - 4*a*c])*x))/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*
d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^
2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (
b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[
b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]))
```

---

**Rubi [A]** time = 6.21881, antiderivative size = 1432, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {992, 935, 1103}

$$-\frac{\sqrt[4]{db^2+\left(\sqrt{b^2-4ac}\,d-ae\right)b-a\left(2cd+\sqrt{b^2-4ac}\,e-2af\right)}\left(b+2cx+\sqrt{b^2-4ac}\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\,\sqrt{\frac{\cdots}{(4f\cdots}}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] -(((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e
- 2*a*f))^(1/4)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^(3/2)*Sqrt[2*a + (b + Sqrt[
b^2 - 4*a*c])*x]*Sqrt[((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)^2*(d + e*x + f*x
^2))/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*
f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]*(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f -
 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)
)/(Sqrt[b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c
]*e - 2*a*f)]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))*Sqrt[(1 - ((b + Sqrt[b^2 -
4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/((b^2*d +
b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e
+ (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b +
Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x)^2))/(1 + (Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - S
qrt[b^2 - 4*a*c]*(c*e - b*f)]*(2*a + (b + Sqrt[b^2 - 4*a*c])*x))/(Sqrt[b^2*
d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)
]*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)))^2]*EllipticF[2*ArcTan[((2*c^2*d - b*c*e
 + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[2*a + (b + S
qrt[b^2 - 4*a*c])*x])/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*d +
Sqrt[b^2 - 4*a*c]*e - 2*a*f))^(1/4)*Sqrt[b + Sqrt[b^2 - 4*a*c] + 2*c*x])],
(2 + ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f))/(Sqrt[b^2*d + b*(Sqrt[
b^2 - 4*a*c]*d - a*e) - a*(2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f)]*Sqrt[2*c^2
*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a*f)]))
/4])/((4*a*c - (b + Sqrt[b^2 - 4*a*c])^2)*(2*c^2*d - b*c*e + b^2*f - 2*a*c*
f - Sqrt[b^2 - 4*a*c]*(c*e - b*f))^(1/4)*Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x
 + f*x^2]*Sqrt[1 - ((b + Sqrt[b^2 - 4*a*c])*(2*c*d - b*e + 2*a*f)*(2*a + (b
 + Sqrt[b^2 - 4*a*c])*x))/((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*(2*c*
d + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)) + ((4*c^
2*d - 2*c*(b + Sqrt[b^2 - 4*a*c])*e + (b + Sqrt[b^2 - 4*a*c])^2*f)*(2*a + (
b + Sqrt[b^2 - 4*a*c])*x)^2)/(((b + Sqrt[b^2 - 4*a*c])^2*d - 2*a*(b + Sqrt[
b^2 - 4*a*c])*e + 4*a^2*f)*(b + Sqrt[b^2 - 4*a*c] + 2*c*x)^2)]))
```

Rule 992

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r +
2*c*x]*Sqrt[2*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*
c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 935

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2
*(a + b*x + c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sq
rt[a + b*x + c*x^2]), Subst[Int[1/Sqrt[1 - ((2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*x^2)/(c*f^2 - b*f*g + a*g^2) + ((c*d^2 - b*d*e + a*e^2)*x^4)/(c*f^2 -
b*f*g + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+bx+cx^2}\sqrt{d+ex+fx^2}}\,dx = \frac{\left(\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\right)\int \frac{1}{\sqrt{b+\sqrt{b^2-4ac}+2cx}\sqrt{2a+\left(b+\sqrt{b^2- \cdots}\right.}}}{\sqrt{a+bx+cx^2}}$$

$$= -\frac{\left(2\left(b+\sqrt{b^2-4ac}+2cx\right)^{3/2}\sqrt{2a+\left(b+\sqrt{b^2-4ac}\right)x}\sqrt{\frac{\left(4ac-\left(b+\sqrt{b^2- \cdots}\right.\right.}{\left(\left(b+\sqrt{b^2-4ac}\right)^2 d-2a\left(b+\sqrt{b^2 \cdots}\right.\right.}}\right.}{\cdots}$$

$$= -\frac{\sqrt[4]{b^2d+b\left(\sqrt{b^2-4ac}d-ae\right)-a\left(2cd+\sqrt{b^2-4ac}e-2af\right)}\left(b+\sqrt{b^2-4ac}+2 \cdots\right.}{\cdots}$$

**Mathematica [A]** time = 2.487, size = 670, normalized size = 0.47

$$-\frac{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(-\sqrt{e^2-4df}+e+2fx\right)\sqrt{-\frac{c\sqrt{b^2-4ac}\left(\sqrt{e^2-4df}+e+2fx\right)}{\left(\sqrt{b^2-4ac}-b-2cx\right)\left(f\left(\sqrt{b^2-4ac}+b\right)-c\left(\sqrt{e^2-4df}+e\right)\right)}}\sqrt{-\frac{c\left(\sqrt{b^2-4ac}\sqrt{e^2-4df}-e\left(\sqrt{b^2- \cdots}\right.\right.}{\left(\sqrt{b^2- \cdots}\right.}}}{\sqrt{a+x(b+cx)}\sqrt{d+x(e+fx)}\left(f\left(\sqrt{b^2-4ac \cdots}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[a + b*x + c*x^2]*Sqrt[d + e*x + f*x^2]),x]
```

```
[Out] -(((-b + Sqrt[b^2 - 4*a*c] - 2*c*x)*(e - Sqrt[e^2 - 4*d*f] + 2*f*x)*Sqrt[-(
(c*Sqrt[b^2 - 4*a*c]*(e + Sqrt[e^2 - 4*d*f] + 2*f*x))/(((b + Sqrt[b^2 - 4*a
*c])*f - c*(e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*Sqr
t[-((c*(4*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 - 4*d*f] - 2*Sqrt[b^2 - 4*a*c]*f
*x + 2*c*Sqrt[e^2 - 4*d*f]*x - e*(Sqrt[b^2 - 4*a*c] + 2*c*x) + b*(-e + Sqrt
[e^2 - 4*d*f] + 2*f*x)))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4
*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x)))]*EllipticF[ArcSin[Sqrt[(((-b + S
qrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(-e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt
[b^2 - 4*a*c] - 2*c*x))]], (2*c*d - b*e + 2*a*f - Sqrt[b^2 - 4*a*c]*Sqrt[e^
2 - 4*d*f])/(2*c*d - b*e + 2*a*f + Sqrt[b^2 - 4*a*c]*Sqrt[e^2 - 4*d*f])])/(
((-b + Sqrt[b^2 - 4*a*c])*f + c*(e - Sqrt[e^2 - 4*d*f]))*Sqrt[(c*Sqrt[b^2 -
 4*a*c]*(-e + Sqrt[e^2 - 4*d*f] - 2*f*x))/(((b + Sqrt[b^2 - 4*a*c])*f + c*(
-e + Sqrt[e^2 - 4*d*f]))*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))]*Sqrt[a + x*(b +
 c*x)]*Sqrt[d + x*(e + f*x)]))
```

---

**Maple [A]** time = 0.836, size = 929, normalized size = 0.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x)
```

```
[Out] 8*(2*b*f^2*x^2-2*c*e*f*x^2+2*x^2*c*f*(-4*d*f+e^2)^(1/2)+2*(-4*a*c+b^2)^(1/2
)*f^2*x^2+2*b*e*f*x+2*x*b*f*(-4*d*f+e^2)^(1/2)-8*c*x*f*d+2*(-4*a*c+b^2)^(1/
2)*e*f*x+2*x*f*(-4*a*c+b^2)^(1/2)*(-4*d*f+e^2)^(1/2)-2*b*d*f+b*e^2+b*e*(-4*
d*f+e^2)^(1/2)-2*d*e*c-2*c*d*(-4*d*f+e^2)^(1/2)-2*(-4*a*c+b^2)^(1/2)*d*f+(-
4*a*c+b^2)^(1/2)*e^2+e*(-4*a*c+b^2)^(1/2)*(-4*d*f+e^2)^(1/2))*EllipticF((-(
f*(-4*a*c+b^2)^(1/2)-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(-2*f*x+(-4*d*f+e^2)^(1/
2)-e)/(f*(-4*a*c+b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(2*f*x+(-4*d*f+e^
2)^(1/2)+e))^(1/2),((f*(-4*a*c+b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)-b*f+c*e)*(f*
(-4*a*c+b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(f*(-4*a*c+b^2)^(1/2)-c*(-
4*d*f+e^2)^(1/2)-b*f+c*e)/(f*(-4*a*c+b^2)^(1/2)-c*(-4*d*f+e^2)^(1/2)+b*f-c*
e))^(1/2))*((-4*d*f+e^2)^(1/2)*(b+2*c*x+(-4*a*c+b^2)^(1/2))*f/(f*(-4*a*c+b^
2)^(1/2)+c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^(1/2)*
((-4*d*f+e^2)^(1/2)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))*f/(f*(-4*a*c+b^2)^(1/2)-c
*(-4*d*f+e^2)^(1/2)-b*f+c*e)/(2*f*x+(-4*d*f+e^2)^(1/2)+e))^(1/2)*(-(f*(-4*a
*c+b^2)^(1/2)-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(-2*f*x+(-4*d*f+e^2)^(1/2)-e)/(
f*(-4*a*c+b^2)^(1/2)+c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(2*f*x+(-4*d*f+e^2)^(1/2
)+e))^(1/2)*(c*x^2+b*x+a)^(1/2)*(f*x^2+e*x+d)^(1/2)/(1/c/f*(-2*f*x+(-4*d*f+
e^2)^(1/2)-e)*(2*f*x+(-4*d*f+e^2)^(1/2)+e)*(-b-2*c*x+(-4*a*c+b^2)^(1/2))*(b
+2*c*x+(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(-4*a*c+b^2)^(1/2)-
c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(c*f*x^4+b*f*x^3+c*e*x^3+a*f*x^2+b*e*x^2+c*d*
x^2+a*e*x+b*d*x+a*d)^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + ex + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + ex + d}}{cfx^4 + (ce + bf)x^3 + (cd + be + af)x^2 + ad + (bd + ae)x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)/(c*f*x^4 + (c*e + b*f)
*x^3 + (c*d + b*e + a*f)*x^2 + a*d + (b*d + a*e)*x), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + bx + cx^2}\sqrt{d + ex + fx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{fx^2 + ex + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(f*x^2 + e*x + d)), x)
```

# 3.123 $\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx$

**Optimal**. Leaf size=652

$$\frac{\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(-\sqrt{23}+11i\right)\left(5x^2+3x+2\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\left(1-\frac{\sqrt{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\left(6-\left(1-i\sqrt{23}\right)x\right)}{-4x-i\sqrt{23}+1}\right)\sqrt{-\frac{11\left(-\sqrt{23}+3i\right)\cdots}{\left(\sqrt{23}+7i\right)\left(-\cdots\right)}\cdots}}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}\sqrt{-\frac{11\left(-\sqrt{23}+3i\cdots\right.}{\left(\sqrt{23}+7i\right)\left(\cdots\right.}\cdots}}$$

```
[Out] (Sqrt[23/11]*(1 - I*Sqrt[23] - 4*x)*Sqrt[-1 + I*Sqrt[23] + 4*x]*Sqrt[6 - (1
 - I*Sqrt[23])*x]*Sqrt[((11*I - Sqrt[23])*(2 + 3*x + 5*x^2))/((7*I + Sqrt[2
3])*(1 - I*Sqrt[23] - 4*x)^2)]*(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23
]))]*(6 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x))*Sqrt[(11 - (41*(I +
Sqrt[23])*(6 - (1 - I*Sqrt[23])*x))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x
)) - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1
- I*Sqrt[23] - 4*x)^2))/(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23]))]*(6
 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x))^2]*EllipticF[2*ArcTan[((-((
3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[6 - (1 - I*Sqrt[23])*x])/Sqrt
[-1 + I*Sqrt[23] + 4*x]], (44 - (41*(I + Sqrt[23]))/Sqrt[11 + I*Sqrt[23]])/
88])/((23 + I*Sqrt[23])*(-((3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[3
 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]*Sqrt[11 - (41*(I + Sqrt[23])*(6 - (1 -
I*Sqrt[23])*x))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)) - (11*(3*I - Sqrt
[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^
2)])
```

---

**Rubi [A]** time = 0.676932, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {992, 935, 1103}

$$\frac{\sqrt{\frac{23}{11}}\left(-4x-i\sqrt{23}+1\right)\sqrt{4x+i\sqrt{23}-1}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(-\sqrt{23}+11i\right)\left(5x^2+3x+2\right)}{\left(\sqrt{23}+7i\right)\left(-4x-i\sqrt{23}+1\right)^2}}\left(1-\frac{\sqrt{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\left(6-\left(1-i\sqrt{23}\right)x\right)}{-4x-i\sqrt{23}+1}\right)\sqrt{-\frac{11\left(-\sqrt{23}+3i\right)\cdots}{\left(\sqrt{23}+7i\right)\left(-\cdots\right)}\cdots}}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{-\sqrt{23}+3i}{\sqrt{23}+7i}}\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}\sqrt{-\frac{11\left(-\sqrt{23}+3i\right)\left(6-\left(1\cdots\right.\right.}{\left(\sqrt{23}+7i\right)\left(-4x-i\cdots\right.}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

```
[Out] (Sqrt[23/11]*(1 - I*Sqrt[23] - 4*x)*Sqrt[-1 + I*Sqrt[23] + 4*x]*Sqrt[6 - (1
 - I*Sqrt[23])*x]*Sqrt[((11*I - Sqrt[23])*(2 + 3*x + 5*x^2))/((7*I + Sqrt[2
3])*(1 - I*Sqrt[23] - 4*x)^2)]*(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23
]))]*(6 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x))*Sqrt[(11 - (41*(I +
Sqrt[23])*(6 - (1 - I*Sqrt[23])*x))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x
)) - (11*(3*I - Sqrt[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1
- I*Sqrt[23] - 4*x)^2))/(1 - (Sqrt[-((3*I - Sqrt[23])/(7*I + Sqrt[23]))]*(6
 - (1 - I*Sqrt[23])*x))/(1 - I*Sqrt[23] - 4*x))^2]*EllipticF[2*ArcTan[((-((
3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[6 - (1 - I*Sqrt[23])*x])/Sqrt
[-1 + I*Sqrt[23] + 4*x]], (44 - (41*(I + Sqrt[23]))/Sqrt[11 + I*Sqrt[23]])/
```

```
88])/((23 + I*Sqrt[23])*(-((3*I - Sqrt[23])/(7*I + Sqrt[23])))^(1/4)*Sqrt[3
 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]*Sqrt[11 - (41*(I + Sqrt[23])*(6 - (1 -
I*Sqrt[23])*x))/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)) - (11*(3*I - Sqrt
[23])*(6 - (1 - I*Sqrt[23])*x)^2)/((7*I + Sqrt[23])*(1 - I*Sqrt[23] - 4*x)^
2)])
```

Rule 992

```
Int[1/(Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]*Sqrt[(d_) + (e_.)*(x_) + (f_.
)*(x_)^2]), x_Symbol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(Sqrt[b + r +
2*c*x]*Sqrt[2*a + (b + r)*x])/Sqrt[a + b*x + c*x^2], Int[1/(Sqrt[b + r + 2*
c*x]*Sqrt[2*a + (b + r)*x]*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

Rule 935

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_.) + (b_.)
*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[(-2*(d + e*x)*Sqrt[((e*f - d*g)^2
*(a + b*x + c*x^2))/((c*f^2 - b*f*g + a*g^2)*(d + e*x)^2)])/((e*f - d*g)*Sq
rt[a + b*x + c*x^2]), Subst[Int[1/Sqrt[1 - ((2*c*d*f - b*e*f - b*d*g + 2*a*
e*g)*x^2)/(c*f^2 - b*f*g + a*g^2) + ((c*d^2 - b*d*e + a*e^2)*x^4)/(c*f^2 -
b*f*g + a*g^2)], x], x, Sqrt[f + g*x]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c,
 d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rubi steps

$$\int \frac{1}{\sqrt{3-x+2x^2}\sqrt{2+3x+5x^2}}\,dx = \frac{\left(\sqrt{-1+i\sqrt{23}+4x}\sqrt{6+\left(-1+i\sqrt{23}\right)x}\right)\int \frac{1}{\sqrt{-1+i\sqrt{23}+4x}\sqrt{6+\left(-1+i\sqrt{23}\right)x}\sqrt{2+3x+5x^2}}\,dx}{\sqrt{3-x+2x^2}}$$

$$= -\frac{\left(2\left(-1+i\sqrt{23}+4x\right)^{3/2}\sqrt{6+\left(-1+i\sqrt{23}\right)x}\sqrt{\frac{\left(24-\left(-1+i\sqrt{23}\right)^2\right)^2\left(2+3x+5x^2\right)}{\left(180-18\left(-1+i\sqrt{23}\right)+2\left(-1+i\sqrt{23}\right)^2\right)\left(-1+i\cdots\right.}}\right.}{\left(24-\left(-1+i\sqrt{23}\cdots\right.\right.}$$

$$= -\frac{\sqrt{\frac{23}{11}}\left(-1+i\sqrt{23}+4x\right)^{3/2}\sqrt{6-\left(1-i\sqrt{23}\right)x}\sqrt{\frac{\left(11i-\sqrt{23}\right)\left(2+3x+5x^2\right)}{\left(7i+\sqrt{23}\right)\left(1-i\sqrt{23}-4x\right)^2}}\left(1-\frac{\sqrt{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\cdots}{1-i\cdots}\right.}{\left(23+i\sqrt{23}\right)\sqrt[4]{-\frac{3i-\sqrt{23}}{7i+\sqrt{23}}}\sqrt{3-x+2x^2}\cdots}$$

**Mathematica [A]** time = 0.61975, size = 390, normalized size = 0.6

$$\frac{\left(-4x+i\sqrt{23}+1\right)\left(10ix+\sqrt{31}+3i\right)\sqrt{\frac{20ix-2\sqrt{31}+6i}{\left(11i+5\sqrt{23}-2\sqrt{31}\right)\left(4ix+\sqrt{23}-i\right)}}\sqrt{\frac{\left(-22-10i\sqrt{23}+4i\sqrt{31}\right)x-\sqrt{713}-i\sqrt{31}-3i\sqrt{23}+63}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(4ix+\sqrt{23}-i\right)}}\text{EllipticF}\left(\sin^{-1}\left(\cdots\right.\right.}{\left(-11i+5\sqrt{23}-2\sqrt{31}\right)\sqrt{\frac{10ix+\sqrt{31}+3i}{\left(11i+5\sqrt{23}+2\sqrt{31}\right)\left(4ix+\sqrt{23}-i\right)}}\sqrt{2x^2-x+3}\sqrt{\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(Sqrt[3 - x + 2*x^2]*Sqrt[2 + 3*x + 5*x^2]),x]
```

```
[Out] ((1 + I*Sqrt[23] - 4*x)*(3*I + Sqrt[31] + (10*I)*x)*Sqrt[(6*I - 2*Sqrt[31]
+ (20*I)*x)/((11*I + 5*Sqrt[23] - 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*S
qrt[(63 - (3*I)*Sqrt[23] - I*Sqrt[31] - Sqrt[713] + (-22 - (10*I)*Sqrt[23]
+ (4*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*
I)*x))]*EllipticF[ArcSin[Sqrt[2]*Sqrt[-((-63 + (3*I)*Sqrt[23] + I*Sqrt[31]
+ Sqrt[713] + 2*(11 + (5*I)*Sqrt[23] - (2*I)*Sqrt[31])*x)/((11*I + 5*Sqrt[2
3] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x)))]], (1197 + 41*Sqrt[713])/484])
/((-11*I + 5*Sqrt[23] - 2*Sqrt[31])*Sqrt[(3*I + Sqrt[31] + (10*I)*x)/((11*I
 + 5*Sqrt[23] + 2*Sqrt[31])*(-I + Sqrt[23] + (4*I)*x))]*Sqrt[3 - x + 2*x^2]
*Sqrt[2 + 3*x + 5*x^2])
```

---

**Maple [A]** time = 3.535, size = 420, normalized size = 0.6

$$\frac{\frac{4i}{23}\left(2i\sqrt{31}+5i\sqrt{23}-11\right)\left(i\sqrt{23}-4x+1\right)^2\sqrt{23}\sqrt{10}}{2i\sqrt{31}-5i\sqrt{23}-11}\sqrt{5x^2+3x+2}\sqrt{2x^2-x+3}\sqrt{-\frac{\left(2i\sqrt{31}-5i\sqrt{23}-11\right)\left(-1+4x+\cdots\right.}{\left(2i\sqrt{31}+5i\sqrt{23}-11\right)\left(i\sqrt{23}-4\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x)
```

```
[Out] 4/23*I*(5*x^2+3*x+2)^(1/2)*(2*x^2-x+3)^(1/2)*(2*I*31^(1/2)+5*I*23^(1/2)-11)
*(-(2*I*31^(1/2)-5*I*23^(1/2)-11)*(-1+4*x+I*23^(1/2))/(2*I*31^(1/2)+5*I*23^
(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2)*(I*23^(1/2)-4*x+1)^2*(I*23^(1/2)*(I*31^
(1/2)+10*x+3)/(2*I*31^(1/2)-5*I*23^(1/2)+11)/(I*23^(1/2)-4*x+1))^(1/2)*(I*2
3^(1/2)*(I*31^(1/2)-10*x-3)/(2*I*31^(1/2)+5*I*23^(1/2)-11)/(I*23^(1/2)-4*x+
1))^(1/2)*23^(1/2)*10^(1/2)*EllipticF((-(2*I*31^(1/2)-5*I*23^(1/2)-11)*(-1+
4*x+I*23^(1/2))/(2*I*31^(1/2)+5*I*23^(1/2)-11)/(I*23^(1/2)-4*x+1))^(1/2),((
2*I*31^(1/2)+5*I*23^(1/2)+11)*(2*I*31^(1/2)+5*I*23^(1/2)-11)/(2*I*31^(1/2)-
5*I*23^(1/2)+11)/(2*I*31^(1/2)-5*I*23^(1/2)-11))^(1/2))/(10*x^4+x^3+16*x^2+
7*x+6)^(1/2)/(2*I*31^(1/2)-5*I*23^(1/2)-11)/((-1+4*x+I*23^(1/2))*(I*23^(1/2
)-4*x+1)*(I*31^(1/2)+10*x+3)*(I*31^(1/2)-10*x-3))^(1/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{5x^2+3x+2}\sqrt{2x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{5\,x^2+3\,x+2}\sqrt{2\,x^2-x+3}}{10\,x^4+x^3+16\,x^2+7\,x+6},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)/(10*x^4 + x^3 + 16*x^2 +
 7*x + 6), x)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{2x^2-x+3}\sqrt{5x^2+3x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x**2+3*x+2)**(1/2)/(2*x**2-x+3)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(2*x**2 - x + 3)*sqrt(5*x**2 + 3*x + 2)), x)
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{5\,x^2+3\,x+2}\sqrt{2\,x^2-x+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5*x^2+3*x+2)^(1/2)/(2*x^2-x+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(5*x^2 + 3*x + 2)*sqrt(2*x^2 - x + 3)), x)
```

# Chapter 4

# Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

### 4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```